# Relativistic Plasma Klystron Amplifier in Connection with Application to High Gradient Accelerators

Han S. Uhm
Naval Surface Warfare Center
10901 New Hampshire Ave, White Oak
Silver Spring, Maryland 20903-5640

Abstract

A novel plasma klystron amplifier is presented in which current modulation of a relativistic electron beam is achieved by the unstable two-stream interaction between the electron beam and a plasma column. A model to describe current modulation is developed. Due to a relatively large growth rate of the instability, required interaction length of the beam and plasma column is short for most applications. A full current modulation can be achieved in the plasma klystron amplifiers.

## I. INTRODUCTION

The generation of intense coherent radiation from the relativistic klystron amplifiers[1-5] (RKA) is one of the renewed interests in the microwave community, because of high-power capability and high efficiency of RKA. High-power relativistic klystron amplifiers have various applications, including the high-gradient RF accelerators and high-power radar. One of the main issues in RKA is enhancement of power and frequency simultaneously. Cavity size and opening should be reduced to increase the excitation frequency. Therefore, a high-power high-frequency klystron amplifier has inherent problems due to reduced size of cavities, including electron emission at the gap opening and shorting, etc. In addition, high-power RF signal is needed in the input cavity for high-power klystrons. In order to eliminate some of these intrinsic problems in the high-power RKA, I present a novel plasma klystron amplifier in which current modulation of the electron beam is accomplished by the two-stream interaction between the beam and plasma column. Because of a large growth rate of the two-stream instability, interaction length of the plasma klystron is short.

As illustrated in Fig. 1, a relativistic annular electron beam with radius of $R_b$ enters the drift tube with radius of $R_c$. In the plasma klystron, there are no intermediate passive cavities, which often present problems associated with self-oscillation and RF breakdown. The electron beam is pre-modulated at the first cavity before it enters a space between the drift tube wall and a plasma column with radius of $R_p$. An axial electric field accompanied by the modulated electron beam excites space charge waves in the plasma column, which acts like a inductive medium amplifying the electrostatic waves. The physical mechanism of self-amplification of the electrostatic waves is the longitudinal two-stream instability of the beam-plasma system.

This work was supported in part by the Independent Research Fund at the Naval Surface Warfare Center and in part by SDIO/IST.

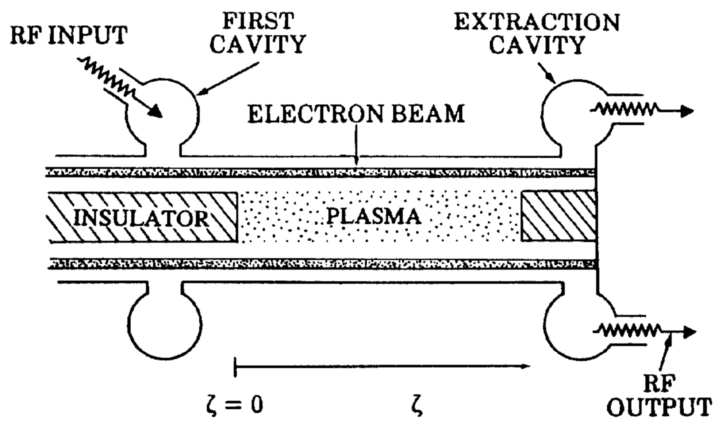


Fig. 1. A Schematic presentation of plasma klystron amplifier.

## II. STABILITY ANALYSIS

Analysis of the two-stream instability of the beam-plasma system is carried out within the framework of the linearized hybrid-Maxwell equations, where beam electrons are described by the Vlasov equation and plasma electrons are described by a cold-fluid model. In the stability analysis, we adopt a normal mode approach in which all perturbations are assumed to vary according to $\delta\Phi(\mathbf{x},t) = \Phi(r)\exp[i(kz - \omega t)]$, where $\omega$ is the eigenfrequency and k is the axial wavenumber. The eigenfunction of the longitudinal two-stream instability is the axial electric field $E_z(r)$.

To solve the eigenvalue equation, we assume that the plasma density is uniform inside and drops abruptly at $r = R_p$. For the present purpose, the stability analysis is restricted to the long-wavelength low-frequency perturbations. Under these conditions, the eigenvalue equation is obtained[6,7] and solved, and the dispersion relation of the two-stream instability is expressed as

$$1 - \frac{\epsilon_b(k^2c^2 - \omega^2)}{(\omega - k\beta_b c)^2} + \frac{\epsilon_p(k^2c^2 - \omega^2)}{\omega^2} - \frac{g\epsilon_b\epsilon_p(k^2c^2 - \omega^2)^2}{\omega^2(\omega - k\beta_b c)^2}, \tag{1}$$

where the parameters $\epsilon_b$ and $\epsilon_p$ are the beam and plasma dielectric effects, and the geometrical factor g is defined by $g = \ln(R_b/R_p)/\ln(R_c/R_p)$. Equation (1) can be used to investigate properties of the longitudinal two-stream instability in an annular electron beam propagating through a plasma column.

Stability analysis of the dispersion relation in Eq. (1) is carried out in a special case when the beam contacts the plasma, i.e., $g = 0$. Then, the last term in Eq. (5) is neglected and the dispersion relation is significantly simplified. Defining the phase velocity of the unstable wave by $\omega/k = \beta_p c$, the instability condition is given by[8]

$$(\epsilon_b^{1/3} + \epsilon_p^{1/3})^3 > \gamma_p^2\beta_b^2, \tag{2}$$

where $\gamma_p^2 = (1-\beta_p^2)^{-1}$. The electron beam velocity in the plasma klystron amplifier is close to the speed of light ($\beta_b \approx 1$) for high power. Instability occurs only when the parameter $\epsilon_p$ is in the order of unity or larger, as we have seen in Eq. (2). In this context, we conclude that the plasma klystron amplifier requires an opaque plasma. The parameter $\epsilon_b$ is much less than unity for most applications, satisfying $\epsilon_b << \epsilon_p$. In this case, the phase velocity of the unstable wave is close to the beam velocity and the dispersion relation in Eq. (1) is further simplified.[8] Defining Doppler-shifted frequency $x = (\omega - k\beta_b c)/kc$, it is shown that the maximum growth rate occurs at the phase velocity satisfying $\beta_p/\beta_b = (\epsilon_p/\epsilon_b)^{1/3}/[1+(\epsilon_p/\epsilon_b)^{1/3}]$ and is given by[8]

$$x_i = Im(x) = \frac{\sqrt{3}}{2\gamma_b}\left(\frac{\epsilon_b\sqrt{\epsilon_p}}{2}\right)^{1/3}. \tag{3}$$

For the present purpose, we consider an example where an annular electron beam with radius of $R_b = 2$ cm and energy of $\gamma_b = 1.67$ enters a drift tube with a radius of $R_c = 2.5$ cm. The drift tube contains a plasma column whose radius is $R_p = 2$ cm and density is $n_p = 6.5 \times 10^{11}$ cm$^{-3}$. For these plasma conditions, $\epsilon_p$ is calculated to be $\epsilon_p = 0.7$. The electron beam current is assumed to be 5.5 kA, which corresponds to $\epsilon_b = 0.037$ for the beam parameters mentioned above. The normalized electron beam velocity is $\beta_b = 0.8$. Note that these beam and plasma parameters satisfy the instability criterion in Eq. (2). Therefore, the normalized growth rate $x_i$ is estimated to be $x_i = 0.17$ obtained from Eq. (3). The plasma column needed for the plasma klystron amplifiers can be produced by thermionic arc discharges[9,10]. These discharge devices are frequently operated at the room-temperature vapor pressure of mercury, which provides plasma densities in the range from $n_p = 10^9$ cm$^{-3}$ to $n_p = 10^{13}$ cm$^{-3}$ for 10-to-30 cm long discharge tubes.

## III. NONLINEAR MODE EVOLUTION

We now describe the current modulation of an electron beam propagating through the space between the plasma column and drift tube wall. An axial electric field accompanied by the pre-modulated electron beam enters this region and initiates excitation of the space charge waves. Strength of this axial electric field is determined in terms of the system parameters prior to this region.[5] The plasma column responds to the initial perturbations of the axial electric field and amplifies the field strength by acting as an inductive medium. The physical mechanism of the field amplification is the two-stream instability mentioned above. We assume the initial condition that a beam segment labeled by $t_0$ enters the region at time $t = t_0$. Then, the axial electric field $E_z(z,t_0)$ acting on the beam segment $t_0$ is expressed as

$$E_z(\zeta,\theta) = -E_0 \exp(x_i\zeta/\beta_b)\sin\theta, \tag{4}$$

where $E_0$ is the initial axial electric field, $\zeta = \omega z/\beta_b c$ is the normalized propagation distance, $\theta = \omega t_0$ is the normalized time and $x_i$ is the normalized growth rate of the instability obtained in Eq. (3).

Energy modulation of the beam segment labeled by $\theta$ is obtained from $d\gamma/dz = -eE_z/mc^2$. Without loss of generality, we assume that the axial coordinate z equals zero at the beginning of the plasma column. Integrating along the propagation distance and neglecting the initial modulation, we obtain the relativistic mass ratio

$$\gamma(\zeta,\theta) = \gamma_b + \epsilon_0 \exp(x_i\zeta/\beta_b)\sin\theta, \tag{5}$$

of the segment $\theta$ at the propagation distance $\zeta$. In Eq. (5), $\epsilon_0 = \beta_b^2 eE_0/mc\omega x_i << 1$. The instantaneous velocity $\beta(\zeta,\theta)c$ of the beam segment $\theta$ is expressed as

$$\frac{\beta_b}{\beta} = 1 + \frac{\gamma_b - \gamma}{\gamma_b(\gamma_b^2 - 1)}, \tag{6}$$

where $\beta_b c$ is the beam velocity at the injection point. Making use of the velocity definition $dz/dt = \beta c$ and the definition $\varphi = \omega t$, we obtain

$$\varphi = \theta + \zeta - \epsilon \exp\left(\frac{x_i}{\beta_b}\zeta\right)\sin\theta, \tag{7}$$

where use has been made of the fact that the parameters $\epsilon = \epsilon_0\beta_b/\gamma_b(\gamma_b^2-1)x_i << 1$. Equation (7) relates the present time t to the initial time $t_0$. Differentiating $\varphi$ in Eq. (7) with respect to $\theta$ gives

$$\frac{d\varphi}{d\theta} = 1 - \epsilon\exp(\frac{x_i}{\beta_b}\zeta)\cos\theta. \tag{8}$$

The beam current at the injection point is a constant value of $I_b$. The beam segment $t_0$ passes the injection point at time $t = t_0$. When this segment arrives at z in time t, it is stretched by a factor of $dt/dt_0$. Thus, the beam current of the segment $t_0$ at z is proportional to $d\theta/d\varphi$. In this regard, the normalized current ratio $F(\zeta,\theta)$ is expressed as

$$F(\zeta,\theta) = \frac{I(\zeta,\theta)}{I_b} = \frac{N(\zeta)}{|d\varphi/d\theta|}, \tag{9}$$

where the normalization constant $N(\zeta)$ is defined by

$$\frac{2\pi}{N(\zeta)} = \int_0^{2\pi}|d\theta/d\varphi|d\theta. \tag{10}$$

The normalization constant $N(\zeta)$ ensures the charge conservation.

Substituting Eq. (8) into Eqs. (9) and (10) gives the current modulation

$$F(\zeta,\theta) = \begin{cases} \sqrt{1-f^2}/(1-f\cos\theta), & \zeta < \zeta_m, \\ N(\zeta)/|1-f\cos\theta|, & \zeta > \zeta_m, \end{cases} \tag{11}$$

where the exponential function $f(\zeta)$ is defined by

$$f(\zeta) = \epsilon\exp(x_i\zeta/\beta_b), \tag{12}$$

the normalization constant $N(\zeta)$ is calculated from Eq. (10) and the propagation distance $\zeta_m$ for maximum current modulation is obtained from $f(\zeta_m) = 1$. It is obvious from Eq. (11) that the modulated beam current has one peak per period until the beam segment $\theta$ reaches $\zeta = \zeta_m$. If the beam propagates further from $\zeta = \zeta_m$, it starts to bunch two peaks per period, thereby providing a possibility of high harmonic modulation.

The propagation distance $z_m$ of the maximum current modulation is expressed as

$$z_m = \frac{\beta_b c}{\omega}\zeta_m = \frac{\beta_b^2 c}{\omega x_i}\ln(\frac{1}{\epsilon}), \tag{13}$$

which has a significant meaning because it determines the length of the klystron amplifier. Remember that the parameter $1/\epsilon$ in Eq. (13) is much larger than unity in general. Several points are noteworthy from Eq. (13). First, the microwave tube length ($z_m$) is inversely proportional to the normalized growth rate ($x_i$) of the two-stream instability. We have observed in the stability analysis that the growth rate $x_i$ is a slowly changing function of the plasma density, but it is sensitive to the beam current. The growth rate can be significantly increased and the tube length can be considerably shortened, by increasing the beam current $I_b$ or the ratio of $R_c/R_b$. Second, the tube length is proportional to $\ln(1/\epsilon)$, indicating that it is a weakly dependent function of the initial energy modulation at the first cavity. In this regard, a relatively low-power microwave input in the first cavity may well excite the two-stream instability in the beam-plasma region and may deliver a highly modulated electron beam to the extraction cavity. Third, the length of the microwave tube is inversely proportional to the frequency. The higher the frequency, the shorter the tube length. Equation (13) also indicates that the tube length can be drastically reduced by decreasing the beam energy. As an example, we consider a system where $\epsilon = 0.01$, $x_i = 0.13$, $\beta_b = 0.8$, and the microwave frequency is 3.5 GHz. Substituting these parameters into Eq. (13), we find the tube length $z_m = 30$ cm. These parameters are easily attainable in the present experimental conditions. There is a broad range of system parameters, which the present technology allows. Obviously, the plasma klystron amplifier has a great potential for high-power high-frequency microwave device. There will be many different plasma configurations, which may be more advantageous for the plasma klystron amplifiers. For example, an electron beam can propagate through a plasma cylinder, providing a large plasma volume.

## IV. REFERENCES


[1] M. Friedman, J. Krall, Y. Y. Lau, and V. Serlin, J. Appl. Phys. **64**, 3353 (1988).

[2] Y. Y. Lau, M. Friedman, J. Krall, and V. Serlin, IEEE Trans. on Plasma Science, **18**, 553 (1990).

[3] M. Friedman, V. Serlin, A. Drobot, and A. Mondelli, IEEE Trans. on Plasma Science, **14**, 201 (1986).

[4] M. Friedman, J. Krall, Y. Y. Lau, and V. Serlin, Rev. Sci. Instrum., **61**, 171(1990).

[5] H. S. Uhm, Phys. Fluids B **5**, 190 (1993).

[6] H. S. Uhm and R. C. Davidson, Phys. Fluids **24**, 1541 (1981).

[7] H. S. Uhm and G. Joyce, Phys. Fluids B **3**, 1587 (1991).

[8] H. S. Uhm, J. Appl. Phys. **56**, 2041 (1984).

[9] N. A. Krall and A. W. Trivelpiece, **Principles of Plasma Physics**, (McGraw-Hill, New York, 1973) Chap. 1.

[10] H. S. Uhm, J. D. Miller, R. F. Schneider, and D. J. Weidman, IEEE Trans. on Plasma Science **19**, 535 (1991).