A Self-Consistent Theory of Ferromagnetic Waveguide Accelerators Driven by Electron Beams

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Abstract

A fully self-consistent theory of ferromagnetic waveguide accelerators driven by a relativistic electron beam is developed. The theoretical analysis is based on Faraday's law, which provides a second-order partial-differential equation of the azimuthal magnetic field, under the assumption that $\mu\epsilon >> 1$. Here μ and ϵ are the permeability and dielectric constant of the waveguide material. The azimuthal magnetic field and axial acceleration field are obtained in forms of integral equations for an arbitrary profile of the drive-beam current I(t).

I. INTRODUCTION

An induced electric field appears whenever the magnetic field changes in time. This induced electric field is an excellent means for charged particle acceleration. One of the most advanced devices for intense electron beam accelerators is the induction linear accelerator (Linac),[1] where each module of many local accelerators applies its electric field to a cluster of traveling electrons. The electric field of each local accelerator in Linac originates from the time varying magnetic field, which is excited by an electrical current carried by a wire. In recent years, there has been a strong progress in the high-current electron-beam technology. Electron beams with an energy of 10 MeV and a current of 10 kA are easily available in the present technology. In addition, a tremendous improvement has been made in the effective control of these electron beams, including the focus, modulation, and a timely termination of the beam current. Thus, the electron beam itself is used as a drive current in the wakefield accelerators, where a short and intense bunch of electrons passes through a plasma[2-4] or dielectric waveguide,[5-7] leaving behind intense electromagnetic field. The axial component of this electromagnetic field accelerates charged particles in the witness beam, which follows the drive electron beam. Based on the transverse magnetic (TM) waveguide modes, a preliminary theory[6,7] has been developed to estimate the acceleration field, which is the fundamental-radial mode in most cases. However, in reality, the acceleration field is a sum of the whole radial modes, which is a complicated function of various physical parameters, including the geometric configuration, the material properties of the waveguide, and so on. In addition, evolution of the acceleration field in time is again a sum of the every radial-mode evolution. In this regard, we develop a fully self-consistent theory of the wakefield accelerators, which consists of a waveguide with a ferromagnetic material.

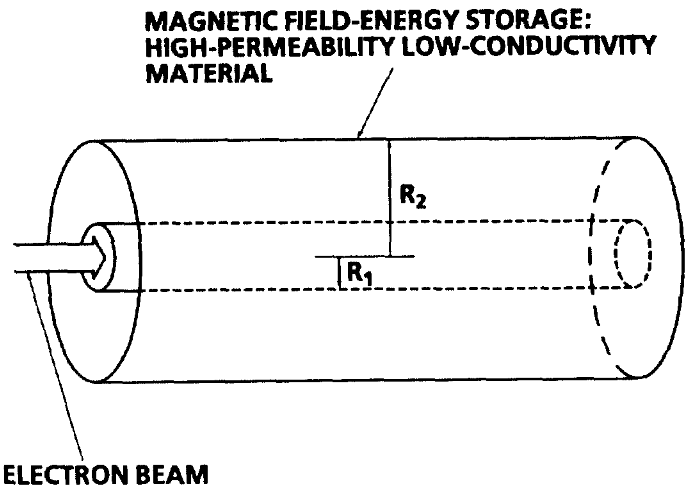

Fig. 1. Ferromagnetic waveguide accelerators.

II. BASIC ASSUMPTIONS

The theoretical model is based on the induced electric field due to decay of the field energy stored in an energy storage device. As shown in Fig. 1, we assume that an electron beam with current I(t) propagates through a hole with radius of R_1 in the field-energy storage with radius of R_2. Note that the electron-beam current I(t) carries both charge and current, which store the electric- and magnetic-field energies in the energy storage device. The line charge density $\lambda(t)$ carried by the current I(t) is given by $\lambda(t) = I(t)/\beta c$, where βc is the beam velocity and c is the speed of light in vacuum. In the subsequent analysis, a polar coordinate system is introduced with the z-axis along the axis of symmetry, r represents the radial distance from the axis and θ is the polar angle. The system is azimuthally symmetric around the axis. Due to a slowly changing current I(t), the azimuthal magnetic field B_θ in the energy storage material is given by

$$B_\theta(r,t) = \frac{2\mu I(t)}{cr}, \qquad (1)$$

This work was supported by the Independent Research Fund at the Naval Surface Warfare Center.

where μ is the permeability of the energy storage material. Similarly, the radial electric field E_r in the range of r satisfying $R_1 < r < R_2$ is given by

$$E_r(r,t) = \frac{2I(t)}{\epsilon\beta cr}, \tag{2}$$

where ϵ is the dielectric constant of the energy storage material. The field energy associated with the magnetic and electric fields in Eqs. (1) and (2) is stored in the energy storage material.

We note from Eqs. (1) and (2) that the induced electric field due to the radial electric field E_r is negligible in comparison with that due to the azimuthal magnetic field B_θ for the energy storage material with $\epsilon\mu >> 1$, which is common in present applications. In this context, in the subsequent analysis, we use the relation

$$\frac{\partial}{\partial r}E_z(r,t) = \frac{1}{c}\frac{\partial}{\partial t}B_\theta(r,t) \tag{3}$$

in evaluation of the induced electric field resulted from a fast-changing drive current.

III. ACCELERATING FIELD FOR DRIVE-BEAM I(t)

As shown in Eq. (3), the induced electric field increases drastically as the drive current decreases quickly. Remember that a high induced electric field is needed for efficient acceleration of charged particles. When the drive current changes quickly, the induced electric field must be determined in a self-consistent manner. Ampere's law in the Maxwell equation is written as

$$\nabla \times B = \frac{\mu\epsilon}{c}\frac{\partial}{\partial t}E + \frac{4\pi\mu}{c}J_T, \tag{4}$$

where B is the magnetic field, E is the electric field, and the total current density J_T represents both the steady-state beam current and the induced current J_{in}. Assuming that the conductivity of the energy storage material is σ, the induced current density is expressed as $J_{in} = \sigma E$. Making use of Faraday's law, the curl of Eq. (4) is expressed as

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)\right] - \frac{4\pi\sigma\mu}{c^2}\frac{\partial}{\partial t}B_\theta - \frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}B_\theta = 0, \tag{5}$$

in the storage device defined by the range of r satisfying $R_1 < r < R_2$. In obtaining Eq. (5), we have neglected the term $(\partial^2/\partial z^2)B_\theta = (1/\beta^2c^2)(\partial^2/\partial t^2)B_\theta$, which is much less than the term proportional to $\mu\epsilon$ in Eq. (5) provided $\mu\epsilon >> 1$.

The solution of Eq. (8) is expressed as

$$B_\theta(r,t) = \int_0^\infty dk a_k J_1(kr)\exp(-\lambda_k t), \tag{6}$$

where $J_1(x)$ is the Bessel function of the first kind of order one and λ_k is the generalized frequency. Substituting Eq. (6) into Eq. (5) and defining

$$\gamma_k = \frac{2\pi\sigma}{\epsilon}, \quad \alpha_k = i\omega_k = \sqrt{\frac{4\pi^2\sigma^2}{\epsilon^2} - \frac{k^2c^2}{\mu\epsilon}}, \tag{7}$$

we find that the generalized frequency λ_k is expressed as

$$\lambda_k = \gamma_k - i\omega_k. \tag{8}$$

Defining the critical wave number k_0 by

$$k_0 = \frac{2\pi\sigma}{c}\sqrt{\frac{\mu}{\epsilon}}, \tag{9}$$

we can express the time profile of the solution in Eq. (6) by

$$q_k(t) = \exp(-\lambda_k t) = \exp(-\gamma_k t)\cdot\begin{cases}\exp(\alpha_k t), & 0 < k < k_0,\\ \cos(\omega_k t), & k > k_0,\end{cases} \tag{10}$$

which satisfies the initial and final conditions, $q_k(t=0) = 1$ and $q_k(t=\infty) = 0$. Substituting Eq. (10) into Eq. (6), the desired solution is expressed as

$$B_\theta(r,t) = \int_0^\infty dk a_k J_1(kr) q_k(t). \tag{11}$$

We now calculate the magnetic field $B_\theta(r,t)$ driven by the current $I(t) = I(t')U(t-t')$, where U(x) is the Heaviside step function defined by $U(x) = 1$ for $x > 0$ and 0, otherwise. It is obvious that $B_\theta = 0$ for $t < t'$ by the causality. The magnetic field at the time $t > t'$ is expressed as

$$B_\theta(t) = \frac{2\mu I(t')}{cr}U[(R_2-r)(r-R_1)] + \int_0^\infty dk a_k J_1(kr) q_k(t-t'), \tag{12}$$

where the first term in the right-hand side represents the steady-state solution and the second term represents the time-transient solution. Note that the time-transient solution in Eq. (12) vanishes at the time $t \to \infty$. In obtaining Eq. (12), we have neglected the steady-state solution outside of the energy storage material, assuming that the magnetic permeability of the material is much higher than unity ($\mu >> 1$).

Making use of the initial condition $q_k(t-t') = 1$ at $t = t'$, we obtain

$$\frac{2\mu I(t')}{cr} U[(R_2-r)(r-R_1)] + \int_0^\infty dk a_k J_1(kr) = 0, \quad (13)$$

from Eq. (12). Multiplying Eq. (13) by $rJ_1(k'r)$ and making use of the orthogonality of the Bessel function

$$\int_0^\infty x dx J_1(\eta x) J_1(\xi x) = \frac{\delta(\xi-\eta)}{\xi}, \quad (14)$$

we obtain

$$a_k = 2\mu \frac{I(t')}{c} [J_0(kR_2) - J_0(kR_1)], \quad (15)$$

where $J_0(x)$ is the Bessel function of the first kind of order zero. Substituting Eq. (15) into Eq. (12), the magnetic field at the time $t > t'$ is therefore expressed as

$$B_\theta(r,t) = \frac{2\mu}{cr} I(t') + \frac{2\mu I(t')}{c} \int_0^\infty dk [J_0(kR_2) - J_0(kR_1)] J_1(kr) q_k(t-t'), \quad (16)$$

for $R_1 < r < R_2$.

It is necessary to evaluate the magnetic field due to the drive beam pulse defined by $I(t) = I(t')U[(t'+\Delta t'-t)(t-t')]$ with the pulse length $\Delta t'$. Paralleling the derivation of Eq. (16), the magnetic field at the time $t > t'+\Delta t'$ is given by

$$\Delta B_\theta(r,t) = -2\mu \frac{I(t')}{c} \int_0^\infty dk [J_0(kR_2) - J_0(kR_1)] J_1(kr) \left(\frac{d}{dt'} q_k\right) \Delta t', \quad (17)$$

which is the magnetic field contributed by a segment $\Delta t'$ of the drive beam current $I(t')$. In obtaining Eq. (17), we have assumed that the pulse length $\Delta t'$ is very small. Integrating Eq. (17) over the time t', we can show that the magnetic field $B_\theta(r,t)$ due to a continuous drive beam is expressed as

$$B_\theta(r,t) = -\frac{2\mu}{c} \int_0^\infty dk [J_0(kR_2) - J_0(kR_1)] J_1(kr) \cdot \int_{-\infty}^t dt' I(t') \frac{\partial}{\partial t'} q_k(t-t'), \quad (18)$$

which determines the magnetic field in the storage material for an arbitrary time profile of the drive beam current $I(t')$.

The induced axial-electric field E_z is proportional to the time derivative of the azimuthal magnetic field as shown in Eq. (3). Substituting Eq. (18) into Eq. (3) gives

$$\frac{\partial}{\partial r} E_z(r,t) = -\frac{2\mu}{c^2} \int_0^\infty dk [J_0(kR_2) - J_0(kR_1)] J_1(kr) \cdot \int_{-\infty}^t dt' \left(\frac{dI}{dt'}\right) \frac{\partial}{\partial t'} q_k(t-t'), \quad (19)$$

where use has been made of the relation $(\partial/\partial t)q_k = -(\partial/\partial t')q_k$. Neglecting the azimuthal magnetic field outside the energy storage material ($r > R_2$), we approximate the boundary condition of the axial electric field by $E_z(r,t) = 0$ at $r = R_2$. Integrating Eq. (19) over the radius r, the axial electric field in the energy storage material ($R_1 < r < R_2$) is given by

$$E_z(r,t) = -\frac{2\mu}{c^2} \int_0^\infty \frac{dk}{k} [J_0(kR_2) - J_0(kR_1)][J_0(kR_2) - J_0(kr)] \int_{-\infty}^t dt' \left(\frac{dI}{dt'}\right) \frac{\partial}{\partial t'} q_k(t-t'). \quad (20)$$

Because we neglect the axial electric field due to the azimuthal magnetic field in the hole ($r < R_1$), the axial electric field E_z, which accelerates the charged particles at the axis, is approximately given by the electric field at $r = R_1$. Equation (20), together with Eq. (18), is one of the main results of this article and can be used to determine the acceleration gradient for a broad range of physical parameters, including properties of the energy storage material, geometric configuration of the system, species of the charged particles, and intensity of the drive-beam current. Specific examples of application of Eq. (20) will be presented in the following papers.

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