Measuring Emittance Using Beam Position Monitors*

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Abstract

The Los Alamos Advanced Free-Electron Laser uses a high charge (greater than 1 nC), low-emittance (normalized rms emittance less than 5π mm mrad), photoinjector-driven accelerator. The high brightness achieved is due, in large part, to the rapid acceleration of the electrons to relativistic velocities. As a result, the beam does not have time to thermalize its distribution, and its transverse profile is, in general, non-Gaussian. This, coupled with the very-high brightness, makes it difficult to measure the transverse emittance. Techniques used must be able to withstand the rigors of very-intense electron beams and not be reliant on Gaussian assumptions. Beam position monitors are ideal for this. They are not susceptible to beam damage, and it has been shown previously that they can be used to measure the transverse emittance of a beam with a Gaussian profile [1]. However, this Gaussian restriction is not necessary, and, in fact, a transverse emittance measurement using beam position monitors is independent of the beam's distribution.

I. INTRODUCTION

The Advanced Free-Electron Laser (AFEL) is a compact, computer-controlled FEL that is intended as a coherent light source, tunable from the infrared to the visible. The accelerator is driven by a photoinjector and produces a high-brightness, 20-MeV beam. In order for it to achieve lasing in the visible regime, the AFEL relies heavily on beam quality, i.e., low emittance, and on the high peak currents that are obtainable with a photoinjector.

Measuring the second-moment properties of electron beams from photoinjectors is not a trivial proposition [2]. At the present, the AFEL uses single-quadrupole scans on an intercepting screen to measure the emittance. However, simulations indicate that this method underestimates the rms emittance by a factor of about four. In fact, this method seems to measure the instantaneous emittance at the center of the beam [2]. While this number is more important to the performance of the laser, the rms quantity is more important for beam transport through the beamline.

In this paper, we will discuss the possibility of using beam position monitors (BPMs) to measure the rms emittance of the AFEL electron beam. What we will show is that the numbers produced by this technique are independent of the beam distribution. Thus, the measurement gives true rms values whose meanings are clear.

II. IMAGE CHARGE DISTRIBUTION

Consider an electron beam pulse traveling down a beam pipe. If, in its rest frame, this pulse has some distribution, $I(\rho,\phi,z)$, that is normalized to the total charge, q_{tot}, then the image charge distribution on the beam pipe is given by

$$\sigma(\phi,z,t) = -\frac{\gamma^2}{\pi a^2}\int_V I(\rho',\phi',\gamma(z'-\beta ct)) \otimes \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} a_n \cos[n(\phi-\phi')] \otimes \frac{J_n\left(x_{nm}\frac{\rho'}{a}\right)}{J_{n+1}(x_{nm})} e^{-\frac{x_{nm}}{a}\gamma|z-z'|} d^3x',\ a_n = \begin{cases}\frac{1}{2}, & n=0\\ 1, & n\neq 0\end{cases},$$

where a is the radius of the beam pipe, β and γ are the usual relativistic parameters, the J_ns are Bessel functions, the x_{nm}s are Bessel function zeros, and the volume of integration is the volume that contains the electron pulse. As γ becomes large, this simplifies to the expression,

$$\sigma(\phi,z,t) = -\frac{1}{2\pi a}\iint_{\substack{\text{area of}\\ \text{pipe}}} I'(\rho',\phi',z-\beta ct) \otimes \left\{1+2\sum_{n=1}^{\infty}\left(\frac{\rho'}{a}\right)^n \cos[n(\phi-\phi')]\right\} da',$$

where $I'(\rho',\phi',z-\beta ct)$ is the pulse distribution in the lab frame, also normalized to q_{tot}[3].

III. BPM SIGNAL

A beam position monitor consists of four electrodes placed around the beam pipe at 90^0 intervals, as shown in Fig. 1. They couple to the beam through the image charge, or wall current, produced on the beam pipe by the electron beam. Their signals can be expanded in powers of $1/a$. In general, the terms of this multipole expansion are dependent on the distribution of the electron beam. However, what we will show is that the terms important for measuring the emittance are distribution independent.

For the case where the electrodes have no angular width and the electron beam distribution is Gaussian, the first four terms of the multipole expansion have previously been determined [1]. It is a simple matter to extend this result to the case of electrodes with angular width (Fig. 1), which we have done. Table 1 gives the first three terms of the multipole expansion in this case, normalized to $q_{tot}/2\pi a$. From these the quantity $\sigma_x^2-\sigma_y^2$ can be determined, and that determination leads to a method of measuring the emittance [1].

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Table 1: Multipole terms for Gaussian beam, normalized to $q_{tot}/2\pi a$

Electrode	R=Right $(\phi=0)$	L=Left $(\phi=\pi)$	T=Top $\left(\phi=\frac{\pi}{2}\right)$	B=Bottom $\left(\phi=\frac{3\pi}{2}\right)$
Monopole	2α	2α	2α	2α
Dipole	$4\sin\alpha\frac{\bar{x}}{a}$	$-4\sin\alpha\frac{\bar{x}}{a}$	$4\sin\alpha\frac{\bar{y}}{a}$	$-4\sin\alpha\frac{\bar{y}}{a}$
Quadrupole	$2\sin 2\alpha\left[\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right]$	$2\sin 2\alpha\left[\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right]$	$-2\sin 2\alpha\left[\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right]$	$-2\sin 2\alpha\left[\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right]$

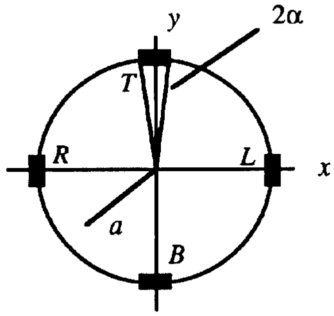

Fig. 1: BPM electrode positions.

Now consider the case of a general beam distribution. A BPM at some position z_0 along our beam pipe has electrodes placed as shown in Fig. 1, with length $2\Delta z$. Then the image charge on an electrode at angular position, ϕ, is

$$q(\phi,t)=-\frac{1}{2\pi a}\int_{\phi-\alpha}^{\phi+\alpha}d\phi\int_{z_0-\Delta z}^{z_0+\Delta z}dz\iint_{\substack{area\ of\\ pipe}}I'(\rho',\phi',z-\beta ct)$$
$$\otimes\left\{1+2\sum_{n=1}^{\infty}\left(\frac{\rho'}{a}\right)^n\cos[n(\phi-\phi')]\right\}da'$$
$$q(\phi,t)=-\frac{1}{2\pi a}\int_{z_0-\Delta z}^{z_0+\Delta z}dz\iint_{\substack{area\ of\\ pipe}}I'(\rho',\phi',z-\beta ct)$$
$$\otimes\left\{2\alpha+4\sum_{n=1}^{\infty}\left(\frac{\rho'}{a}\right)^n\frac{\sin n\alpha}{n}\cos[n(\phi-\phi')]\right\}\rho'd\rho'd\phi'.$$

The integration over z is complicated by two things: the beam bunch is moving, so that the integration depends on t, and there is the possibility that the electrode length may be shorter than the beam pulse length. For simplicity, we will fix t so that the center of the charge, βct, corresponds to the center of the BPM, z_0. This is the point at which the charge peaks. Also, we will assume that the electrode length is large enough relative to the bunch length that we can take $\Delta z\to\infty$. (For the AFEL, the electron pulses are about 3 mm long, so even 1-cm-long electrodes are adequate for this assumption to be good.)

The integration over ρ' and ϕ' can be changed to an integration over x' and y'. Since we have assumed no special distribution, it is perfectly acceptable to make the substitution

$$I'(\rho',\phi',z-\beta ct)\to I'(x'-\bar{x},y'-\bar{y},z-z_0).$$

This substitution indicates that we are now writing the beam distribution in Cartesian coordinates with the beam's center at $(\bar{x},\bar{y},z_0)$. Then we can use the expansion

$$2\alpha+4\sum_{n=1}^{\infty}\left(\frac{\rho'}{a}\right)^n\frac{\sin n\alpha}{n}\cos[n(\phi-\phi')]=2\alpha$$
$$+4\frac{\sin\alpha}{a}(x'\cos\phi+y'\sin\phi)$$
$$+2\frac{\sin 2\alpha}{a^2}\left[\left(x'^2-y'^2\right)\cos 2\phi+2x'y'\sin 2\phi\right]$$
$$+\frac{4}{3}\frac{\sin 3\alpha}{\alpha}\left[\left(x'^3-3x'y'^2\right)\cos 3\phi+\left(3x'^2y'-y'^3\right)\sin 3\phi\right]$$
$$+\text{higher order terms}$$

to convert the rest of the integral to Cartesian coordinates [3].

The beam distribution is always zero outside the pipe; therefore, when integrating over x' and y', we can make the limits of integration $+\infty$ and $-\infty$. Then the peak image charge on a BPM electrode is

$$q_{peak}(\phi)=-\frac{1}{2\pi a}\int_{z_0-\infty}^{z_0+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}I'(x-\bar{x},y-\bar{y},z-z_0)$$
$$\left\{2\alpha++4\frac{\sin\alpha}{a}(x\cos\phi+y\sin\phi)\right.$$
$$+2\frac{\sin 2\alpha}{a^2}\left[\left(x^2-y^2\right)\cos 2\phi+2xy\sin 2\phi\right]$$
$$+\frac{4}{3}\frac{\sin 3\alpha}{\alpha}\left[\left(x^3-3xy^2\right)\cos 3\phi+\left(3x^2y-y^3\right)\sin 3\phi\right]$$
$$\left.+\text{higher order terms}\right\}dxdydz,$$

where we have dropped the primes for convenience. Making use of the following integrals:

$$\int_{z_0-\infty}^{z_0+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}I'(x-\bar{x},y-\bar{y},z-z_0)\,dxdydz=q_{tot},$$

$$\int_{z_0-\infty}^{z_0+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xI'(x-\bar{x},y-\bar{y},z-z_0)\,dxdydz = q_{tot}\bar{x},$$

$$\int_{z_0-\infty}^{z_0+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} yI'(x-\bar{x},y-\bar{y},z-z_0)\,dxdydz = q_{tot}\bar{y},$$

$$\int_{z_0-\infty}^{z_0+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^2I'(x-\bar{x},y-\bar{y},z-z_0)\,dxdydz$$

$$= \int_{z_0-\infty}^{z_0+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[(x-\bar{x})^2 + 2x\bar{x} - \bar{x}^2\right]$$

$$\otimes I'(x-\bar{x},y-\bar{y},z-z_0)\,dxdydz$$

$$= q_{tot}\left[\langle(x-\bar{x})\rangle^2 + 2\bar{x}^2 - \bar{x}^2\right]$$

$$= q_{tot}\left(\sigma_x^2 + \bar{x}^2\right), \ (\sigma_x \text{ is the rms halfwidth in x})$$

$$\int_{z_0-\infty}^{z_0+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} y^2I'(x-\bar{x},y-\bar{y},z-z_0)\,dxdydz$$

$$= q_{tot}\left(\sigma_y^2 + \bar{y}^2\right),$$

$q_{peak}(\phi)$ becomes

$$q_{peak}(\phi) = -\frac{q_{tot}}{2\pi a}\left\{ 2\alpha + 4\frac{\sin\alpha}{a}(\bar{x}\cos\phi + \bar{y}\sin\phi) \right.$$
$$+2\frac{\sin 2\alpha}{a^2}\left\{\left[\left(\sigma_x^2-\sigma_y^2\right)-\left(\bar{x}^2-\bar{y}^2\right)\right]\cos 2\phi\right.$$
$$\left.+2\langle xy\rangle\sin 2\phi\right\} + \frac{4}{3}\frac{\sin 3\alpha}{a^3}\left[\left(\langle x^3\rangle - 3\langle x^2y\rangle\right)\cos 3\phi\right.$$
$$\left.\left.+\left(3\langle x^2y\rangle - \langle y^3\rangle\right)\sin 3\phi\right] + \text{higher order terms}\right\}.$$

The angled brackets indicate an rms average. Substituting in $\phi = 0, \pi/2, \pi$, and $3\pi/2$, to get the peak charge for each electrode, one finds that the first three terms in the multipole expansion are identical to those for the Gaussian beam in Table 1.

IV. EMITTANCE MEASUREMENT

Measuring emittance using BPMs is difficult. Most often, it is the lack of an adequate signal to noise ratio that is the main cause for concern. However, we have discovered a further problem that we believe is associated with the very-short beamline of the AFEL and the nature of the measurement.

The matrix equation $\vec{Q} = \bar{\bar{M}} \cdot \vec{\sigma}_1$, where

$$\bar{\bar{M}} \equiv \begin{bmatrix} (R_{11})_1^2 & (2R_{11}R_{12})_1 & (R_{12})_1^2 & -(R_{33})_1^2 & -(2R_{33}R_{34})_1 & -(R_{34})_1^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (R_{11})_n^2 & (2R_{11}R_{12})_n & (R_{12})_n^2 & -(R_{33})_n^2 & -(2R_{33}R_{34})_n & -(R_{34})_n^2 \end{bmatrix},$$

is what we are setting up when we use BPMs to measure the emittance [1]. The elements of the vector $\vec{Q}$ are the measurements, and the vector $\vec{\sigma}_1$ is what we wish to determine [1]. The R_{ij}s are the elements of the transfer matrix between the point where you want to know the emittance and the BPM that is making the measurement.

On the AFEL, the distance from the end of the linac to the BPM that we wish to use for our emittance measurement is about 1.5 m, with four quads along the way. Our first inclination was to vary one of those quads to generate our measurements. However, the matrix produced by doing this proved to be highly unstable. It had a condition number of about 10^4, which means that any error in our measurements could be amplified by that factor when we solved our matrix equation. What we ended up having to do was use two or more quads in concert, so that our transfer matrix acted as a "filter." By setting the quads to appropriate values, we can make all but one of the numbers in a $\bar{\bar{M}}$ matrix row zero, or very small. This allows most of the terms in the vector $\vec{\sigma}_1$ to dominate a number of measurements. As a result, we can reduce the condition number of $\bar{\bar{M}}$ so that it is close to unity, which is as small as it can be.

Why do poorly conditioned matrices arise? For the AFEL, with its short beamline, that situation is partly a resolution problem. Mostly, though, it comes from the fact that not all the elements in one row of the $\bar{\bar{M}}$ matrix are independent. A long beamline will help, but it is not necessarily the answer. One must be careful when making measurements. In general, it has been our experience that adjusting quads at random produces very poorly conditioned matrices, even when more than one quad is turned on at the same time.

V. CONCLUSION

A photoinjector-driven accelerator presents unique challenges for emittance measurements. By using BPMs for this purpose, we circumvent the need for knowledge of the actual distribution. However, in order for this technique to work, we still need to improve the signal to noise ratio of the BPMs, and this is a problem we have not yet addressed.

VI. REFERENCES

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