# Generation of Intensive Long-wavelength Edge Radiation in High-energy Electron Storage Rings

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## Abstract

Computation results on bending magnet edge radiation (ER) in 2.5 GeV electron storage ring Siberia-2 are reported. Special features of the ER in high-energy electron storage rings are discussed. This radiation differs from standard synchrotron radiation as well as from shortwavelength ER in proton storage rings. Emission peculiarities of the long-wavelength ER in high-energy electron storage rings are considered. The ER is shown to possess certain features of long-wavelength bremsstrahlung.

#### I. INTRODUCTION

Electromagnetic edge radiation is generated by ultrarelativistic charged particles in the region of magnetic field change at bending magnet edge.

It was shown both theoretically and experimentally [1] - [3], that in high-energy proton synchrotrons the shortwavelength ER (at  $\lambda \ll \lambda_c$ , where  $\lambda_c$  is critical wavelength of standard synchrotron radiation from uniform field of bending magnet) is much more intensive than the standard synchrotron radiation (SR).

The ER in electron synchrotrons and storage rings was considered in [4] - [8]. The electron ER intensity exceeds corresponding SR intensity level at  $\lambda >> \lambda_c$ ; spectral angular characteristics of the ER are much different from those of the standard SR.

In this paper, the features of ER at  $\lambda > \lambda_c$  are discussed. First the computation results of the ER characteristics for the Siberia-2 electron storage ring (under construction in Kurchatov Institute, Moscow) are presented, and thereafter the peculiarities of the ER formation are analytically considered. The calculations were made in the approximation of infinitely thin electron beam (ER computation results obtained with due regard for finite beam emittance are discussed in [9]).

## II. COMPUTING THE EDGE RADIATION CHARACTERISTICS

Fourier transformation of electric field emitted by single electron in its motion along the trajectory  $\vec{r}(\tau)$  may be given by the relation obtained from Fourier transformations of delayed potentials [10]

$$\vec{E}_{\omega} = \frac{ie\omega}{c} \int_{-\infty}^{+\infty} \frac{\vec{\beta} - \vec{n}}{R} \exp\left[i\omega(\tau + R/c)\right] d\tau , \qquad (1)$$

where  $\vec{\beta} = (d\vec{r}/d\tau)/c$  is relative velocity of electron;  $\vec{n} = \vec{R} / R$ ,  $\vec{R} = \vec{r}^* - \vec{r}$ ,  $R = |\vec{R}|$ ;  $\vec{r}^*$  denotes observation point position,  $\omega$  is radiation frequency, *e* is charge of electron; *c* is the speed of light, *i* is unit imaginary number.

Spectral angular distribution of the radiation energy is

$$\frac{d\varepsilon}{d\Omega d\omega} = \frac{cr^{*2}}{4\pi^2} \left| \vec{E}_{\omega} \right|^2,\tag{2}$$

where  $r^* = |\vec{r}^*|$  [10].

It is practically more convenient to consider number of photons emitted by all the electron beam per time unit, dN/dt. If longitudinal bunch length exceeds the observed radiation wavelength, then the radiation of different electrons is known to be essentially incoherent. Therefore it is obtainable for infinitely thin electron beam,

$$\frac{dN}{dtd\Omega(d\omega/\omega)} = \frac{\alpha c^2 r^{*2} I}{4 \pi^2 e^3} \left| \vec{E}_{\omega} \right|^2,$$
(3)

where I is electron current,  $\alpha$  is the fine-structure constant. In line with approximation (3), spectral angular distribution of the beam radiation is defined by that of one electron.

Let the y-axis be coincident with straight section axis in the storage ring; x is horizontal axis and z is vertical one. Electromagnetic radiation of ultra-relativistic particle is directed forward for the most part [10]. Hence one can set in Eq.(1)  $|\beta_x| <<1$ ,  $|n_x| <<1$ ,  $|n_z| <<1$ . The electron is assumed to move in median plane, therefore  $\beta_z=0$ . With regard to it, the exponent phase in (1) may be shown up as

$$\begin{cases} \omega(\tau + R/c) \approx \Phi_0 + \Phi; \\ \Phi = (\pi/\lambda) \int_0^s \left[ \gamma^{-2} + n_c^2 + (n_x - \beta_x)^2 \right] ds', \end{cases}$$
(4)

where the trajectory length  $s=\tau\beta c$  is used as integrating variable;  $\gamma$  is the electron reduced energy ( $\gamma >> 1$ );  $\Phi_0$  does not depend on s. Squared magnitudes of  $\sigma$ - and  $\pi$ component of  $\vec{E}_{\omega}$  are given by the expressions

$$\begin{cases} \left| (E_{\omega})_{\sigma} \right|^{2} = \frac{4\pi^{2}e^{2}}{c^{2}\lambda^{2}} \left| \int_{-\infty}^{+\infty} \frac{n_{x} - \beta_{x}}{R} \exp(i\Phi) ds \right|^{2}; \\ \left| (E_{\omega})_{\pi} \right|^{2} = \frac{4\pi^{2}e^{2}}{c^{2}\lambda^{2}} \left| \int_{-\infty}^{+\infty} \frac{n_{z}}{R} \exp(i\Phi) ds \right|^{2}. \end{cases}$$

$$(5)$$

In accordance with the definition of  $\vec{n}$ :  $n_x=(x^*-x)/R$ ,  $n_z=z^*/R$ ;  $x^*$ ,  $z^*$  are Cartesian coordinates of observation point in the detector screen; x(s) and  $\beta_x(s)$  are defined from the equation of motion,

$$x(s) \approx \int_{0}^{s} \beta_{x}(s') ds' + x(0), \quad \beta_{x}(s) \approx \frac{1}{r_{0}} \int_{0}^{s} f(s') ds' + \beta_{x}(0), \quad (6)$$

where  $r_0 = -mc^2 \gamma (eB_0)$ ;  $B_0$  is uniform field in bending magnet; f(s) is magnetic field form-factor,  $f(s) = B(s)/B_0$ .

In storage rings the ER measurement geometry is such that a superposition of radiation emitted at two adjacent bending magnet edges is observed (see Fig.1). Therefore in a number of papers [4] - [7] general attention was paid to the interference effect. On the other hand, the characteristics of interfering ER in many respects are defined by the



Figure 1. Edge radiation detection scheme.

peculiarities of radiation generated at single bending magnet edge. Relations (3) - (6) allow to calculate the characteristics of single bending magnet ER as well as the ER from two adjacent bending magnet edges.

The convergence of integrals in (5) is provided by the phase  $\Phi$ ; therefore Eqs.(5) are not convenient for immediate computation. The integrals may be represented as

$$\int_{-\infty}^{+\infty} A \exp(i\Phi) ds \approx \int_{s_1}^{s_2} A \exp(i\Phi) ds + \left(\frac{A}{i\Phi'} + \frac{A'\Phi' - A\Phi''}{\Phi'^3} + \dots\right) \exp(i\Phi) \bigg|_{s_2}^{s_1},$$

where A denotes pre-exponent factors; prime means differentiation with respect to s; the series in braces is obtained through sequential integration by parts of the residuals

 $\int_{-\infty}^{1} A \exp(i\Phi) ds \text{ and } \int_{s_2}^{\infty} A \exp(i\Phi) ds.$  The actual numerical

integration limits  $(-\infty < s_1 < s_2 < +\infty)$  should be chosen to make the series effectively converge. The use of only a few terms of the series makes the computation much easier.

When computing the ER characteristics, the following parameters of the Siberia-2 storage ring were used:  $\gamma=5\cdot10^3$ , I=100mA,  $|r_0|=1960$ cm. The function B(s) was determined according to measurements performed in the Institute of Nuclear Physics (Novosibirsk).

In line with the results of computing the single bending

a)

magnet ER for  $\lambda > \lambda_c$ , intensive peaks appear in the angular distribution of radiation. In terms of angles respective to straight section axis, the horizontal one  $\xi$  and vertical one  $\zeta$ , the peaks are located at  $\xi \cong \pm \gamma^{-1}, \zeta = 0$  for  $\sigma$ component and at  $\xi = 0, \zeta = \pm \gamma^{-1}$  for  $\pi$ - component (the origin of coordinates is set in the intersection of the electron trajectory and the magnet edge). This result correlates well with ER analysis at  $\lambda \to \infty$  [6]. The peak intensity considerably exceeds the intensity of the standard SR at  $\lambda \gg \lambda_c$ . The ER peaks are symmetrical with respect to median plane at any wavelength, whereas the peak symmetry with respect to vertical plane appears only at  $\lambda \gg \lambda_c$ . Fig.2 shows the angular distributions of radiation at  $\lambda = 400$ nm. In this figure the standard SR corresponds to large horizontal angle ( $\xi \gg \gamma^{-1}$ ).

There are not peaks at  $\lambda < \lambda_c$  and the radiation intensity at small angles  $(|\xi| < 2\gamma^{-1})$  is below the level of standard SR.

Intensity distributions of the  $\lambda$ =400nm radiation generated at two adjacent bending magnet edges are presented in Fig.3. Geometrical parameters used in the computation are shown in Fig.1. The combined intensity distribution comprises the system of concentric interference circles well-known in optics.

## III. PECULIARITIES OF ER FORMATION IN LONG-WAVELENGTH REGION

In this chapter some approximate formulae revealing the mechanism of the ER formation at  $\lambda > \lambda_c$ , are given.

At large distance from the emission region transversal components of the vector  $\vec{n}$  may be treated as coincident with observation angles,  $n_x \cong \xi$ ,  $n_z \cong \zeta$ . In terms of it, the relations (3) - (5) may be modified as



Figure 2. Angular distribution of single bending magnet edge radiation at  $\lambda$ =400nm: a)  $\sigma$ - component; b)  $\pi$ - component.



Figure 3. Intensity distribution of interfering edge radiation at  $\lambda$ =400nm: a)  $\sigma$ - component; b)  $\pi$ - component.

$$\frac{dN}{dtd\Omega(d\lambda/\lambda)} \approx \frac{\alpha\gamma^2 I}{\pi^2 e} \left| F_{\sigma} \vec{e}_{\sigma} + F_{\pi} \vec{e}_{\pi} \right|^2;$$
(7)

$$\begin{cases} F_{\sigma} = \frac{i\pi}{\lambda\gamma} \int_{-\infty}^{+\infty} (\xi - \beta_x) \exp(i\Phi) ds; \\ F_{\pi} = -\frac{i\pi\zeta}{\lambda\gamma} \int_{-\infty}^{+\infty} \exp(i\Phi) ds; \end{cases}$$
(8)

$$\Phi \approx \frac{\pi}{\lambda} [(\gamma^{-2} + \zeta^2 + \xi^2)s - 2\xi_0^{\dagger}\beta_x ds' + \int_0^{\dagger}\beta_x^2 ds'].$$
(9)

To analyze the convergence of integrals (8), one should consider the structure of the phase  $\Phi$ . Characteristic length unit may be easily found for each term of the phase in Eq.(9) (the term's contribution to  $\Phi$  value is of the order of one at this length). The length units are the following at  $\gamma|\xi|\sim 1$ ,  $\gamma|\zeta|\sim 1$ :  $\lambda\gamma^2$  for the first term,  $(\lambda|r_0|\gamma)^{1/2}$  and  $(\lambda|r_0|^2)^{1/3}$ for the second and third terms respectively. Therewith, it is obtainable from the  $\lambda_c$  definition that at  $\lambda > \lambda_c$ :

 $\lambda \gamma^2 > (4\pi/3)^{1/2} (\lambda |r_0| \gamma)^{1/2} > (4\pi/3)^{2/3} (\lambda |r_0|^2)^{1/3} > (4\pi/3) |r_0| \gamma^{-1};$ the relation signs become opposite at  $\lambda < \lambda_c$ .

Let the characteristic length of magnetic field dropping be  $\Delta s$ . Analysis of Eqs. (8), (9) reveals that if  $\Delta s$  is much smaller than the smallest length unit of the phase (i.e., in the trajectory region  $\Delta s$  the phase change is much less then one),  $\Delta s$  value can be ignored in (8), (9) and the magnetic field can be regarded as the step-changing one,  $f(s)=\Theta(s)$ , (10)

where  $\Theta(s)$  is the step function.

One can find from the outlined considerations that in the case  $\Delta s < dr_0 | \gamma^1$  Eq. (10) is reasonable approximation for  $\lambda > \lambda_c$ ; also, it is reasonable for  $\lambda < \lambda_c$  if  $\Delta s < <\lambda \gamma^2$ . Otherwise, in the case  $\Delta s > |r_0| \gamma^1$  Eq.(10) may be reasonable only at  $\lambda > \lambda_c$ , if the following relation takes place:  $\Delta s < <(\lambda | r_0 |^2)^{1/3}$ .

It is worth mentioning that the requirement  $\Delta s << \langle \lambda | r_0 |^2 \rangle^{1/3}$  is much weaker then the  $\Delta s << \langle r_0 | \gamma^1$  one being traditionally used [6] as the restriction on application of the approximation (10). In the case of the Siberia-2 storage ring  $\Delta s > |r_0| \gamma^1$ , but values  $\Delta s$  and  $(\lambda | r_0 |^2)^{1/3}$  are comparable even in VUV wavelength region.

In the  $\lambda \gg \lambda_c$  limit the following asymptotic expansion of Eqs. (8) is valid in approximation (10) for  $\gamma |\xi| \sim 1$ ,  $\gamma |\zeta| \sim 1$ :

$$\begin{cases} \left|F_{\sigma}\right|^{2} \approx \frac{\gamma^{2}\xi^{2}}{\left[1+\gamma^{2}\xi^{2}+\gamma^{2}\zeta^{2}\right]^{2}} + 2c_{1}p^{3/3}\frac{\gamma\xi}{1+\gamma^{2}\xi^{2}+\gamma^{2}\zeta^{2}} + (c_{1}^{2}+c_{2}^{2})p^{2/3};\\ \left|F_{\sigma}\right|^{2} \approx \frac{\gamma^{2}\zeta^{2}}{\left[1+\gamma^{2}\xi^{2}+\gamma^{2}\zeta^{2}\right]^{2}} - 2c_{3}p^{2/3}\frac{\gamma^{2}\zeta^{2}}{1+\gamma^{2}\xi^{2}+\gamma^{2}\zeta^{2}}, \end{cases}$$
(13)

where  $p = \pi |r_0|/(\lambda \gamma^3) = 3\lambda_c/(4\lambda)$ ;  $c_1$ ,  $c_2$ ,  $c_3$  are the constants:  $c_1 = \pi/(3^{1/3}\Gamma(1/3))$ ,  $c_2 = \pi/(3^{5/6}\Gamma(1/3))$ ,  $c_3 = \pi/(3^{7/6}\Gamma(1/3))$ ;  $\Gamma$  is the gamma-function.

Approximate relations (13) qualitatively agree with the computation results (Fig.2). As may be seen from Eqs. (13), the angular positions and widths of the ER peaks no longer depend on  $\lambda$  at  $\lambda \gg \lambda_c$ . Moreover, it is obtainable that for ER

$$\frac{dN}{dtd\Sigma(d\lambda/\lambda)} \bigg|_{\xi^2 + \zeta^2 = \gamma^{-2}}^{\lambda \to \infty} \frac{\alpha \gamma^2 I}{4\pi^2 e}, \text{ while for the SR } \frac{dN}{dtd\Sigma(d\lambda/\lambda)} \bigg|_{\xi > \gamma^{-1}}^{\lambda \to \infty} 0.$$

This makes the ER similar to the soft bremsstrahlung possessing the corresponding feature [10].

By these means, the ER peaks at  $\lambda > \lambda_c$  arise in consequence of the sudden change in magnetic field, from nil to the constant value, along the electron trajectory in high energy electron storage rings. The peaks appear in the case that the effective trajectory region of forming the radiation is larger than the length of magnetic field dropping.

For the radiation emitted at two adjacent bending magnet edges, the following relations may be easily obtained from Eqs. (8), (9), (10) at  $\lambda \gg \lambda_c$ :

$$\begin{split} &|F_{\sigma}| \approx \frac{4\gamma^{2}\xi^{2}}{(1+\gamma^{2}\xi^{2}+\gamma^{2}\zeta^{2})^{2}} \sin^{2}[\frac{\pi l}{2\lambda}(\gamma^{-2}+\xi^{2}+\zeta^{2})]; \\ &|F_{\pi}| \approx \frac{4\gamma^{2}\zeta^{2}}{(1+\gamma^{2}\xi^{2}+\gamma^{2}\zeta^{2})^{2}} \sin^{2}[\frac{\pi l}{2\lambda}(\gamma^{-2}+\xi^{2}+\zeta^{2})], \end{split}$$
(14)

where *l* is inter-magnet distance. Eqs. (14) are valid for far removed detector  $(r^*\gg l)$  and small angles  $(\not|\xi|\sim 1, \not|\zeta|\sim 1)$ . The similar relations were obtained in [6] at somewhat different modeling assumption. The qualitative agreement of Eqs. (14) and computation results (Fig. 3) is evident.

In line with Eqs. (14), the highest amplification of the ER due to the interference takes place at  $l=\lambda\gamma^2(2n-1)/2$ , n=1,2,... This amplification is unattainable at  $l<\lambda\gamma^2/2$ , and in the limit  $l\ll\lambda\gamma^2$  the ER is suppressed.

### **IV. SUMMARY**

The ER features at  $\lambda > \lambda_c$  in high-energy electron storage rings look very attractive to SR users in spectral range extending from VUV to IR. The electron beam diagnostics seems to represent a distinct promising application of the edge radiation [9].

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