Development on Multistrip Monitor for Nonintercepting Measurement of Beam Geometric Moments

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Abstract

The wall current distribution created by a charged particle passing through a beam pipe gives information on the beam profile moments [1]. A coordinate translation has been made to obtain beam centroid moments. The beam current, position and profile moments (in terms of $\sum_i \rho_i^n \cos n(\theta - \phi_i)$) can be obtained by performing a spatial Fast Fourier Transform on the wall current distribution of a cylindrical multistrip monitor [2].

Computer simulations of the measurement and their results are presented. The accuracy of the measurement is predicted to be better than 0.1%.

I. INTRODUCTION

Beam profile moments provide beam profile information and are very useful for beam emittance [1] and instability studies. A multistrip monitor provides a non-intercepting tool for that purpose. The derived parameters are moments, which not only depend on beam intensity, but also on beam shape and size.

Moment descriptors have various forms. Some examples of moments include geometric, complex, radial and orthogonal moments. For the particular problem studied here, which has rotational symmetries, radial moments are adopted in this paper; however, we shall introduce the subject with the more familiar geometric moments.

The definition of the two dimensional geometric $(p+q)$th order moments of a density distribution function $\rho(x,y)$ in plane x,y is defined in terms of Riemann integrals [3] as:

$$M_{pq} = \int\int x^p y^q \rho(x,y) dx \ dy, \qquad p,q = 0,1,2,... \tag{1}$$

It is assumed that $\rho(x,y)$ is a piecewise continuous, bounded function, and that it can have non-zero values only in a finite part of the x,y plane; then moments of all orders exist and the uniqueness theorem can be proved.

Uniqueness Theorem: The double moment sequence M_{pq} is uniquely determined by $\rho(x,y)$; and conversely, $\rho(x,y)$ is uniquely determined by the set M_{pq}.

II. MOMENT MATCHING APPROACH

We could always obtain a continuous function $g(x,y)$, whose moments exactly match those of $f(x,y)$ up to a given order N_{max}, assuming that we have the set of moments M_{pq} [4]. The more higher order moments we have, the more accurate and closer to the original function $f(x,y)$ the $g(x,y)$ must be.

$$\begin{aligned} g(x,y) &= g_{00} + g_{10}x + g_{01}y + g_{20}x^2 + g_{11}xy + g_{02}y^2 \\ &+ g_{30}x^3 + g_{21}x^2y + g_{12}xy^2 + g_{03}y^3 + ... \end{aligned} \tag{2}$$

The coefficients g_{pq} should be determined so that the moments of $g(x,y)$ match the moments, M_{pq}, of $f(x,y)$, according to the following expression:

$$\int_{-1}^{+1}\int_{-1}^{+1} x^p y^q g(x,y) dx \, dy = M_{pq} \,. \tag{3}$$

So, if we have the whole set of the second or third degree moments, we could reconstruct the beam up to the accuracy of that order [4].

There are some other orthogonal polynomials that can be used for image reconstruction, which are more convenient than the moment matching approach [4].

III. WALL-CURRENT DISTRIBUTION AND MOMENTS

When a charged particle beam passes through a conducting pipe, an image current will be produced on the wall. If the particle is an ultra-relativistic one, or if the beam is very long, then the electrical field will be in a plane which is perpendicular to the direction of motion. The wall current distribution in the cross-sectional plane is a two-dimensional problem, and can be described with the field produced by a line current. The wall-current distribution is not only determined by the beam position, but also by the density distribution of the beam and the total beam current.

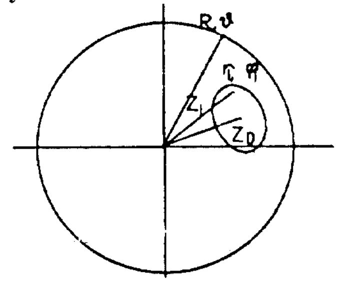

Fig.1: Wall current due to a line current in a conducting cylinder and coordinate translation.

For a delta function line current ρ_i, at point (r_i, ϕ_i), the image current density, J, on a conducting cylinder of radius R at point (R,θ)(see Fig.1) is given by:

$$J_{image}(r,\phi,R,\theta) = \frac{\rho_i}{2\pi R}\frac{(R^2 - r_i^2)}{(R^2 + r_i^2 - 2Rr_i\cos(\theta - \phi_i))} \tag{4}$$

Expanding in powers of r_i/R gives:

$$J_{image}(r,\phi,R,\theta) = \frac{\rho_i}{2\pi R}\left[1 + 2\sum_{n=1}^{\infty}(\frac{r_i}{R})^n \cos n(\theta - \phi_i)\right] . \tag{5}$$

The above expansion represents a series of azimuthal components. The waveiength of the dipole component(n=1) is the circumference of the beam pipe $2\pi R$. The induced current density at one observed location on the wall will be the integral of the above equation over all the beam particles:

$$\begin{aligned} I_{\text{image}}(r,\phi,a,\theta) &= \frac{1}{2\pi R}\left[\sum_i \rho_i + 2\left(\sum_{n=1}^{\infty}\frac{\cos n\theta}{R^n}\right.\right. \\ &\times \sum_i \rho_i r_i^n \cos n\phi_i + \left.\left.\sum_{n=1}^{\infty}\frac{\sin n\theta}{R^n}\sum_i \rho_i r_i^n \sin n\phi_i\right)\right] . \end{aligned} \tag{6}$$

The above equation shows that the wall-current density created by a beam passing through the pipe is actually the sum of moments closely related in form to the radial moments with both degree and angular dependence of n. The FFT components of each order give the moments of that order in the forms of $\sum_i \rho_i^n \cos n\phi_i$ and $\sum_i \rho_i^n \sin n\phi_i$.

But the moments we obtained here are the moments around the center of the beam pipe(P_{nn}). Only the moments around the beam centroid have the property of invariance, and can be used to describe the beam.

IV. CENTROID MOMENTS

A coordinate translation has to be done to obtain the centroid moments,Fig.1. Let us use the complex form to express the moments. From the FFT, the moments we obtained are: $P_0 = \sum_i \rho_i$

$$P_{nn} = \sum_i \rho_i r_i^n \cos n\phi_i + i \sum_i \rho_i r_i^n \sin n\phi_i = \sum_i \rho_i Z_i^n \quad (7)$$

where $Z_i = x_i + iy_i$ is the distance from the particle to the center of the pipe. $Z_0 = x_0 + iy_0$ is the distance from the centroid to the center of the pipe. So, the distance from the particle to the centroid is: $Z_i - Z_0$.

The centroid moments M_{nn} are:

$$M_0 = P_0 = \sum_i \rho_i \quad (8)$$

$$\begin{aligned} M_{nn} &= \sum_i \rho_i (Z_i - Z_0)^n = \sum_i \rho_i \sum_{k=0}^{n} \frac{n!(-1)^{n-k}}{k!(n-k)!} Z_i^k Z_0^{n-k} \\ &= (-1)^n Z_0^n P_0 + \sum_{k=0}^{n-1} \frac{n!(-1)^{n-k}}{k!(n-k)!} Z_0^{n-k} P_{kk} + P_{nn}. \end{aligned}$$

V. THE BASIC BEAM INFORMATION FROM A CYLINDRICAL MULTISTRIP MONITOR

The wal-current distribution on a cylindrical multistrip monitor immediately yields basic beam parameters such as current, position, orientation and information related to size.

A. Phase angle of each higher order moment

The FFT of the wall current distribution only gives the value of $Re(M_{nn}), Im(M_{nn})$ and ϕ_n directly. Here, $Re(M_{nn})$ represents the real part of the second order moment, and $Im(M_{nn})$ represents the imaginary part.

Although the azimuthal moments are the sum of the density value at each point multiplied by its distance r_i^n and $\cos n\phi_i$ or $\sin n\phi_i$, where ϕ_i is the phase angle of the particle at that place, the collective effect of the sum is that the moment has a phase angle ϕ_n, which represents the orientation of the image component of that order. For example, for the quadrupole moment, ϕ_2 represents the orientation of the best-fit beam ellipse, and for the sextupole, ϕ_3 represents the orientation of the triangle, etc. So, from the FFT, $n\phi_n$ can be determined by $tan^{-1}(Im(M_{nn})/Rm(M_{nn}))$, and ϕ_n as well. In the following, we use ϕ_n to represent the phase angle in each FFT component of the corresponding order and use M_{nn} to represent the centroid moments.

B. DC component

The first component, i.e. zero order of the FFT, is a constant value, which represents the dc current of the beam with a factor of πR. The dc current is also called the zero degree moment.

C. Position

The first order component of the FFT is $\frac{1}{2\pi R^2} \sum_i \rho_i r_i \cos(\theta - \phi_i)$, which gives the first degree moment around the center of the pipe as $Re(M_{11}) = \sum_i \rho_i r_i \cos\phi_i, Im(M_{11}) = \sum_i \rho_i r_i \sin\phi_i$. ϕ_1 represents the phase angle in the FFT data, which is the total effect of the position shift of all the particles.

Therefore the position of the centroid in a polar coordinate is $r = (\sqrt(Re(M_{11})^2 + Im(M_{11})^2))/M_0$, and $\phi_1 = tan^-1(Im(M_{11})/Re(M_{11}))$, or in the xy plane, $x = r\cos\phi_1$, $y = r\sin\phi_1$.

D. Quadrupole, sextupole and higher order moments

The second order component from FFT provides information about the best-fit ellipse. The third order component gives the sextupole moment; the fourth order gives the octupole moment. The number of moments that can be obtained depends on how many strips the monitor has. The moments are all in the form of:

$$Re(M_{nn}) = \sum_i \rho_i r_i^n \cos n\phi_i \quad Im(M_{nn}) = \sum_i \rho_i r_i^n \sin n\phi_i \quad (9)$$

The coefficients of the second order component from the FFT give the second degree moments, i.e. quadrupole moments and phase. It also can be written as:

$$\begin{aligned} Re(M_{22}) &= \sum_i \rho_i r_i^2 (\cos^2\phi_i - \sin^2\phi_i) = \sum_i \rho_i x_i^2 - \sum_i \rho_i y_i^2 \\ Im(M_{22}) &= 2\sum_i \rho_i x_i y_i \quad \phi_2 = \frac{1}{2} tan^{-1} \frac{Im(M_{22})}{Re(M_{22})} \end{aligned} \quad (10)$$

For a uniformly distributed elliptical beam whose principal axes are the same as x and y, the following formulae are valid:

$$\int\int \rho x^2 dx\, dy = \frac{1}{4}\pi\rho a b^3 \quad \int\int \rho y^2 dx\, dy = \frac{1}{4}\pi\rho a^3 b \quad (11)$$

where $2a$ and $2b$ are the long and short axes of the ellipse respectively, and $\int\int \rho xy dx\, dy = 0$. So,

$$\frac{M_{22}}{M_0} = \frac{1}{4}(a^2 - b^2) \quad \frac{M_{20}}{M_0} = \frac{1}{4}(a^2 + b^2) \quad (12)$$

where M_{20} is the second degree moment with 0 angular dependence, i.e. $\sum_i \rho_i r_i^2$. The half-axis a and b can be determined therefore.

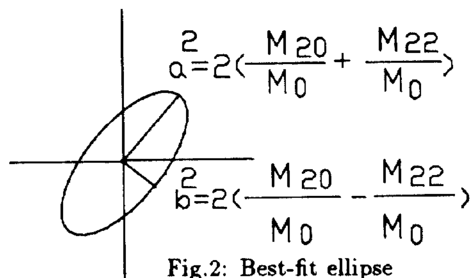

Fig.2: Best-fit ellipse

E. Width signal

Although we have obtained M_{22} and the phase of the beam, M_{20}, which contains $a^2 + b^2$, is missing from our FFT data. A Monte Carlo computer simulation has been done with elliptical beams of uniform distribution. All had the same $a^2 - b^2 = 0.75$, but a/b varied from 1.5 to 2, 3 and 4. The wall-current distribution has been calculated for both centered and off center beams. As long as the positions and phases of the beams are the same, the difference between the corresponding wall-current distributions is of order 10^{-4}, which is at the level of statistical fluctuation of the Monte Carlo calculation. Therefore, M_{20} or a/b has to be measured with another method.

If the beam aspect ratio is known from the beam optics or some other measurement, then the best fit ellipse can be determined. With Teague's moment matching formulae or other orthogonal polynomials, one could reconstruct a beam shape to the accuracy of that order. For a thin beam, say, b is 1/3 or even 1/4 of a, $a^2 - b^2$ is approximately a^2, so the width signal a can be obtained immediately.

A round beam does not have high order moments, so the cylinder multistrip monitor will indicate this. When a round beam is in the center of the pipe, the distribution on the wall is uniform.

VI. COMPUTER SIMULATION

A Monte Carlo calculation has generated an elliptical beam, which is 0.25 cm vertically off the center of the beam pipe with a size of 2 cm × 1 cm and $\phi_2 = \pi/2$. Since the moments of each order can be calculated, the accuracy of the method can be determined. The comparison shows the accuracy can be better than 0.1%. Fig.3 is the reconstruction of the generated beam up to second degree moment.

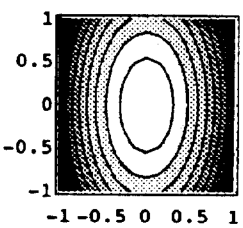

Fig.3: Reconstruction of computer generated beam.

The result of the computer simulation is listed in the following table for the first 6 orders:

Table: Monte Carlo Simulation Results

n	FFT Ampl	phase	ctr. moments	dir. calc.
1	0.2502696	1.570795	-7.96E-09	0.
2	0.2500159	3.12109	-0.18738	-0.1875
3	0.1564581	4.712302	-1.104E-04	0.
4	0.1446773	6.282875	0.070239432	0.0702232
5	0.1183089	1.570440	1.4728506E-05	0.
6	0.1101785	3.142132	-0.032884017	-0.0328926

VII. SENSITIVITY

The wall-current distribution formula [6] gives the amplitude of the second order component of the FFT as $\frac{1}{\pi R^3}\sum_i \rho_i r_i^2 \cos 2(\theta-\phi_i)$. The sensitivity to measuring this component is:

$$\frac{2}{R^2}\left[\frac{|\sum_i \rho_i r_i^2 \cos 2(\theta-\phi_i)|}{\sum_i \rho_i}\right] = \frac{(a^2-b^2)}{2R^2} \qquad (13)$$

Assume a voltage across the resistor of the monitor is induced by a sizeless beam with same intensity, position, phase and time structure. If the aspect ratio of the measured beam is 2 and the short half axis b is 1/10 of the pipe radius, then the signal we could pick up is 0.015 of this voltage value, which not only depends on beam structure but also on the frequency response of the monitor. The signal amplitude also depends on the sensed current, which is determined by the width of the strip used.

If the wall-current monitor has the frequency response needed, and assuming that the peak current is 10 mA, with 10 ohm resistors, 16 strips, we will have about 6.25 mV across each resistor. With a 40 dB amplifier, the signal will be 625 mV. With an aspect ratio of 2, $b/R = 10$, we will have a signal changing from -9mV to +9 mV around 625 mV.

An advantage of the method is that because the variation in the amplitude of the signals depends on moments, we can get a measurable signal for a beam that is small in size but high in intensity.

VIII. ERROR DUE TO NOISE

In the pipe, there may be other sources of electrical charges, such as residual gas etc., which cause errors in the signal. Considered as white noise, their effect will be uniformly distributed in the area of the whole pipe. In the round pipe, this will give an extra uniform distribution to the wall-current, which only changes the zero degree moment, not the others. When we use the zero degree moments as a dc level for normalization to get beam size information, there will be an error.

There may be some other noise sources due to grounding or RF etc, and one should try to eliminate them before doing the FFT.

IX. SOME SUGGESTIONS

The FFT analysis of the wall-current distribution has a unique advantage; it can distinguish the moments of each order very clearly. A concern with some present BPM systems is correction for non-linearities. But if the beam pipe is round, using the FFT, it is very easy to obtain the position of the beam centroid without any non-linearities. Also, the zero order coefficient of the FFT gives the dc component right away. One does not need to do BPM mapping anymore, just place a round pipe antenna in the center of the pipe to calibrate the correction coefficients for all the pick-up strips. This will save a lot of work.

To avoid noise problems, some filtering, which usual electronics have already used, may be needed before doing the FFT.

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XI. REFERENCES

[1] R.Miller et al:*Nonintercepting emittance Monitor*, Proc. XII High Energy Accelerator Conference, 1983, p602.

[2] Yan Yin:*A multistrip beam profile high order moment monitor* ,Proc. XVth Int. Conf. on High Energy accelerators, Hamburg, 1992, p260.

[3] M.K.Hu: *Visual pattern recognition by moment invariant* IRE Trans.Info.Theory,IT-8,179-187(Feb.1962).

[4] Michael Reed Teague:*Image Analysis via the general theory of moments*, J.Opt.Soc.Am.,Vol.70,No.8,920-930,Aug.1980.