Optimum Operation of Gyrotwistrons

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Abstract

Relativistic gyrotwistrons, which can, in principle, achieve efficiencies in excess of 50%, are promising sources for driving particle accelerators. The optimum operating point for these devices represents a tradeoff between maximizing the available energy and minimizing the deleterious effects of velocity spread. The former increases with Doppler upshift and pitch ratio, while the effects of velocity spread are made worse by both of these quantities. We discuss the issues that go into deciding how to achieve maximum efficiency, with particular attention paid to the role of beam current.

I. INTRODUCTION

One possible source for driving particle accelerators is the relativistic gyrotwistron,[1] in which the output cavity of a standard gyroklystron amplifier is replaced by a traveling wave section. There are two issues involved in designing a gyrotwistron: stability and efficiency. In another paper ("Stability of Gyrotwistrons," by P.E. Latham, G.S. Nusinovich, and J. Cheng, these proceedings), we consider issues of stability; here we ask the question: what parameters yield the optimum efficiency? In addressing this question, we make the following assumptions:

1. The electron beam voltage and current are fixed.
2. The electron beam has a perpendicular velocity spread, $\Delta\beta_{\perp 0}/\beta_{\perp 0}$, that is independent of magnetic field and pitch ratio α ($\alpha \equiv \beta_\perp/\beta_z$).
3. The axial wave number, k_z, is chosen so that the beam and wave are initially in resonance; i.e. at the beginning of the output waveguide,

$$\omega - s\Omega_c - k_z\bar{v}_z = 0$$

where ω is the operating frequency, s is the harmonic, Ω_c is the initial cyclotron frequency and $\bar{v}_z$ is the initial average velocity.

The first and third assumptions are fairly reasonable; the second needs some explanation. This assumption stems from the observation that the perpendicular velocity spread is an adiabatic invariant. Thus, if a given value of $\Delta\beta_{\perp 0}/\beta_{\perp 0}$ can be achieved at some value of the pitch ratio, it can be achieved at any other value of α by adiabatically varying the magnetic field. Since an electron gun can be constructed to produce a beam over an extremely broad range of magnetic fields, it follows that the perpendicular velocity spread is independent of both magnetic field and pitch ratio. Of course, in a particular electron gun this will be true only over a range of magnetic fields.

The task now is, in principle, largely numerical: simply optimize efficiency with respect to initial pitch ratio and magnetic field, with the axial wave number chosen to satisfy the resonance condition given above. This is, however, a big job; especially since the efficiency needs to be optimized with respect to the magnetic field **profile**. To narrow down the parameter range, in this paper we estimate, using simple physical arguments, the parameters which are likely to yield high efficiency. This will give us a good starting point for a numerical investigation, which will be carried out in future work. In the next section we discuss the issues relevant to high efficiency gyrotwistrons, culminating with a single graph (at each harmonic) which displays our results. Section III contains our summary and conclusions.

II. DISCUSSION AND RESULTS

As mentioned, the optimum operating point for a gyrotwistron represents a tradeoff between maximizing the available energy and minimizing the effects of velocity spread. The available energy is given by the maximum single particle efficiency,[2]

$$\eta_{sp} = \frac{\gamma_0 + 1}{2\gamma_0}\frac{\alpha^2}{1+\alpha^2}\frac{\omega}{s\Omega_c}\ .$$

where γ_0 is the initial relativistic factor. The single particle efficiency, which is easily derived by noting that the Hamiltonian depends on a single phase

0-7803-1203-1/93$03.00 © 1993 IEEE

and thus has two conserved quantities, has a simple physical interpretation: the available perpendicular energy increases with pitch ratio, and the available axial energy increases with increasing Doppler upshift (the $\omega/s\Omega_c$ term).

With no velocity spread, typically one is able to achieve efficiencies on the order of 70% of the single particle efficiency. However, with velocity spread the picture changes dramatically. A good measure of the effect of velocity spread is the spread in phase which it induces. To estimate this, we write down the zeroth order equation[1] for the slow gyrophase ψ:

$$\frac{d\psi}{d\xi} = \frac{1 - s\Omega_c/\omega - k_z v_z/\omega}{\sqrt{I}\beta_z}$$

where I is the normalized current and $\xi = \sqrt{I}(\omega/c)z$ is the normalized length with c the speed of light. The normalized current for the TE_{mn} mode is

$$I \equiv \hat{I}\,[\bar{\beta}_\perp k_\perp c/s\Omega_c]^{2(s-1)}\ \frac{(k_\perp c/\omega)^2}{k_z c/\omega}\frac{1-k_z\bar{v}_z/\omega}{\bar{\beta}_z^2}$$

$$\hat{I} \equiv \frac{4eI_b}{m_e\gamma_0 c^3}\left[\frac{s^{(s-1)}}{(s-1)!2^s}\right]^2 \frac{J^2_{m\pm s}(k_\perp r_g)}{(\nu_{mn}^2-m^2)J_m^2(\nu_{mn})}$$

where I_b is the beam current, e and m_e are the electron charge and mass, respectively, $k_\perp$ is the perpendicular wave number, r_g is the guiding center radius, ν_{mn} is the nth root of the derivative of the Bessel function: $J'_m(\nu_{mn}) = 0$. In these units the current appears only in the zeroth order equation of the phase, so we can estimate the effects of the current by looking exclusively at this term. Bars denote an average over the initial velocity distribution.

The phase spread induced by the velocity spread, which we denote Δ_ψ, is defined to be

$$\Delta_\psi \equiv \Delta\beta_z \frac{\partial}{\partial\beta_z}\left(\frac{d\psi}{d\xi}\right)_{\beta_z=\bar{\beta}_z} .$$

A straightforward calculation yields

$$\Delta_\psi = \left[\frac{k_z c/\omega}{(1-k_z\bar{v}_z/\omega)}\right]^{1/2} \frac{(s\Omega_c/\omega)^{s-1}(1-s\Omega_c/\omega)}{\bar{\beta}_\perp^{(s-1)}(k_\perp c\ \omega)^s}\frac{\bar{\alpha}^2}{\sqrt{\hat{I}}}\frac{\Delta\beta_{\perp 0}}{\beta_{\perp 0}}$$

where we used $\Delta\beta_z/\bar{\beta}_z = \bar{\alpha}^2\Delta\beta_{\perp 0}/\beta_{\perp 0}$ (recall that $\Delta\beta_{\perp 0}/\beta_{\perp 0}$ is independent of both pitch ratio and magnetic field).

For large Doppler upshift, i.e. k_z near one, the most important scaling in this expression is the term $(k_\perp c/\omega)^{-s}$, which indicates a rapid increase in Δ_ψ as k_z increases. This is consistent with the well known fact that large Doppler upshifted devices are sensitive to velocity spread. However, the sensitivity comes not so much from the $k_z v_z$ term in the resonance condition, but from the weak coupling that occurs when the perpendicular wavenumber becomes small. Note that a slow wave device would have significantly different scaling, as the coupling would not vanish at large k_z. For small k_z, the dominant terms are $\sqrt{k_z c/\omega}$ and $(1 - s\Omega_c/\omega)$. The former arizes because the normalized current is inversely proportional to k_z, so the coupling increases as the axial wavenumber decreases. The latter term, which is proportional to $k_z v_z$ because of the resonance condition, reflects the fact that devices operating near cutoff are insensitive to velocity spread.

The other dependences in this expression are fairly easy to understand: Δ_ψ increases as the beam current decreases and velocity spread increases. The dependence on the pitch ratio is slightly more complicated: because of the $\bar{\beta}_\perp^{(1-s)}$ term, Δ_ψ decreases as α decreases only up to the second harmonic. At the third harmonic Δ_ψ depends weakly on the pitch ratio, and at the fourth harmonic and above Δ_ψ actually increases as $\bar{\alpha}$ decreases. Thus, harmonics above two are not good candidates for operation at small pitch ratio.

In Fig. 1 we plot level curves of both the single particle efficiency and Δ_ψ for the first, second and third harmonics (Figs. 1a, 1b and 1c, respectively). The value $\Delta_\psi = 0.30$ corresponds roughly to the parameters of the University of Maryland experiment[3] (voltage near 425 kV, current near 160 A, pitch ratio near 1, and perpendicular velocity spread near 6%.) At these parameters, numerical simulations indicate that we can achieve gyrotwistron efficiencies on the order of 35%.

The salient feature of these plots is that at a fixed pitch ratio, Δ_ψ increases rapidly as we move into the regime of high single particle efficiency; adjacent level curves of Δ_ψ increase by a factor of four. For instance, to move from the level curve $\Delta_\psi = 0.30$ to $\Delta_\psi = 1.20$ and still extract a large fraction of the available energy, either the velocity spread would have to decrease by a factor of 4 or the current would have to increase by a factor of 16. Both of these are extremely difficult technologically. Thus, since the maximum pitch ratio is generally limited by instabilities and reflected particles, there are severe constraints on the achievable

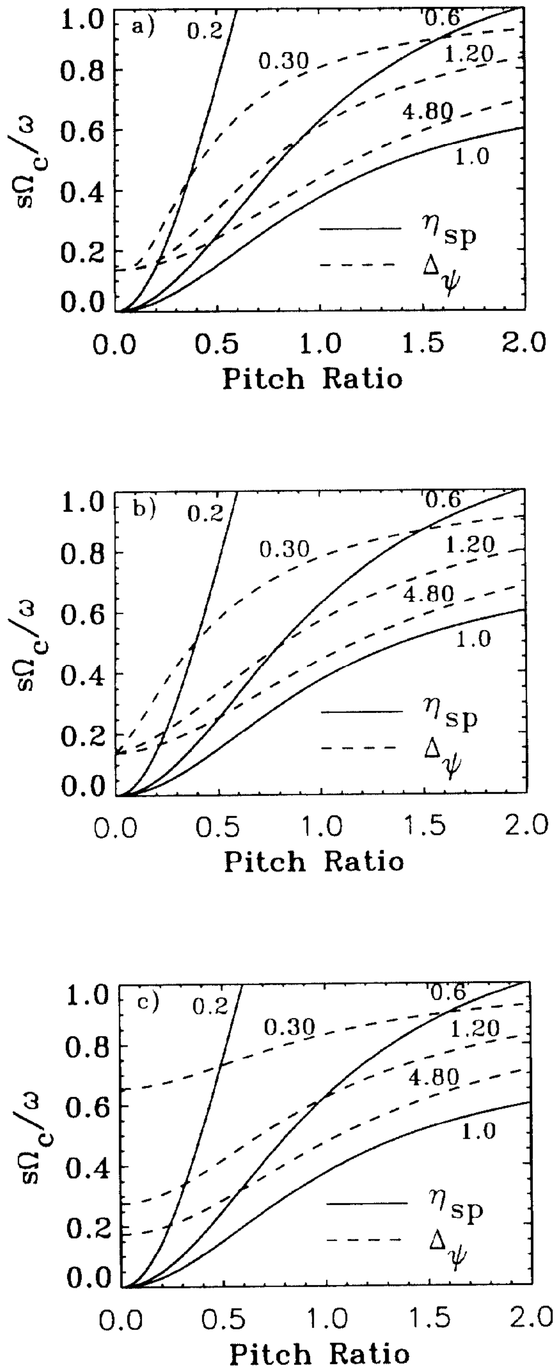

Figure 1: Level curves of single particle efficiency (solid lines) and Δ_ψ (dashed lines). a) TE_{01}, fundamental operation; b) TE_{11}, second harmonic; c) TE_{21}, third harmonic.

efficiency in a realistic gyrotwistron.

III. SUMMARY AND CONCLUSIONS

We have examined, using simple physical arguments, the parameters that lead to optimum operation of a gyrotwistron. The two key concepts in this analysis were the single particle efficiency, which is a measure of the available beam energy, and the phase spread induced on the beam by the velocity spread. We found that large beam current could offset the effects of velocity spread and thus increase efficiency. Thus, high current devices will, in general, yield large efficiencies than those with low current (assuming that the other parameters are fixed).

This work was supported by the U.S. Department of Energy.

IV. REFERENCES

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