Dynamic Closed Orbit Correction

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Abstract

This paper discuss the high speed method of the orbit correction. The speed is affected by many time-constants such as vacuum chamber, magnet, position sensor, power supply, controller, and beam itself. The beam effect is the major concern of this paper. The damping effect implies that the transfer function of the orbit correction contains at least a pole. In the DC case, the transfer function becomes the response matrix of the closed orbit. In the fast feedback application, this pole has to be taken into account to avoid instability. If the zero-pole compensation is possible, the correction speed can be increased. On the other hand, the local bump method doesn't change the periodic boundary condition of the orbit. Therefore, the local bump responds immediately without damping transient. The linear combination of the local bump is fast, but less degree of freedom. The general method to increase the correction speed and the eddy current induced sextupole component are discussed.

I. Introduction

In the control system, the frequency response of each subsystem should be identified. The controller is then designed with proper gain and phase compensation to have a satisfactory dynamic response and accuracy. The modeling of the subsystem plays an important role in the design phase. In this paper, The model of the accelerator from the control point of view is discussed and treated with the beam position monitor (BPM) together. The transfer function between the input steering field and output reading of the beam position contains several time-constants, such as damping time and betatron frequency which are known by every accelerator physicists. Here, all of these physical nature is organized into a engineering presentation.

There are many methods to correct the beam position. The major concern is the frequency response of these methods. The speed means the fast setting and the high feedback gain. Both characters are wanted for the good dynamic response in a feedback control system. Two methods, bump and response matrix, are selected. The bump is fast in sense of accelerator response. Therefore, the controller increase the gain at high frequency region. The response matrix is accurate and suitable for the slow operation. The combined method joins the advantages and provides the fast and accurate feedback.

II. Kick Response

The betatron oscillation induced by a kick pulse is

$$x_{mn} = \sqrt{\beta_m \cdot \beta_n} \cdot \sin(\phi_m - \phi_n) \cdot \theta \qquad .....(1)$$

where m,n are the location indices for the beta function and the phase in respect to the observation as well as the kick. θ is the strength of the kick. If we take the damping effect into account[1], the particle oscillation amplitude of the *k*th turn of a circular accelerator is written by

$$x_{mn}(k) = e^{-kT_0/\tau} \cdot \sqrt{\beta_m \cdot \beta_n} \cdot \theta \cdot \sin(2\pi\nu k + \phi_{mn}) \qquad .....(2)$$

The damping factor $e^{-kT_0/\tau}$ express the time relationship between the revolution time T_0 and the damping time τ in average. ν is the characteristic tune of the accelerator, and $\phi_{mn} = \phi_m - \phi_n$. The observation, turns by turns, is actually a discrete form. We can find the z transform

$$X_{mn}(z) = \sum_{k=-\infty}^{\infty} x(k) \cdot z^{-k} \qquad .....(3)$$

in a standard text book [2], with a general expression

$$H_{mn}(z) = \frac{X_{mn}(z)}{\theta(z)} = \sqrt{\beta_m \cdot \beta_n} \cdot \mathrm{Im}\left[\frac{z e^{j\phi_{mn}}}{z - e^{-T_0/\tau} \cdot e^{j2\pi\nu}}\right]$$
$$|z| > e^{-T_0/\tau} \qquad .....(4)$$

This transfer function is used to measure the betatron tune with the knock-out method. The betatron oscillation has a resonant amplitude when the excitation frequency approaches the betatron frequency. The name "knock-out" is no longer true for an electron machine with damping.

If the concerned frequency is much less than the revolution frequency, we take the short revolution time limit $T_0 \to 0$. The observation is treated as a continuous signal, whose corresponding Laplace transform is[2]

$$H_{mn}(s) = \sqrt{\beta_m \beta_n} \cdot \mathrm{Im}\left[\frac{e^{j\phi_{mn}}}{s + \frac{1}{\tau} - j\omega_t}\right] \qquad .....(5)$$

where $\omega_t = 2\pi\nu/T_0$ is the angular velocity of the betatron oscillation. In this continuous-signal approach, the kick pulse is becoming a δ-function $\theta\delta(t - t_0)$. The transfer function of the impulse response of the accelerator is just like a damped second order low-pass filter with a pair conjugate poles at $\left(\frac{1}{\tau} \pm j\omega_t\right)$. The betatron frequency ω_t is the upper limitation of the feedback frequency, since the phase changes very fast when the excitation frequency approaches ω_t.

III. Response Matrix

We define the discrete unit step kick of the kth turn

$$u(k) = \begin{bmatrix} 0 & k < 0 \\ 1 & k > 0 \end{bmatrix} \quad .....(6)$$

The Z transform of this function is

$$u(z) = \frac{z}{z-1} \qquad (|z| > 1) \quad .....(7)$$

The final state of the unit step response is obtained by using the final value theorem.

$$\lim_{k\to\infty} x_{mn}(k) = \lim_{z\to 1}(z-1)\cdot H_{mn}(z)\cdot u(z)\cdot\theta$$

$$= \sqrt{\beta_m\beta_n}\cdot\theta\cdot \mathrm{Im}\left(\frac{e^{j\phi_{mn}}}{1-e^{-T_0/\tau}\cdot e^{j2\pi\nu}}\right) \quad .....(8)$$

Wiedemann has exact the same expression in the Chapter 7 of his book [1]. He took the short revolution time limit $T_0 \to 0$ to get the well known formulation of the response matrix.

$$A_{mn} = \frac{\sqrt{\beta_m\beta_n}}{2\sin(\pi\nu)}\cos(\pi\nu + \phi_{mn}) \quad .....(9)$$

In many standard textbook, this formula is solved by the periodic boundary condition without damping effect[3]. It holds true as long as $T_0 << \tau$. In case of $T_0 \sim \tau$, the equation 8 has to be applied. For a super big synchrotron light source with many insertion devices, this condition may occur. With some rearrangement, we can prove that the denominator is always greater then zero.

$$1 - 2e^{-T_0/\tau}\cos(2\pi\nu) + e^{-2T_0/\tau}$$

$$= 2e^{-T_0/\tau}\left(\cosh\left(\frac{T_0}{\tau}\right) - \cos(2\pi\nu)\right) \quad > 0 \quad .....(10)$$

It implies that the resonant line disappear. The damping effect has a positive contribution to avoid resonant.

IV. Local Bump

A local bump doesn't change the closed boundary condition[3]. Only the orbit inside the bump is changed. There is no betatron oscillation propagate to the next turn. Hence, the response of the step local bump is immediate without oscillation and damping. The transfer function of the local bump is merely a matrix without poles and zeros.

$$H_{mi} = \sum_n \sqrt{\beta_m\beta_n}\sin(\phi_{mn})\cdot a_{ni} \quad .....(11)$$

Where a_{ni} describes the tri-diagonal bump matrix

$$a_{ni} = \delta_{n,i-1}\frac{\sin(\phi_{i,i+1})}{\sqrt{\beta_{i-1}}} + \delta_{n,i}\frac{\sin(\phi_{i+1,i-1})}{\sqrt{\beta_i}} + \delta_{n,i+1}\frac{\sin(\phi_{i-1,i})}{\sqrt{\beta_{i+1}}} \quad ....(12)$$

Since the bump is created by three kicks, the column number is less than row number by 2. This also means that the degree of freedom for bump correction is less than for matrix.

V. BPM

The beam position monitor functions like a radio receiver. The signal coming from the buttons is mixed with local oscillation and filtered to low frequency. Then, we have the observation

$$O_{mn}(t) = \sum_{k=0}^{t>kT_0} f_b(t-kT_0)x_{mn}(k) \quad .....(13)$$

where $f_b(t-kt_0)$ is the impulse response of the BPM. This formula is rather difficult to evaluate. We try a first order low pass filter to make this calculation clearly. Assume that the low pass filter has the same time constant as the damping time constant of the accelerator. The observation of the equation 2 becomes

$$O_{mn}(t) = \sum_{k=0}^{t>kT_0} e^{-(t-kT_0)/\tau}\cdot x_{mn}(k) \quad .....(14)$$

For the low frequency application, we approximate again with the same limit $T_0 \to 0$.

$$\lim_{T_0\to 0} O_{mn}(t) = e^{-t/\tau}A_{mn} \quad .....(15)$$

The transfer function of the observation turns out to be

$$H^{o}_{mn} = \frac{A_{mn}}{s + 1/\tau} \quad .....(16)$$

the response matrix with a first order low pass filter.

VI. Fast Feedback

A combined correction method of bump and response matrix is considered here to apply on the global feedback system. The fundamental idea is to use bump method at high speed and response matrix at low speed. The response matrix has more degrees of freedom, which allows a better correction of the closed orbit distortion. The local bump is usually applied at the local correction of the photon port with a high setting rate. We want to merge the bump method into global feedback system. From the analysis of last section, the betatron oscillation raised by corrector setting will not affect the observation, if we use BPM filter to smoothing reading. However, the betatron is damped with the speed of the damping time constant which is independent of the smoothing time constant.

The concern is slow damping time, which is in the range of a couple of millisecond. The fast setting from the response matrix will activate the betatron oscillation. The photon beam position detectors will pick up the betatron oscillation and force the local bump at photon port to correct it. From this point of view, we have to reduce the correction gain at high frequency region (> 100 Hz), if the response matrix is applied. In the SRRC case, this attenuation will be contributed from the vacuum chamber.

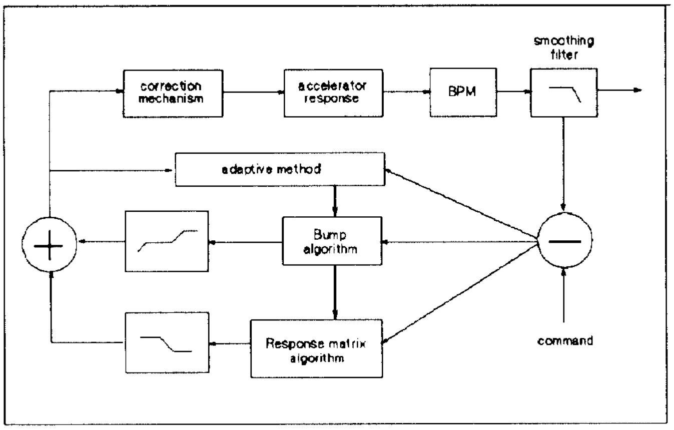

Figure 1. System Block Diagram

Figure 1. is the block diagram of this combined correction method. The BPM readings pass through the smoothing filter and are distributed to each correction algorithm. After the calculation, the setting values are compensated in the high frequency region for the bump method, and in the low frequency region for the response matrix. The sum of two setting values is send to the correction magnet. We compare the sum setting and the BPM reading to estimate the parameter changes. The new parameter values modify both algorithms with the adaptive method[4].

The compensation of the bump method starts with a high pass edge, since the DC accuracy of bump is worse than that of the response matrix. The gain at higher frequency region is enhanced by another rising edge, which attempt to compensate the attenuation from the vacuum chamber. This enhancement should be carefully adjusted in respect to he speed of the correction power supply. On the contrary, the setting from the response matrix is enhanced below the cut-off frequency raised from the vacuum chamber response.

The correction mechanism transfer the setting current of magnet to steering angle. The response contains two poles. One of them is coming from the vacuum chamber; the other depends on the character of the power supply to drive the inductive load. The modern MOS technology provides the high speed capability to fulfill this requirement.

VII. Vacuum Chamber

The eddy current is the energy dissipated part of the field equation when the alternative field penetrates a metallic chamber. The transfer function is more or less like a low pass filter of the first order. The time constant is proportional to the product $\Delta \cdot \sigma \cdot a$ of the thickness Δ, the conductivity σ and the perpendicular dimension "a" of the chamber in respect to the field[5]. This time constant is not the limitation of the feedback speed indeed, since the phase lag is stable and less than 90 degree. Driving over this cut-off frequency requires more power to keep the same field. However, the high frequency component of the feedback signal exists rarely because of the shield effect. The power consumption is small as long as the signal is small. The speed is acutely limited by the second pole of the correction power supply and the time delay of the controller.

The other concern is the sextupole component. We take a low order approximation of the lost field[6]

$$\Delta B(x) \approx \Delta B(0)\cdot\left(1-{x^2}/{a^2}\right). \qquad .....(17)$$

The jitters of the beam is in the range of hundred μm. Its spectrum concentrates most in the region lower than the chamber cut-off frequency. Since the lost field is proportional to the jitters and its frequency, the contribution of the sextupole can be estimated by equation 17.

VIII. Reference

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