Propagation of Short Electron Pulses in Underdense Plasmas

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Abstract

Our program for an experimental plasma wake field accelerator (PWFA) to take place at the Argonne Wakefield Accelerator (AWA) facility, in the recently proposed blow-out regime[1] relies on the propagation of an intense electron beam through an underdense plasma with a minimum of degradation. This paper presents a near-equilibrium model of beam propagation using the Maxwell-Vlasov equations governing the beam's transverse behavior. Numerical results are presented which use this model simultaneously with the plasma electron cold fluid equations. A solenoidal magnetic field, which is necessary for high density plasma containment, also provides an initial beam equilibrium to begin the calculation. We compare the equilibrium model with a discrete beam particle simulation, which verifies the basic conclusions of the equilibrium model, and shows the collisionless damping approach to equilibrium in the beam head. The initial matching requirements for the beam's entry into the plasma are examined. We also discuss the possibility of performing an adiabatic lens experiment.

Introduction

The recently proposed nonlinear blow-out regime for the PWFA[1], in which all of the plasma electrons become expelled from the region of the beam, has the advantage of linear focusing for the trailing (accelerated) bunch and a very high acceleration gradient. This regime requires a very intense, high quality drive beam. Since the acceleration can be prolonged by using a more energetic drive beam, the long term propagation of this beam through the plasma needs to be addressed.

When an intense electron beam propagates through a plasma in the underdense regime ($n_o < n_{beam}$, n_o is the plasma density), a sufficiently long bunch length will cause complete rarefaction of the plasma electrons, forming an ion channel. This is called the ion focusing regime (IFR) due to the intense magnetic self-focusing forces of the beam as it propagates through the ion channel. For a fully rarefied ion channel, the equilibrium beam radius σ_r is given by,

$$\sigma_r = \left[\frac{\varepsilon_n}{\sqrt{2\pi r_e n_o \gamma}} \right]^{1/2}$$

where ε_n is the normalized emittance, r_e is the classical electron radius, and γ is the Lorentz factor. The beam can be divided into three qualitatively different regions. The extreme leading edge, or *head* of the beam receives no focusing from the plasma, causing it to expand. The *body* of the beam, which travels in the completely rarefied ion channel, receives the maximum focusing force. In the *transition region* between these two, the beam evolution cannot be described with linear optics, due to the remaining population of plasma electrons. For short pulses ($\sigma_z \approx k_p^{-1}$, $k_p = \omega_p/c$), the evolution of the beam head and the transition region are very important in determining the effective propagation of the beam over long distances.

For the ultra-relativistic case, the beam head expands freely, due to its finite emittance, retarding the response of the plasma electrons at later time steps. This diminishes the focusing force for the next beam slice, leading to what is called emittance driven erosion. A simple 1-d model predicts that after some initial expansion, the erosion happens at a very slow rate. In addition, previous particle-in-cell simulation work by Krall, *et al.* [2] has shown that for emittance-driven erosion, the point on the beam where the plasma becomes completely rarefied, termed the pinch point, moves very slowly, and a near equilibrium develops. Inspired by this result, we develop a model in which beam physics is described by a Maxwell-Vlasov equilibrium and the plasma electrons are described by fully relativistic cold fluid equations. This model also includes the effects of a solenoidal magnetic field. Such a field is required for the containment of the plasma in the PWFA, which can in some cases help to further stabilize this erosion.

Numerical Treatment of Beam Near Equilibrium

The response of the plasma due to the beam's electromagnetic field is modeled using a technique developed by Breizman [3]. This model relies on a wake field type assumption: any plasma perturbation translates at the beam velocity. Further, the beam velocity is taken to be the speed of light. The plasma electron currents can then be modeled by the cold fluid equation,

$$\frac{\partial \mathbf{p}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{p} = -e\left(\mathbf{E} + \tfrac{1}{c}(\mathbf{v} \times \mathbf{B})\right)$$

and the continuity equation. Implicit in using a cold fluid model is the fact that the plasma electron velocity quickly becomes much greater than the initial temperature of the plasma. The plasma can be treated as

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a fluid only when the motion is laminar. This is satisfied to a very good approximation in the first half cycle of the plasma motion, making the cold fluid model well suited to studying the effects on the beam. These equations are used along with the Maxwell equations for $\nabla\times E$ and $\nabla\times B$. The speed of light condition, $\partial/\partial z = -\partial/\partial ct$ is used to eliminate the time variable in these equations, which yield a self consistent instantaneous representation of the plasma disturbance. We use conducting wall boundary conditions in the radial direction and further assume that the plasma is quiescent ahead of the beam.

The body of the beam is assumed to be initially matched to the linear focusing force of the ion channel. For the transition region, the nonlinear focusing brings about a mixing of the transverse phase space in just a few betatron periods. If we assume that the emittance increase, which is very difficult to compute, can be neglected, the phase space can be described by the equilibrium Maxwell-Vlasov equation, setting $\partial/\partial t = 0$:

$$\frac{\partial}{\partial t} f(r,p_r) = \frac{p_r}{\gamma m}\frac{\partial f(r,p_r)}{\partial r} + F_r(r)\frac{\partial f(r,p_r)}{\partial p_r} = 0$$

where $f(r,p_r)$ is the distribution in the transverse phase space, p_r is the transverse momentum, γ is the Lorentz factor, m is the electron mass, and F_r is the radial force arising from the plasma fields. In addition, $f(0,0)$ must be constant due to conservation of phase space area.

The matching of the beam body implies that the head will undergo betatron oscillations. For the purpose of the model, we assume that the head is itself matched to the solenoid. This is in part justified by the small number of particles which feel no plasma focusing.

In Figure 1, this method is applied to a 10^{14} cm^{-3} plasma, using the future AWA beam parameters [4]: 100 nC charge per bunch, $\sigma_z = .7$ mm, $\gamma = 300$, and $\varepsilon_n = 400\,\pi$ mm-mrad. The final peak density mimics the initial gaussian density quite well. This run produced a longitudinal wake field capable of sustaining greater than a 1 GeV/m acceleration.

Initial Matching: Time Dependent Aspects

The equilibrium model of the last section relies on a trumpet shaped beam, an condition which may be difficult to produce from a longitudinally uncorrelated profile. In this section we examine the requirements for initial matching into the focusing channel and present computational results which reveal the efficacy of such matching. In order to minimize the betatron oscillations of the beam head, the largest possible initial beam radius must be used. This is important when the period of these oscillations is longer than the stopping distance of the beam through the plasma. It is therefore advantageous to use the focusing properties of the plasma in such a way as to reduce the radius of the beam body, while leaving the beam head unaffected. The initial ramp in the plasma density at the start of any actual device can be useful for this task. We have studied the effect of a plasma profile which builds up as one side of a gaussian and remains flat afterward. Although this does not represent the ideal focusing scheme, it is a good approximation of the experimental situation.

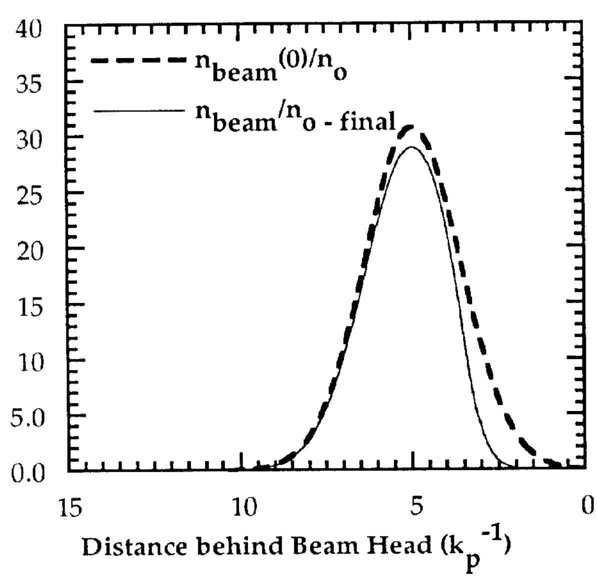

Figure 1. Final (equilibirum) and initial beam density normalized to plasma density using AWA beam parameters, $n_o = 10^{14}$ cm^{-3}, and a solenoidal field of B=2000 Gauss. The matched equilibrium is achieved very early in the the beam pulse.

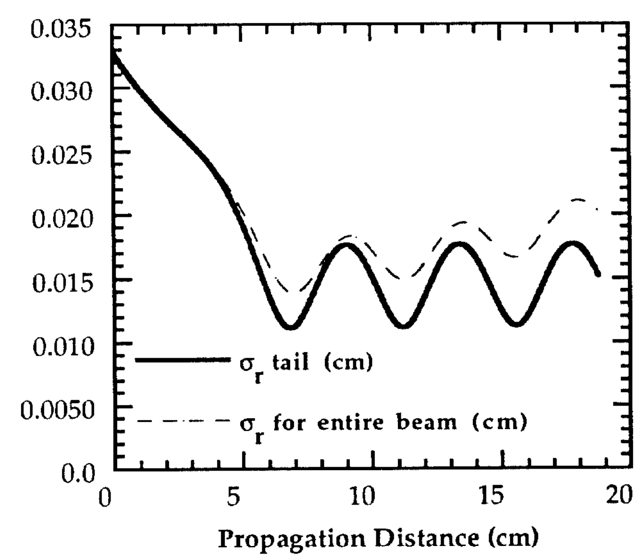

Figure 2. Matching of the beam into plasma with a gaussian ramp, $\sigma = 2$ cm. For this run, the beam has a virtual waist of 0.026 cm at a position 4 cm into the device. The magnetic field was 2000 Gauss.

To simulate these time dependent effects, we used the same numerical techniques as in the last section to treat the plasma dynamics. The beam in this case was composed of a set of discrete particles, treated self-consistently with the plasma dynamics. It is assumed that the variation in n_0 over the extent of the beam can be neglected. Figure 2 shows results, using the AWA beam parameters, for ramp which builds up as a gaussian from $n_0 = 10^{12}$ cm^{-3} to $n_0 = 10^{14}$ in 8 cm, and remains flat afterward. A slight mismatch of the beam body is tolerable since it does not diminish the radial force seen by the plasma electrons, which have already left the region of the beam.

Through observing the evolution of the phase space we have witnessed a new mechanism for the suppression of erosion of the beam head. The presence of any erosion diminishes the focusing strength of a given z-slice very slowly. Thus, the slice's σ_r is adiabatically increased until it becomes matched to the solenoid, approximating the equilibrium discussed in the last section. In the case of a weak solenoid, this predicts that a small portion of the beam head will become lost, while the rest of the beam propagates in a near equilibrium.

Adiabatic Focusing

There has been great interest in using a plasma lens for the final focusing in a linear collider[5]. Chen *et al.* [6] has proposed making such a lens adiabatic, with a smoothly increasing focusing strength. The advantages of adiabaticity are to reduce the effects of synchrotron radiation, lessening impact of the chromatic aberrations, and diminishing the sensitivity to initial optical mismatch. Because the equations governing the plasma motion have a simple scaling with n_0, the results of a sub-GeV, experiment can yield a good prediction for larger γ and an n_0 of 10^{18} cm^{-3} In addition, as n_0 increases, the condition $\sigma_z >> k_p^{-1}$ becomes easier to satisfy. Thus, a proof of principle experiment at low energy would circumvent some of the experimental difficulties of an actual final focus experiment. The AWA beam, with its high peak current, and accompanying access to the underdense regime, can be used to experimentally study such a device. The discrete particle method developed above has been used to numerically study the adiabatic lens. The AWA beam is intense enough to cause blowout at very early times after the arrival of the beam. In addition, the charge and energy can be scaled down to study a large parameter space.

The work by Chen, *et al.*, suggests the use of a lens in which the β function is linearly decreased. This lens has the feature that the plasma density is increasing very rapidly near the end. We have studied a lens whose density increases only exponentially, which we believe is more realistic experimentally. Figure 3 presents simulation results for such a lens. The body of the beam is focused, in good agreement with the theoretical limit assuming a completely rarefied ion channel. Note that the σ_r averaged over the entire beam never becomes small due to phase differences and the insufficient focusing of the beam head.

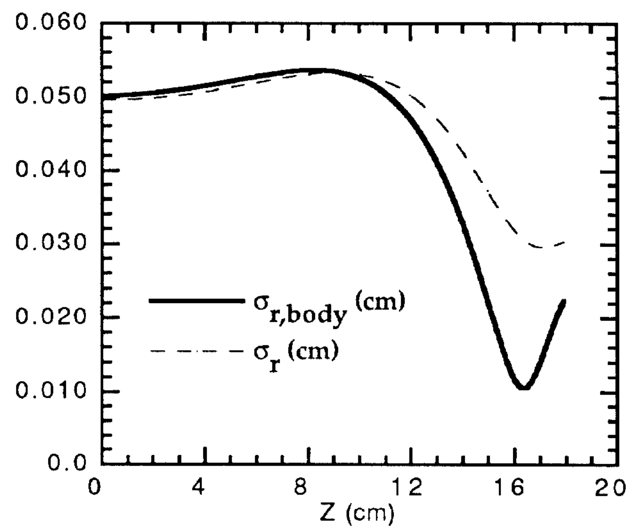

Figure 3: Adiabatic lens for the AWA beam, n_0 starts at 2.5 x10^{11} and increases to 5 x10^{13} at z=15 cm. The body of the beam (the core population containing half the beam) is focused quite efficiently, down to 100 microns at z=16 cm.

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