NUMERICAL SIMULATIONS OF DRIVING BEAM DYNAMICS IN THE PLASMA WAKEFIELD ACCELERATOR*

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I. INTRODUCTION

Novel plasma based acceleration devices[1,2] have become the subject of active research because of their ability to support acceleration gradients in excess of 10 GeV/m. The plasma wakefield accelerator (PWFA) is one such device which consists of an intense electron beam (the primary beam) whose purpose is to excite a plasma wave which, in turn, accelerates a trailing electron bunch (the secondary beam). Two issues of current interest in the PWFA are 1) the equilibrium and stability of the driving beam and 2) the effect of the wakefield on the quality of the trailing electron bunch. The PWFA is currently the subject of experiments to be performed at the Argonne National Laboratory (ANL)[3] and the University of California, Los Angeles (UCLA)[4].

In the UCLA experiment, a question of particular interest is the equilibrium state of the driving electron beam. Two intriguing suggestions have been made. The first is that in the limit that the beam density greatly exceeds the plasma density, the plasma electrons will be completely expelled from the axis. It was recently pointed out[5] that the resulting ion channel will have a focusing force that varies linearly with radius, preserving the emittance of the trailing electron bunch. The second is that, in parameter regimes of interest, the driving beam will experience a severe radial pinching force. Specifically, Ref. 4 suggested that the electron beam could be pinched via ion focusing in the plasma by as much as a factor of 10 (from r_b=300μm to r_b=30μm). What such extreme pinching does to the beam and whether or not an equilibrium state at r_b=30 μm an be achieved are questions that have yet to be addressed. These ideas suggest that a highly nonlinear wakefield with favorable focusing properties could be produced by a driving electron beam in a tightly focused equilibrium state. Furthermore, this mode of operation may be accessible in new PWFA experiments to be performed at ANL and UCLA. In order to investigate these assertions, we first consider the envelope equation for an electron beam propagating in a plasma with $n_b \geq n_p$. We then compare numerical solutions of this equation to results obtained via two-dimensional axisymmetric (r,z) particle simulation using the FRIEZR particle simulation code.

II. THE ENVELOPE EQUATION

Consider the envelope equation of Lee and Cooper[6] for an axisymmetric relativistic electron beam:

$$\beta^2\frac{d^2R}{dz^2}+\frac{\beta^2}{\gamma}\frac{d\gamma}{dz}\frac{dR}{dz}+\frac{U}{R}-\frac{4\epsilon^2}{R^3}=0, \tag{1}$$

where R is the rms radius of the beam, γ is the relativistic factor, and the beam is assumed to be moving in the z-direction with $v_z \gg v_\perp$. We have used as the definition for the rms emittance, $\varepsilon=\varepsilon_{n,rms}/(\beta\gamma)$.

Each term of Eq. (1) has a physical meaning. The first two terms above account for inertial effects. The third term accounts for self-field effects,

$$U=\frac{1}{c^2}\int_0^\infty rdr[\frac{2\pi rJ_b(r)}{I_b}]\frac{e}{\gamma m}[E_r-\beta B_\theta], \tag{2}$$

where J_b is the beam current density, I_b is the beam current, and E_r and B_θ are electric and magnetic fields. The fourth term represents the expansion of the beam due to emittance. In applying Eq. (1) to this problem, we have assumed that $r_w > c/\omega_p > R$, where , $\omega_p=(4\pi n_o e^2/m)^{1/2}$, n_o is the initial plasma density, and r_w is the radius of the plasma chamber. This condition ensures that return currents do not flow within the electron beam. If we make the additional assumption that $n_b \geq n_o$, the plasma electrons will be expelled from the region of the beam.

For a beam with a Gaussian radial profile, we have

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$$U=-\frac{|I_b[kA]|}{17\beta\gamma^3}+\frac{\omega_p^2}{2\gamma c^2}R^2, \quad (3)$$

where our notation is such that $I_b < 0$ for an electron beam. Assuming $d\gamma/dz=0$, Eq. (1) becomes

$$\frac{d^2R}{dz^2}+\frac{1}{R}\frac{eI_b}{\beta\gamma^3mc^3}+\frac{\omega_p^2}{2\gamma c^2}R-\frac{4\epsilon^2}{R^3}=0. \quad (4)$$

With Eq. (4), it is straightforward to solve for the equilibrium radius of a beam with given γ, n_o and ε. For γ>>1 we have

$$R_{eq}=(8\frac{\epsilon^2_{n,rms}}{\gamma}\frac{c^2}{\omega_p^2})^{1/4} \quad (5)$$

As an example, we consider an electron beam with γ=40 (20 MeV), charge Q=1nC, initial rms radius σ_r=100 μm, longitudinal half-width at half maximum σ_z=600μm, and ε_n=10mm-mrad propagating through a plasma of density $n_o=2.0\times10^{14}$ cm^{-3}. In this case, $n_{b,0}=1.67\times10^{14}$ cm^{-3} and $-I_b$= 0.252 kA. With these values, Eq. (5) gives R_{eq}=41 μm indicating that the beam is mismatched. Once the beam begins to pinch, we will have $n_b > n_o$. Thus, we can integrate Eq. (4) to obtain an estimate of the beam dynamics, shown in Fig. 1. This figure shows severe pinching of the beam, with a minimum radius of 17 μm. One might expect significantly strong variations in the wakefield generated by such a beam.

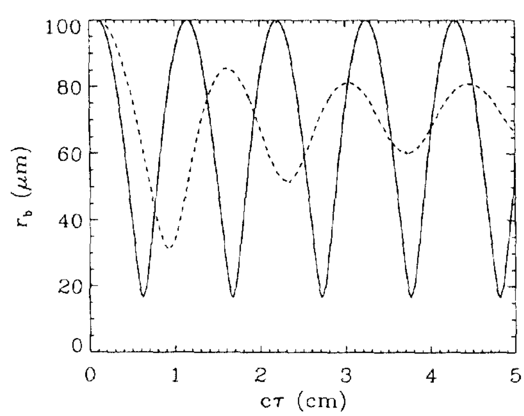

Fig 1. Beam radius R versus time τ from a numerical solution of Eq. (4) (solid line) and from simulation at fixed ζ=0.1 cm (dashed line).

III. SIMULATIONS

To address this question in greater detail, we resort to particle simulation. The FRIEZR particle simulation code is an axisymmetric (r,z) fully electromagnetic, fully relativistic particle code that makes use of "speed of light" coordinates through a change of variables: ζ=ct-z, τ=t. The plasma is represented by particle electrons imbedded in an immobile neutralizing ion background. The relativistic beam, also represented by particles, is injected into the plasma with density

$$n_b(r,\zeta)=n_{bo}\exp(-\frac{r^2}{\sigma_r^2})\sin(\frac{\pi\zeta}{3\sigma_z}). \quad (6)$$

In the (ζ,τ) coordinates, the head of the relativistic beam remains near ζ=0 with the tail of the beam at ζ=0.18 cm. The simulation proceeds for 5 cm, during which the beam energy remains approximately constant (γ_{final}=36). Figure 1 (dashed line) shows the beam radius versus τ at ζ=0.1 cm. It is evident that the beam is being pinched and is undergoing mismatch oscillations. Figure 2 shows the beam density n_b, the plasma density n_p, and the axial electric field E_z on axis at ct=5.0 cm. In this figure, $\delta n=n_p-n_o$, $E_N=0.1E_{wb}$, $E_{wb}=mc\omega_p/e$=1.4 GV/m. Also, the beam is moving towards the left and E_z>0 is accelerating. The envelope equation solution and the particle simulation result are in rough qualitative agreement at early times (τ<1cm). The results then diverge because the plasma electrons are not completely expelled from the axis (see Fig. 2) and because the beam emittance grows as the radial oscillations damp via phase-mixing. This can be seen in Fig. 3, which shows beam emittance at ζ=0.1 cm plotted versus τ. An interesting and somewhat surprising result is contained in this figure, which also shows the peak accelerating electric field $E_{z,peak}$ versus τ. While the beam radius is varying by 50%, the accelerating field is varying by only 20%. We speculate that this occurs for two reasons. Firstly, because the plasma is not entirely expelled from the axis and because the plasma electron density varies over the length of the beam, the simulation shows that the frequency of the radial mismatch oscillations varies as a function of ζ. Secondly, in the limit that $n_b >> n_p$, the wake is driven by a large radial electric field that pushes the plasma electrons to r >> R. At such large radii, E_r is function only of the amount of beam charge enclosed within this radius. Additional simulations have confirmed that the wake amplitude is insensitive to the details of the beam profile in this limit.

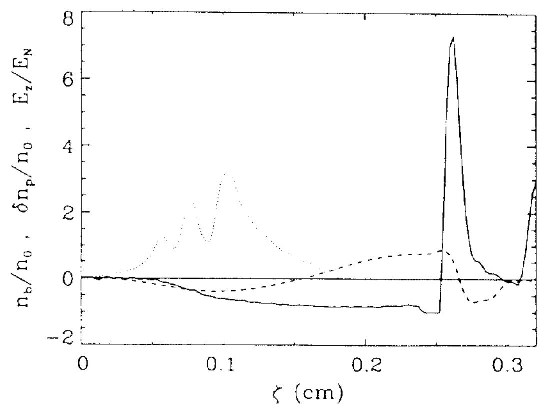

Fig 2. Beam density n_b/n_o (dotted line), plasma density $\delta n_p/n_o$ (solid line), and axial electric field E_z/E_N (dashed line) plotted versus ζ on axis at $c\tau$=5.0 cm.

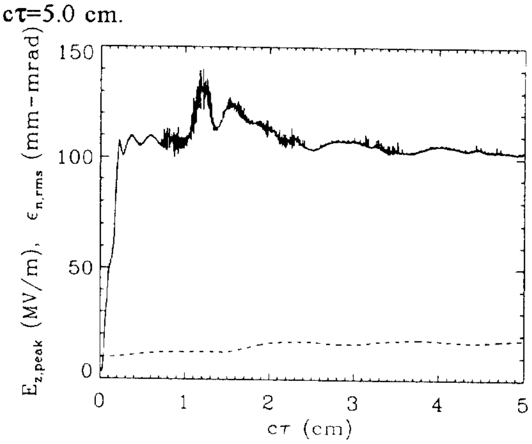

Fig 3. Beam emittance ε_n (dashed line) at fixed ζ = 0.1 cm and peak accelerating field $E_{z,peak}$ (solid line) versus τ.

To avoid the oscillations in the wake amplitude that were observed above, we performed a simulation identical to that of Figs. 1-3, but with $R=R_{eq}$=41 μm as given by Eq. (5). The results are given in Fig. 4, which shows R and $E_{z,peak}$ plotted versus τ. This figure shows that, after a transient of 1.5 cm, the beam-plasma system remains in a quasi-equilibrium state, with a slow expansion of the beam radius and a constant peak accelerating field. The radial expansion occurs because a) the beam loses energy ($\gamma_{final} \approx 35$) and b) the head of the beam, which is not pinched, slowly erodes. Erosion reduces the effectiveness with which the beam expels the plasma electrons such that the plasma electron density inside the beam increases slowly over the length of the simulation. There is no increase in emittance in this case. Note that $E_{z,peak}$ vs. τ in Fig. 4 shows a shorter transient and a significantly higher average value than the corresponding plot in Fig. 3.

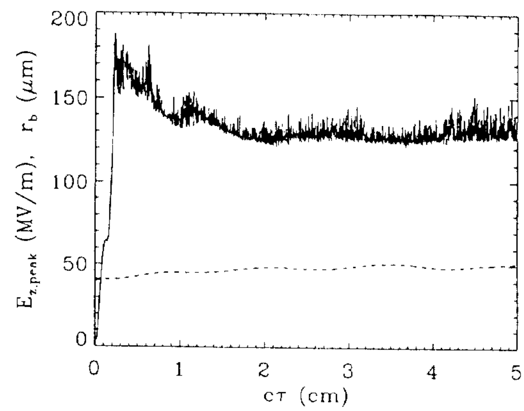

Fig. 4. Beam radius R (dashed line) at fixed ζ = 0.1 cm and $E_{z,peak}$ (solid line) versus τ for a simulation with initial $R=R_{eq}$ = 41 μm.

IV. CONCLUSIONS

The radial pinching forces that we have observed in our simulations can be especially severe when $n_b > n_p$, where n_b and n_p are the beam and plasma densities, respectively. This parameter regime is of interest because of the highly nonlinear wakefields that can be generated. In addition, the filamentation (or Weibel) instability that was observed in the simulations of Keinigs and Jones[7] and Su et al.[8] is avoided in this limit. Our simulations suggest that a highly nonlinear wakefield with favorable focusing properties can be generated by an electron beam in a tightly-focusing equilibrium state.

V. REFERENCES

[1] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**,267 (1979).
[2] P. Chen, J. M. Dawson, R. W. Huff and T.Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).
[3] M. Rosing et al., Conf. Record of the 1991 IEEE Particle Accelerator Conference, p. 555.
[4] J. Smolin, T. Katsouleas, C. Joshi, J. Davis and C. Pellegrini, Bull. Am. Phys. Soc. **35**, 2103 (1990).
[5] J. B. Rosenzweig, B. Breizman, T. Katsouleas and J. J. Su, Phys. Rev. A **44**, R6189 (1991).
[6] E. P. Lee and R. K. Cooper. Part. Accel. **7**, 83 (1976).
[7] R. Keinigs and M. E. Jones, Phys. Fluids **30**, 252 (1987).
[8] J. J. Su, et al., IEEE Trans. Plasma Sci. **PS-15**, 192 (1987).

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