Photonic Band Gap Resonators for High Energy Accelerators*

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Abstract

We have proposed that a new type of microwave resonator, based on Photonic Band Gap (PBG) structures, may be particularly useful for high energy accelerators. We provide an explanation of the PBG concept and present data which illustrate some of the special properties associated with such structures. Further evaluation of the utility of PBG resonators requires laboratory testing of model structures at cryogenic temperatures, and at high fields. We provide a brief discussion of our test program, which is currently in progress.

I. INTRODUCTION

The use of high Q cavity resonators has become an integral part of the accelerator technology applicable to present and future experiments in high energy particle physics. Currently, the resonators in use or under construction, are based on geometric structures where the normal modes are readily understood as a consequence of the electric field satisfying the boundary conditions imposed by the metal walls of the cavity. The nature of both the fundamental and higher order modes can often be qualitatively visualized, even though accurate evaluation of the mode frequencies may be numerically demanding. In contrast, the resonant cavities that we have proposed for potential use in a future generation of accelerators are based on what has been termed Photonic Band Gap (PBG) structures, and they are sufficiently different from both the traditional metal walled cavities or the diverse types of dielectric resonators, that they have to be analyzed and evaluated in their own right. Because the criteria for establishing the resonant modes in a PBG structure are so different, they (presently) cannot be designed or evaluated with the level of intuition normally applicable to the traditional cavity designs. Indeed, the difference in mode densities may be one of the principal advantages of PBG structures, with the possibility, for example, of designs that have negligible or even no higher order modes.

In this paper, for the convenience of the reader, we present a physical explanation of the PBG structure and its key properties, followed by illustrative data and numerical simulations. We conclude with a brief discussion of a typical configuration for a PBG cavity suitable for an accelerator, and an outline of our test program. We have presented a more detailed introduction to the idea of utilizing PBG structures as accelerator cavities [1]. We refer the reader to several prior articles that may also be specifically useful [2,3].

II. A PHYSICAL EXPLANATION OF THE PBG RESONATOR

The principal component of a PBG resonator is a photonic lattice; that is, a configuration which has a periodically varying dielectric constant in at least one direction, and is uniform in all other potential directions. We define the dimension of the PBG element as the number of directions in which the dielectric function varies periodically. A 1-D PBG structure, for example, could be a waveguide filled with a set of dielectric slabs periodically spaced along its length. A 2-D PBG system could be a lattice of very long parallel dielectric rods. A 3-D PBG structure could be composed of dielectric scatterers placed, for example, on a diamond lattice.[4] The dimension of the photonic lattice plays an important role in determining the electromagnetic mode characteristics of the PBG resonator.

Any actual PBG resonator will contain a dielectric lattice terminated in some way (e.g., conducting walls or absorber). While it is difficult to solve the general boundary value problem, Maxwell's equations for an infinite periodic dielectric lattice can be solved numerically with relative ease, and the solutions obtained reflect the dominant properties of any significantly large, but finite, section of such material. The essential characteristic of a periodically varying dielectric medium, common to any dimension, is that regions of frequency exist for which no propagating modes are present for waves traveling in a particular set of directions in the lattice. These frequency regions are called band gaps. In general, one finds band gaps for every direction of propagation for which there is periodic modulation of the dielectric constant. However, if there is a frequency region where these band gaps overlap for all the possible propagation directions, then the system is said to possess a complete photonic (i.e., electromagnetic wave) band gap. In 1-D some complete band gaps are guaranteed for any periodicity in the dielectric constant, since there is only one

*This work supported by the National Science Foundation DMR-93-02913, and the Department of Energy, contracts DE-FG-03-93ER40793 and DE-AC-03-76SF00515.

0-7803-1203-1/93$03.00 © 1993 IEEE

direction of propagation. In higher dimensions, whether or not a complete PBG exists depends on the type of lattice, filling factor, dielectric mismatch, and scatterer structure.

Once we have identified an infinite dielectric lattice with a complete PBG, we may then ask how a finite section of such a lattice will behave. Rather than having absolute forbidden frequency regions, a finite lattice will now have modes in the band gap region which grow or decay in some direction with exponential dependence. As a practical example, if we imagine varying the frequency of a wave incident on a lattice and measuring the power which is transmitted, we would find regions of nearly perfect transmittance, usually designated as pass bands, separated by regions of strong attenuation corresponding to the band gaps. If we are to apply the solutions obtained from the infinite lattice to a finite lattice, we require that the length scale of that lattice be at least several times larger than the largest attenuation length in the lattice.

Having defined a PBG structure, how can it be useful for devices requiring a cavity-like resonance? Let us now restrict our discussion to a specific 2-D geometry. Our PBG structure simply consists of a periodic array of dielectric cylinders, with the axes of the cylinders perpendicular to a pair of bounding conducting plates on top and bottom. This configuration may be tested (either in the laboratory or via numerical simulation) and it is found that indeed there are regions of frequencies for which the transmission through a finite length of the structure is exponentially attenuated for waves incident from any direction. We will see later that it is quite practical to find such configurations for 2-D systems at microwave frequencies, and that the characteristic attenuation lengths can be comparable to the lattice constant.

We now consider a sample of the structure that is made with any circumferential geometry, as long as the distance from boundary to center is many times the value of the longest attenuation length for the frequency range of interest. One can make a perturbation to the dielectric region near the center of this lattice, and arrange to couple energy into that region via a small probe placed in a hole drilled through one of the metal plates above the perturbed site. We know that no energy radiating from the probe will propagate radially outward, because waves in all directions are exponentially attenuated for frequencies within a complete PBG. Thus, in general, the energy incident via the probe will be fully reflected. However, if the perturbation to the dielectric is strong enough, it may be possible that for some frequency, occurring within the PBG region, the electromagnetic fields may just match onto the exponentially decaying waves perfectly, for all directions, and constitute a resonant mode of that system. Indeed, we find that we can make configurations with the properties just described. The perturbation is termed a "defect", and the resonant mode is a defect mode. In this special circumstance we would find that energy can be coupled into the "cavity" where the electromagnetic fields corresponding to that mode will build up until the losses equal the incident power flow. As it turns out, completely removing a cylinder from an otherwise periodic lattice often produces a defect mode with the desired properties.

To utilize the preceding type of resonance to accelerate an electron beam we consider modes where the electric rf field is everywhere normal to the metal plates with a maximum at the center (i.e. a monopole character). The bunched electron beam, suitably phased, would enter via a hole in one plate, and emerge with increased energy through a similar hole in the other plate. As with other types of resonant cavities, there would have to be provisions for coupling drive power into a cavity, which in turn could feed many other resonant cavities all at the same frequency, and suitably coupled by adjustments to the intercavity apertures. An illustration of a possible 3 section, 2π accelerator modular unit based on a triangular periodic lattice is presented in Figure 1. As we shall discuss, the triangular lattice appears to be particularly advantageous as a PBG-defect resonant cavity for accelerator applications.

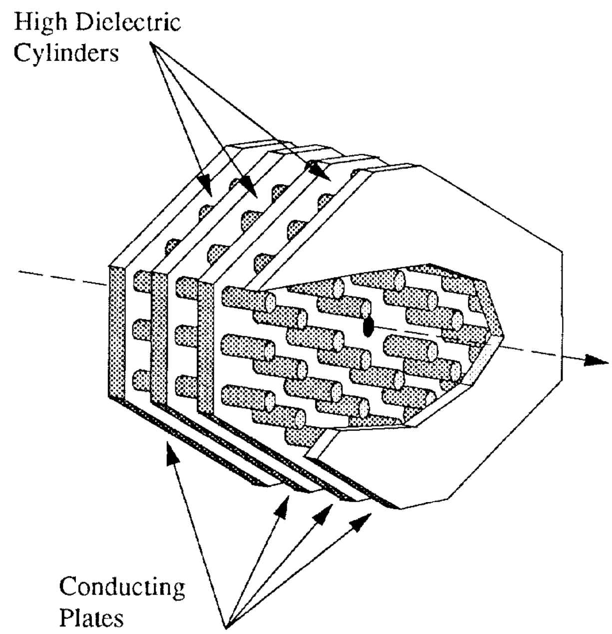

Figure 1. A schematic view of the proposed 2π accelerator unit. In this example the unit consists of three triangular photonic lattices, separated by superconducting sheets. Each of the lattices has a cylinder removed to allow the formation of a defect mode with an electric field maximum in the center. Holes drilled through the conducting plates would allow a particle beam to be accelerated through the unit.

III. NUMERICAL SIMULATIONS AND ILLUSTRATIVE EXPERIMENTS

As a first approach to designing a potential PBG accelerator cavity, we need to determine whether the structure has complete photonic band gaps. This information can be found by computing what has come to be termed the photonic band structure. Since we are concerned with 2-D configurations, and we wish to accelerate particles from one plate to the next, we restrict our attention to modes in which the electric fields are polarized along the cylinder axes (TM modes). Thus, the wave equation we solve reduces to:

$$\nabla^2 E(\bar{x}) = -\frac{\omega^2}{c^2}\varepsilon(\bar{x})E(\bar{x}) \quad (1)$$

where the dielectric function satisfies

$$\varepsilon(\bar{x}+\bar{d}) = \varepsilon(\bar{x}) \quad (2)$$

The vector $\bar{d}$ is any primitive lattice vector. The methods for solving Eq. (1) are well-known [1-5]; the solutions are Bloch waves, which have the form

$$E(\bar{x}) = u_{\bar{k}}(\bar{x})e^{i\bar{k}\cdot\bar{x}} \quad (3)$$

where $u_{\bar{k}}(\bar{x}+\bar{d}) = u_{\bar{k}}(\bar{x})$. The vector $\bar{k}$ indexes solutions, and is referred to as the wave vector. For each value of $\bar{k}$ there is a discrete set of solutions with a discrete set of frequencies $\{\omega_n(\bar{k})\}$. The solutions for a given n are continuous as a function of the wave vector, forming sheets in reciprocal space. These sheets are known as bands, and n, the band index, refers to a given sheet. The bands, due the periodicity of the lattice in coordinate space, are also periodic in reciprocal space; it is thus sufficient to view the solutions in a restricted region of reciprocal space called the Brillouin Zone (BZ). Because the real lattice has fourfold rotational and reflection symmetries, only the solutions for a single octant of the square BZ are unique. A plot of the mode frequencies $\{\omega_n(\bar{k})\}$ corresponding to lattice vectors along the boundary of the BZ comprises the photonic band structure. The Brillouin Zones and band structures for lattices with other symmetries can be similarly defined.

When we calculate the band structure for a given lattice configuration, we expect to learn at what frequencies complete band gaps occur, and how large the band gaps are. An example of a photonic band structure calculation is shown in Figure 2, where we find three band gaps in the spectrum within the lowest fourteen bands. We and others [5,6] have systematically studied the behavior of band gaps for 2-D lattices over a large variation of dielectric constants and filling factors, and for a variety of lattice types. The lattice configurations include the square and triangular lattices with dielectric cylinders at the lattice sites, as well as the inverse cases of dielectric hosts with holes (ε=1) at the lattice sites.

Experimental confirmation of photonic band gaps can be readily obtained through transmission experiments. As discussed above, waves incident on a photonic lattice with frequencies corresponding to the band gap region of the lattice, decay into the lattice with exponential dependence. Thus, band gaps in the band structure will be manifest as regions of attenuation in a transmission measurement. A schematic diagram of our test apparatus is found in reference [3]. We are able to make simple transmission measurements with the equipment, as well as make measurements of the electric energy density of standing wave modes. In Figure 3 we present the transmission spectrum through a square lattice

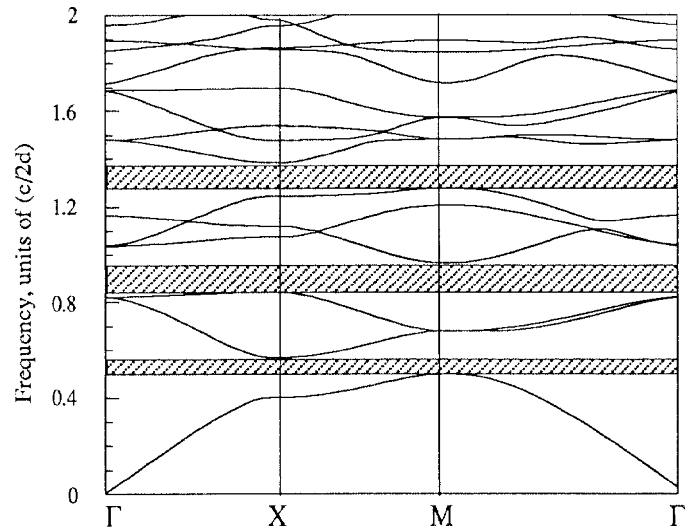

Figure 2. The photonic bandstructure for the square lattice of cylinders with dielectric constant ε=9. The lattice cylinder diameter is 1 cm, and the lattice spacing is 1.27 cm. The three band gaps are indicated by the shaded area.

along the (10) direction. The sample was a 7 X 19 array of cylinders, with dielectric constant ε=9, set in a precision drilled Styrofoam template. Microwave absorber was placed surrounding the scattering region, which minimized reflection back into the lattice. Note the sharp attenuation at frequencies corresponding to the gap region in the calculated bandstructure of Figure 2. The transmittance is reduced by over 40 dB, and has reached the noise floor of the microwave sweeper (a Hewlett-Packard 8756A scalar network analyzer). The configuration used for Figure 3 also had one central cylinder removed. Note the appearance of the sharp resonance in the gap, corresponding to the resonant defect mode.

In Figure 4 we present a detailed mapping of the electric energy density (εE^2) as a function of the distance around a removed cylinder from a square lattice. The mode corresponds to a resonance similar to the one shown in Figure 3, except the lattice spacing is 1.33 cm. The defect mode shown is a monopole mode (antinode in the center), has the four fold symmetry of the lattice, and is well localized. The fields decay most gradually along the (10) and related symmetry directions. A plot along a cut in these directions logarithmically revealed the 1/e decay length to be approximately 0.6 lattice constants. We will compare this value with numerical simulations in Section IV.

IV COMPLEX BANDSTRUCTURE

In addition to the Bloch type of solutions with real wave vector $\bar{k}$, the wave equation also has solutions with real frequency corresponding to complex values of $\bar{k}$. These solutions will exist only when the periodicity of the lattice is broken, for example at a surface or defect. The analytic

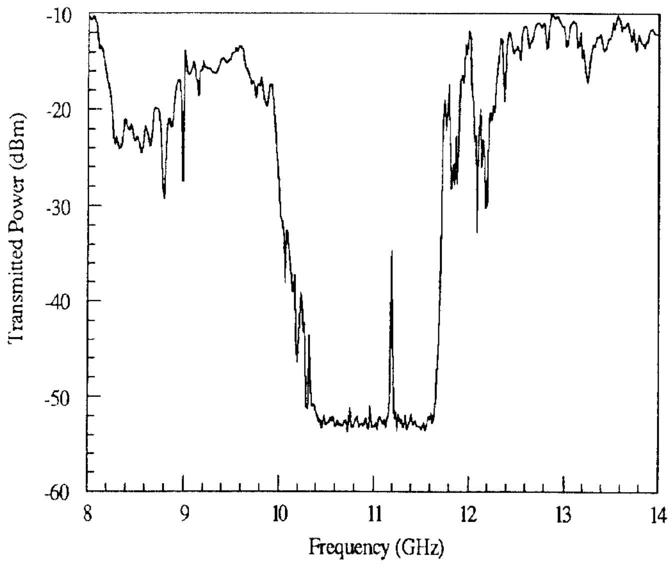

Figure 3. Transmittance vs. frequency of microwaves through a square lattice of 7 x 19 cylinders. The cylinders have radius a=1 cm, and lattice constant d=1.27 cm (0.5"). The dielectric constant of the cylinders is $\varepsilon=9$. The gap which is shown corresponds to the second photonic band gap in Figure 2. The sharp spike in the band gap occurs only after a single central cylinder is removed and is the resonance of interest.

properties of the solutions to the Scrodinger equation with a periodic potential have been rather thoroughly analyzed [7]. For illustration we restrict the propagation vector to lie along the (10) direction of the lattice. In Figure 5 we present the calculated complex bandstructure for the (10) direction of a square lattice. The dimensions of the lattice are the same as the lattice used to make the defect mode in Figure 4. Real frequency lines with complex k must either form loops

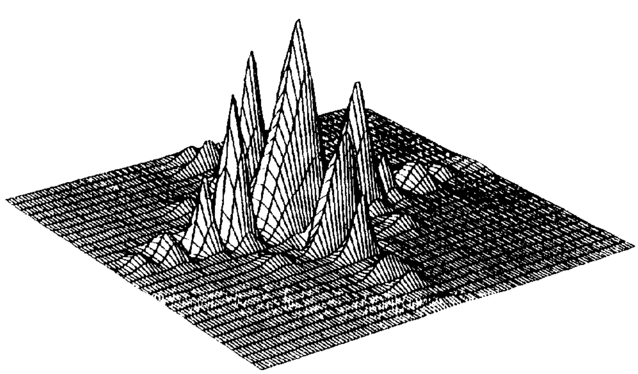

Figure 4. A spatial map of the electric energy density of a defect mode corresponding to the resonance shown in the band gap in Figure 3. All parameters of the lattice are the same as those for Figure 3, except for the lattice constant which in this case was d = 1.33 cm.

connecting one band to another, or must come up from minus infinity and connect to a band. The trajectory of any given real frequency line must increase monotonically with frequency; the collection of these real frequency lines form paths which wind their way through the bandstructure. If we select any given frequency, we find each path gives us no more than one solution at that frequency.

The complex bandstructure provides us with relatively quick insight which can be useful in many instances. As an example, when we consider a lattice geometry for possible use as a PBG structure, we can find from the complex bandstructure not only the size of the gaps, but also the attenuation length of the given gap. The longest attenuation length available to the system will dictate the minimum lateral dimension of the structure; parameters can thus be roughly optimized to find a smallest structure. Note that in the second gap there are three real frequency paths shown with imaginary k (there are, of course, infinitely many solutions with imaginary k at any frequency); however, the smallest imaginary k has a mid-gap value of 0.83, corresponding to a field decay length of $\lambda=1.21$ lattice constants. This is in good agreement with the power decay length of 0.6 lattice constant along the (10) direction of the defect mode, measured from the experimental data above. While the complex bandstructure is important for insight and for certain calculations such as surface modes and transmission spectrums, it is necessary to perform a complete calculation to verify the existence of a desired defect mode, and then to evaluate near field shape, symmetry, etc., of the

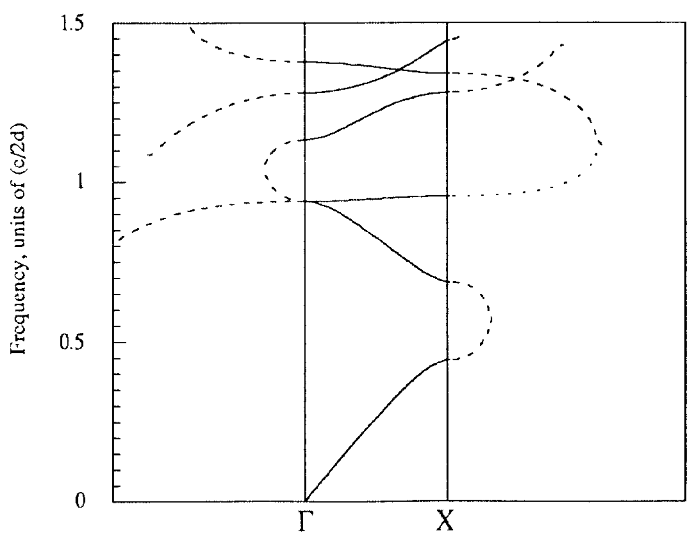

Figure 5. Complex band structure for the (10) direction. The parameters for this calculation match those used for the lattice used in Figure 4. The solid lines between Γ and X correspond to pure traveling waves. The dotted lines on either side of that region correspond to the imaginary (i.e., attenuative) part of the complex wave vector.

mode. Calculations such as these have been successfully carried out with very good accuracy for both two- and three-dimensional structures [8].

V. DISCUSSION AND FUTURE EXPERIMENTS

Extensive numerical simulation studies are required to design an optimum PBG resonant structure. An important criterion will be to find a structure that has no resonant higher order modes. As another example, we find that the exponential decay of the fields for a triangular lattice can be ~30% faster than that of the square lattice with similar parameters. This in turn means that one can have a smaller physical structure for a given design value of unloaded Q (The periphery of a PBG resonator has absorber so as to reduce the Q of all other frequencies, and this in turn means that the unloaded Q will be set by the net Poynting energy flow to the periphery of the finite PBG lattice). Using superconducting niobium plates and high purity sapphire for the dielectric cylinders, we can expect to achieve intrinsic unloaded Q values of $>10^9$. While such high unloaded Q values are required for the regions cooled to liquid helium temperature, we note that the loaded Q for other superconducting designs is typically only $\sim 10^6$. For the structures discussed, we can expect to reach such Q values with a radius of <10 lattice constants.

Our immediate experimental program is to determine several key properties via measurements in a cryogenic apparatus. These include the demonstration of unloaded Q $>10^9$, operation at high gradients (>10 MV/m), and an investigation of the frequency stability, tunability, intercavity coupling, and external power coupling. One may expect particular difficulties due to dielectric breakdown at high field strengths. Once the cryogenic tests are successful, we plan to place a modest multi-cavity unit on a beam line and determine for the presently available superconducting cavities. However, we feel this effort is particularly worthwhile because the properties of PBG structures are so very different the limitations set by multipaction, charging, etc. Clearly there are formidable problems to be investigated and solved in order to make PBG resonant structures a practical replacement than those of the usual resonant cavities. We suggest that other interesting applications may arise, particularly as the special features of PBG structures and resonators become fully appreciated.

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