FFT-Oriented Feedback

F.-J. DECKER

Stanford Linear Accelerator Center, Stanford, California 94309*

Abstract

In the SLC many feedback systems keep the beam under control. Here we concentrate on feedback systems which operate at 120 Hz or lower frequencies, so software can be used to make some decisions. The linac steering feedback uses beam position monitor data and corrects magnet settings to keep the beam orbit to the desired values (mostly flat). Looking at the Fast Fourier Transformation (FFT) of the data, a reduction in the zero and very low frequency component is observed, while on the other hand noise at higher frequencies is amplified. To improve this situation, the FFT can be used to alter the feedback so a flat spectrum can be achieved, indicating the lowest white noise level. If there is a spike at e.g. 2 or 7 Hz or another feedback is oscillating, this feedback would adjust itself till this spike is reduced to the white noise level (not beyond). Different simulation results for the frequency response are presented.

1 Introduction

At the Stanford Linear Collider (SLC) the beam position and angle is control by feedbacks at many different locations. A step function in the position of the injected beam into the linac would cause an over-correction if all the feedback would start to correct the situation in a simple way. One way is to link the feedback together and share the information in a cascaded way [1]. Here we will describe another approach. The history of the beam itself carries a lot of information, which can be used to predict the position (and angle) of the next pulse. Even if some upstream feedback is oscillating or there is a "time slot" separation (60 Hz oscillation), the fast Fourier transformation (FFT) shows which component is the highest and should be suppressed. The main idea is that the feedback should not have a fixed frequency response but vary its response due to the measured FFT spectrum. So instead of trying to reduce some zero-frequency component, which might not be there at the time, and amplifying e.g. some 2 Hz noise, the new FFT-oriented feedback would recognize the situation and would only reduce the highest peaks in the FFT spectrum.

Different insights of averaged response curves and their time behavior might be even interesting for the current cascaded feedback.

*Work supported by the Department of Energy contract DE-AC03-76SF00515.

2 General Feedback Issues

A feedback should keep a status, say the position of a beam, in a fixed state. Any deviations should be brought back as soon as possible, so the rms around this state is minimized. White noise cannot be reduced with a feedback, but any deviation like more low-frequency components can be suppressed. How a feedback works in general can be understood by looking at its frequency response.

2.1 Frequency Response

Let's take the position of beam pulses as an example. Many pulses jitter around zero or slowly drift away. The feedback tries to predict the position of the following pulse by using one or many pulse positions of the past. Applying the prediction might cause a reduction or amplification of the next position amplitude. This depends on the response type of the feedback and on the frequency. Fig. 1 shows a typical behavior of some simple feedbacks for mainly low frequency noise. Looking only at the last point, x_1 dotted, the feedback will reduce it to zero at low frequency, at $1/6 * 120\,\text{Hz} = 20\,\text{Hz}$ is the cross-over and the resulting amplitude will be twice as much at 60 Hz. For an average of the last two points, $(x_1 + x_2)/2$ dashed, the response doesn't overshoot that much, and so on till an average of the last six points, $(x_1 + x_2 + ... + x_6)/6$ dashed again, has three oscillations.

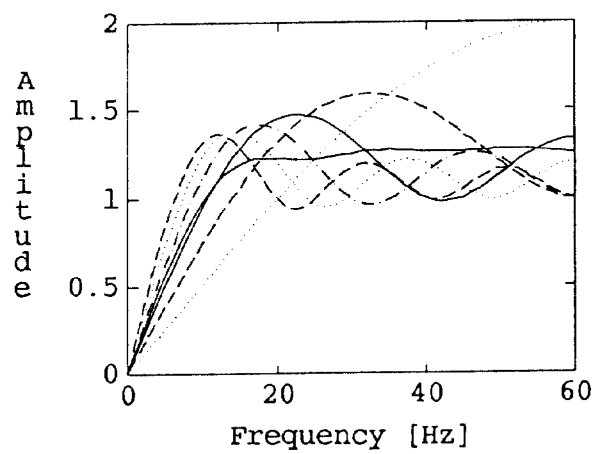

Figure 1: Feedback Response Function.

At 120 Hz all frequencies between 0 and 60 Hz can be recognized. Besides the wanted reduction at low frequency, a simple feedback looking at the last, or the last two, ... or the last six pulses gives always some oscillations. The average of these six possibilities (solid) shows a flat amplification.

An interesting behavior is an average of all these averages (solid), which is pretty flat for higher frequencies and doesn't show the overshoot directly after the first crossover. This average gives additionally a time structure, later points are weighted stronger and earlier points are damped in such a natural way that the "necessary" overshoot is flat. This feedback would reduce any drifts up to about 10 Hz and amplify white noise above that by about a factor of 1.25; (60 pulses yield a reduction up to 3 Hz and an amplification of less than 4 %).

2.2 Higher Frequencies

These feedbacks can be also used to reduce high frequency noise by changing the response function. Instead of predicting the last (x_1), the second last (x_2), ... or (x_4) top in Fig. 2, the feedback can guess minus the last, minus the second last, ... pulse (middle). By determining the frequency of the oscillation it would be possible to reduce any oscillation by just reacting on the last two points: For low frequencies up to 20 Hz x_1 is right, between 20 and 40 Hz x_2 (dashed middle) and between 40 to 60 Hz $-x_1$ will reduce all below the unit amplitude. By knowing the frequency f and taking $x_1 \cos\phi - x_2 \sin\phi$ with $\phi = 2 * \pi * f/120\,\mathrm{Hz}$ a reduction down to 0.55 or lower is achieved.

The goal is to obtain the frequency amounts by making an FFT or by other techniques and use the achieved information to adjust the response of a feedback that it can reduce all pulse to pulse variations to a flat white noise level (lowest possible amount).

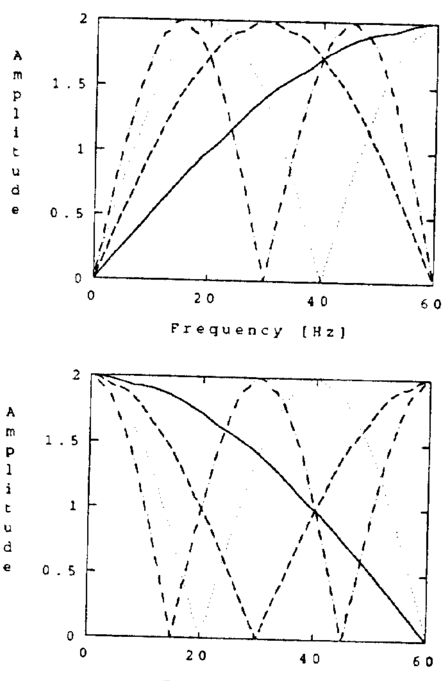

Figure 2: Other Response Functions.

Reacting on the last (x_1), second last (x_2), ... (top), or on minus the last, minus the second last, ... pulse (bottom) has different response functions.

3 Noise and Oscillations

The random white jitter noise of a beam position is difficult to improve, but any frequency component which sticks out in the Fourier transformation and is quite stable indicates some oscillation (or offset, drift for $f \approx 0\,\mathrm{Hz}$). Fig. 3 show a measurement of the beam angle at a linac feedback in the SLC.

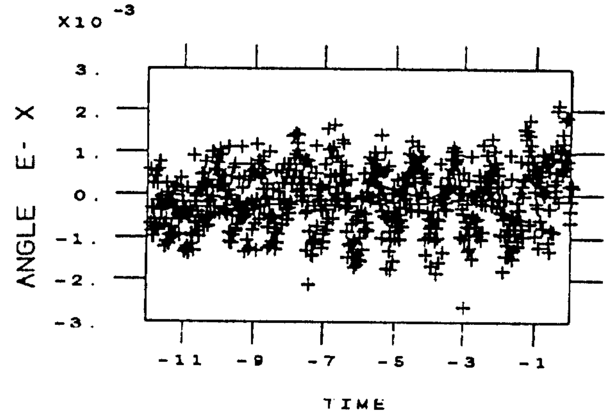

Figure 3: Oscillating Beam Angle.

Besides the noise jitter, the angle of the beam is oscillating with an amplitude of about one sigma of the noise increasing the rms. The sources are mainly feedbacks having too much amplification at these frequency.

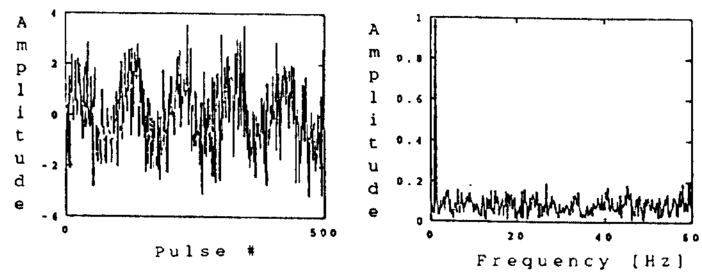

Figure 4: Simulation of Noise plus 1 Hz Oscillation.

The jitter noise and oscillation in the time frame (left) and its frequency amplitudes (right).

A simulation with a white noise jitter (Gaussian sigma = 1) and a 1 Hz oscillation (amplitude = 1) is shown in Fig. 4. The left shows the noisy oscillation for 500 pulses. The square-root of the FFT (not the power spectrum) is shown on the right and gives the amplitude of each of the 256 frequency bins. The 256 noise bins at about $A_f = 0.0625$ give an rms-amplitude

$$\sigma = \sqrt{\sum_{Bin} A_f^2} \tag{1}$$

of one. The oscillation would add about 1/2 "rms" in quadrature. But since the distribution of a sin is not Gaussian at all, but more like double horned corresponding to the two crests of the sin curve, the gaussian fit to the projection is about 30 % bigger, indicating that the single oscillation had about an 0.8 amplitude effect.

4 FFT-Orientation

The so far mentioned examples give some ideas how to implement the information of an FFT. Here some techniques and the comparison with Notch filters are given.

4.1 Simple Approach

Following the example with the last two pulses, here is a way to determine the weights of the desired correction. The last pulses x_1, x_2 ... x_n are multiplied by an $(n+1)*n$ matrix M, which contains the information about the oscillation frequency

$$M_{mk} = \cos(\pi/n * (m-1) * k) * 2/n, \qquad (2)$$

with $M = M/2$ if $m = 1$ or $m = n+1$.

For $m = 0$ it is just an averaging over the last n pulses (no averages over different n to keep it simple); for $m = n$ it is $-x_1$, $+x_2$, $-$, $+$ ..., which would be a high number if there is a big time slot separation (60 Hz oscillation). The square of these numbers and a sin term are used as weights for the different predictions. This method can be compared with Notch-filters at zero, $f/2n$, f/n, ... $f/2$ and a weight for each filter. An averaging like in Fig. 1 is also possible for a Notch filter.

4.2 FFT Weights

An FFT of the last n pulses results in also n numbers, which are the amplitudes and the phases at the corresponding $n/2(+1)$ frequencies. For instance, 6 pulses have an amplitude and phase value for 10, 20, 30, 40, 50 Hz and only an amplitude value for 0 and 60 Hz, since zero and Nyquist (60 Hz) frequency don't have a phase information. The amplitudes can directly be used as weights. Fig. 5 shows an example where eight+1 frequencies are totally suppressed.

The overshoot at low and high frequency needs more investigation, the rest is at least less than 55 % of the original amplitude. Going with this scheme to more pulses doesn't reduce these peaks dramatically but adds new points with total suppression. A time dependent averaging might reduce more the peaks, since the slope near the suppressed frequency is not as steep.

The interesting feature of this feedback is that it can suppress oscillations of any frequency below their initial amplitude. This means that the response of the feedback has no parts with amplification.

4.3 Future Work

The FFT-oriented feedback which adjusts the feedback response to the amounts in the FFT has a high goal to achieve. It should get the maximum out of the past pulses to achieve the lowest spread around the desired value.

The time dependent averaging should get the right damping. Also the behavior of higher order correction [2] may be considered. They achieve a flat curve at the suppressed frequency on the expense of a much worse amplification beyond the crossover point.

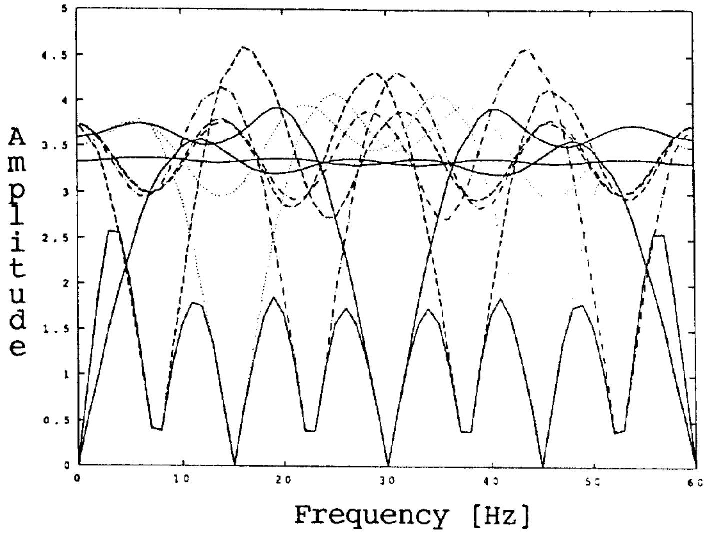

Figure 5: FFT-Oriented Feedback Response.

The lower solid curve shows the response by using the FFT information of oscillations with an amplitude of 3. The other curves are Notch filter type responses for eight pulses.

The response to steps or varying frequencies has to be checked. The many frequency components during a step may be ideally handled by an FFT-oriented feedback. But even whether it is good for frequent changes or better for stable oscillations can be adaptive: Taking many pulses into account slows down the response to fast changes, but these changes would tell the feedback to consider less pulses.

5 Conclusion

The study of a feedback which response is adjusted by the amount of the FFT of the last pulses has shown to reduce every single frequency component. Additionally averaging schemes were found which don't show oscillations in the amplification regime which should be more stable than the currently used response.

Acknowledgement

I would like to thank T. Himel and L. Hendrickson for many helpful discussions, and L. Hendrickson for reading the manuscript.

References

[1] T. Himel, S. Allison, P. Grossberg, L. Hendrickson, R. Sass, H. Shoaee, SLAC, *Adaptive Cascaded Beam-Based Feedback at the SLC*, PAC, Washington, May 1993.

[2] R. Steining, *Sampled Feedback in the Linac Collider*, SLAC, CN-14, 1980.