Wakefield Accelerator Driven by a Relativistic Electron Beam in a Ferromagnetic Waveguide

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Abstract

A novel high-gradient wakefield accelerator is presented in which the drive-beam current leaves behind a high-gradient wakefield, accelerating the witness beam to very high energy. The theoretical analysis is based on Faraday's law, which provides a second-order partial-differential equation of the azimuthal magnetic field, under the assumption that $\mu \varepsilon > 1$. The accelerating field can be more than one half of one gigavolt/meter in an appropriate choice of system parameters.

I. INTRODUCTION

In recent years, there has been a strong progress in the high-current electron-beam technology. Electron beams with an energy of 10 MeV and a current of 10 kA are easily available in the present technology. In addition, a tremendous improvement has been made in the effective control of these electron beams, including the focus, modulation, and a timely termination of the beam current. Thus, the electron beam itself is used as a drive current in the wakefield accelerators, where a short and intense bunch of electrons passes through a $plasma^{1-3}$ or dielectric waveguide, ⁴⁻⁶ leaving behind intense electromagnetic field. The axial component of this electromagnetic field accelerates charged particles in the witness beam, which follows the drive electron beam. Based on the transverse magnetic (TM) waveguide modes, a preliminary theory^{5,6} in a dielectric waveguide accelerator has been developed to estimate the acceleration field, which is the fundamental-radial mode in most cases. However, in reality, the acceleration field is a sum of the whole radial modes, which is a complicated function of various physical parameters, including the geometric configuration, the material properties of the waveguide, and so on. In addition, evolution of the acceleration field in time is again a sum of the every radial-mode evolution. In this regard, I develop a fully self-consistent theory of the wakefield accelerators, which consists of a waveguide with a ferromagnetic material. As will be seen later, the accelerating field is proportional to the square root of the parameter μ/ε , where μ and ε are the permeability and dielectric constant of the waveguide material. The higher the permeability, the higher the accelerating field.

II. WAKEFIELD FOR THE ACCELERATION

The theoretical model is based on the induced electric field due to decay of the field energy stored in an energy storage device. We assume that an electron beam with current I(t) propagates through a hole with radius R_1 in the field-energy storage with radius of R_2 . The energy storage device is a waveguide with a ferromagnetic material. Whenever the drive-beam current I(t) decreases, the induced electric field $E_z(r,t)$ appears in the system. The induced axial-electric field E_z is calculated from the Faraday's law and given by

$$E_{\mathbf{z}}(R_{1},t) = -\frac{2\mu}{c^{2}} \int_{0}^{\infty} \frac{dk}{k} \left[J_{0}(kR_{2}) - J_{0}(kR_{1}) \right]^{2}$$
$$\cdot \int_{-\infty}^{t} dt' \left(\frac{dI}{dt'} \right) \frac{\partial}{\partial t'} q_{k}(t-t') , \qquad (1)$$

where $J_0(x)$ is the Bessel function of the first kind of order zero,

$$q_k(t) = \exp\left(-\frac{2\pi\sigma}{\epsilon}t\right)\cos\left(\frac{kc}{\sqrt{\mu\epsilon}}t\right)$$
 (2)

is the time function and σ is the residual conductivity in the material although it is very small (zero in a practical sense). Substituting Eq. (2) into Eq. (1) and carrying out partial integrations in time and radial coordinate, I obtain the accelerating field

$$E_{z}(t) = -\frac{2}{c} \sqrt{\frac{\mu}{e}} \int_{-\infty}^{t} dt' \left(\frac{dI}{dt'}\right) \int_{0}^{\infty} dk \left[J_{0}\left(kR_{2}\right)\right]$$
$$- J_{0}\left(kR_{1}\right)]^{2} \sin\left[\frac{c\left(t-t'\right)}{\sqrt{\mu e}}k\right],$$
(3)

where the abbreviation $E_z(t)$ represents $E_z(0,t)$. In obtaining Eq. (3), I have neglected the terms proportional to the residual conductivity σ .

For convenience in the subsequent analysis, the normalized times τ_1 and τ_2 are defined by

$$\tau_1 = \frac{c(t-t')}{2R_1\sqrt{\mu\varepsilon}}, \qquad \tau_2 = \frac{c(t-t')}{2R_2\sqrt{\mu\varepsilon}}, \qquad (4)$$

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where we note that $\tau_1 = R_2 \tau_2 / R_1$. For $R_2 >> R_1$, the normalized time τ_1 is much longer than the time τ_2 . Making use of Eq. (4), we can rewrite Eq. (3) by

$$E_{z}(t) = -\frac{2}{cR_{1}} \sqrt{\frac{\mu}{e}} \int_{-\infty}^{t} dt' \left(\frac{dI}{dt'}\right) \int_{0}^{\infty} dx J_{0}^{2}(x)$$

$$\cdot \left[\sin\left(2\tau_{1}x\right) + \frac{R_{1}}{R_{2}}\sin\left(2\tau_{2}x\right) \right]$$

$$- \frac{2R_{1}}{R_{2}} \frac{J_{0}(R_{1}x/R_{2})}{J_{0}(x)} \sin\left(2\tau_{2}x\right) \right].$$

In the remainder of this article, the analysis is restricted to the case when the inner radius R_1 of the energy storage device is much less than the outer radius R_2 , i.e., $R_1 < < R_2$. In this limit, we note several points from Eqs. (4) and (5). First, the term proportional to $\sin(2\tau_1 x)$ in the integrand in Eq. (5) dominates. The corrections associated with other terms are of the order $(R_1/R_2)^{1/2}$ or less. Second, the peak values of the integration over the variable x in Eq. (5) occur around the time t satisfying $\tau_1 \sim 1$, $\tau_2 \sim 1$ and $\tau_2 \sim (R_1/R_2)^{1/2}$, which correspond to the contributions from the terms proportional to $\sin(2\tau_1 x)$, $(R_1/R_2)\sin(2\tau_2 x)$ and $J_0(R_1 x/R_2)$, respectively, in the right-hand side of Eq. (5). In the early stage, the term proportional to $sin(2\tau_1 x)$ dominates. In this regard, we keep the term proportional to $sin(2\tau_1 x)$ in Eq. (5), neglecting other terms. If needed, the corrections associated with other terms can be calculated in a straightforward manner.

The integration over the variable x is carried out by making use of the integral⁷

$$\pi \int_{0}^{\infty} dx J_{0}^{2}(x) \sin(2\tau_{1}x) - \begin{cases} K(\tau_{1}), & \tau_{1} < 1, \\ \frac{1}{\tau_{1}} K(\frac{1}{\tau_{1}}), & \tau_{1} > 1, \end{cases}$$
(6)

where K(x) is the elliptical function of the first kind defined by

$$K(x) = \frac{\pi}{2} \left[1 + \left(\frac{1}{2}\right)^2 x^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 x^4 + \cdots \right]$$
 (7)

After carrying out a straightforward calculation, I show that the acceleration field E_2 in Eq. (5) is approximated by

$$E_{z}(t) = -\frac{2}{\pi c R_{1}} \sqrt{\frac{\mu}{e}} \int_{-\infty}^{t} dt' \left(\frac{dI}{dt'}\right)$$

$$\cdot \left[K(\tau_{1}) U(1-\tau_{1}) + \frac{1}{\tau_{1}} K\left(\frac{1}{\tau_{1}}\right) U(\tau_{1}-1)\right],$$
(8)

where U(x) is the Heaviside step function. Equation (8) can be used to calculate the acceleration-gradient field for a broad range of system parameters, where the drive current changes fast. Note that the drive-beam current I(t) in Eq. (8) is not specified yet. In order to investigate the long pulse-driven accelerator, we consider the drive current defined by

$$I(t) = \begin{cases} I_{m}, & t < 0, \\ I_{m}(1 - \frac{t}{\Delta t}), & 0 < t < \Delta t, \\ 0, & t > \Delta t, \end{cases}$$
(9)

where the parameter Δt is the termination time of the drive beam current. In reality, the drive-beam current I(t) at t < 0 increases very slowly to I_m at t = 0. Thus, Eq. (9) is a good approximation. Substituting Eq. (9) into Eq. (8), and making use of the definitions in Eq. (4) and

$$\tau = \frac{ct}{2R_1\sqrt{\mu\epsilon}}, \quad \eta = \frac{c\Delta t}{2R_1\sqrt{\mu\epsilon}}, \quad (10)$$

the acceleration field can be expressed as

$$E_{z}(t) - \frac{2I_{m}}{\pi R_{1}c} \sqrt{\frac{\mu}{\epsilon}} q(\tau) , \qquad (11)$$

where the function $q(\tau)$ for the drive current in Eq. (9) is defined by

$$q(\tau) - \frac{1}{\eta} \int_{0}^{\tau} d\tau' U(\eta - \tau') [K(\tau_{1}) U(1 - \tau_{1}) + \frac{1}{\tau_{1}} K(\frac{1}{\tau_{1}}) U(\tau_{1} - 1)], \qquad (12)$$

and $\tau' = \tau - \tau_1$.

Figure 1 presents plots of the function $q(\tau)$ versus the normalized time τ obtained from Eq. (12) for $\eta = 0.05$ (solid line), 0.1 (broken line), 0.2 (dotted line), and 0.4 (thin broken line). Several points are noteworthy in Fig. 1. First, the shorter the normalized termination time the higher the peak value of the function q. Second, the peak value of the function $q(\tau)$ is about 2.5 even for a relatively slow termination time. This peak value occurs at $\tau = 1$. Third, the function $q(\tau)$ is always positive for the choice of the drive current in Eq. (9). Fourth, the value of the function q in the range of τ satisfying $0 < \tau < \eta$ increases linearly with time τ . As we note from Eq. (9), the drive current decreases linearly to zero in this range of τ . Because the q value of this tail portion of the drive beam increases with time, the termination slope stiffens further. This mechanism may decrease the normalized termination time η as time goes by. Finally, we emphasize that the time duration of the high acceleration field is quite broad. This property is important for a long witness beam. In the limit when the normalized termination time η is much less than unity, i.e., $\eta << 1$, Eq. (12) is approximated by

$$q(\tau) = \begin{cases} K(\frac{\tau}{2}) \frac{\tau}{\eta}, & 0 < \tau < \eta, \\ K(\tau - \frac{\eta}{2}), & \eta < \tau < 1, \\ 1.4 + \frac{1}{2} \ln(\frac{2}{\eta}), & 1 < \tau < 1 + \eta, \\ \frac{1}{\tau} \ln(\frac{1}{\tau}), & \tau > 1 + \eta, \end{cases}$$
(13)

which agrees reasonably well with the numerical result in Fig. 1 even for $\eta = 0.4$.



Fig. 1. Plots of the function $q(\tau)$ versus the normalized time τ obtained from Eq. (12) for $\eta = 0.05$ (solid line), 0.1 (broken line), 0.2 (dotted line), and 0.4 (thin broken line).

As an example, I assume that the current termination parameter is equal to $\eta = 0.05$, for which the peak value of the function q is 3.2 and the accelerating field is given by

$$E_{m} - \frac{2I_{m}}{R_{1}C} \sqrt{\frac{\mu}{\epsilon}}.$$
 (14)

Assuming that the drive current $I_m = 20$ kA, the hole radius $R_1 = 0.4$ cm and $\mu/e = 4$, we find from Eq. (14) that the accelerating field is given by $E_m = 0.6$ gigavolt/meter, which is very encouraging number. The current termination parameter $\eta = 0.05$ corresponds to the real termination time of $\Delta t = 2.6$ picosecond for $\mu \varepsilon = 4$. As shown in Eq. (9), the risetime of the drive-beam current must be considerably longer than the termination time. The risetime of 26 picosecond may be enough for present example. The accelerating field in Eq. (14) for a ferromagnetic waveguide is six times of that in a dielectric waveguide⁵ for similar system parameters. I remind the reader that the whole pulse length in the example is less than 1 cm, thereby practically indicating that the drive beam is an intense bunch of electrons. The total charge of the drive-beam current in the example is

less than 300 nanocoulomb. Tailoring the beam pulse as mentioned above is very important to achieve a high accelerating gradient. Obviously, the wakefield accelerator in a ferromagnetic waveguide has a great potential for high gradient acceleration of electrons.

In order to achieve the high acceleration field, we must overcome two technical problems. First, the magnetic field in the energy storage material is limited by the saturation field B_{g} , which is expressed as

$$B_s - \frac{2\mu I_m}{CR_1}.$$
 (15)

Once the magnetic field in the storage material is saturated by the drive current I_m , any additional increase of the current does not help much. Eliminating the current I_m in favor of the saturation field B_s , we rewrite Eq. (14) as

$$E_m - \frac{B_s}{\sqrt{\mu \varepsilon}}, \qquad (16)$$

for the saturation current satisfying Eq. (15). Equation (16) clearly indicates that acceleration field is linearly proportional to the saturation field of the material. The higher the saturation field the higher the acceleration field. Second, reversal time of the magnetization in the storage material must be on the order of nanosecond or less. A sub-nanosecond reversal time has been accomplished several decades ago for a small amount of high-permeability materials with μ of 100 or more.⁸ However, the wakefield accelerator for a high-current beam requires a bulk storage material. Establishing a short reversal time of the magnetization in a bulk material may require further research on the material science.

III. REFERENCES

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