# Performance Optimization of Pure Permanent Magnet Undulators

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## Abstract

An application of the Simulated Annealing technique to the problem of optimizing the performance of Pure Permanent Magnet Undulators is described. This method has been applied to the optimization of the first two undulators for Elettra. Predictions are shown to be in good agreement with the actual measurements of undulator sections which have been performed so far.

# I. INTRODUCTION

The problem of optimizing the field quality of Pure Permanent Magnet Undulators (PPMU) has been discussed by several authors [1]. All of the proposed solutions are based on a more or less accurate ability to predict the field resulting from the superposition of a large number of permanent magnet blocks. Therefore, the first step for any optimization process usually consists in a precise characterization of the individual blocks.

Another important issue is to define suitable quantities to be used as 'quality parameters' of the undulator field. For devices which have to be installed in low emittance storage rings for the production of high brilliance synchrotron radiation, attention must be paid not only to the spectral purity of the emitted light, but also to the possible disturbing effect on the stored electron (positron) beam.

To actually perform optimization, an appropriate method has then to be found for sorting the available blocks, in order to increase as much as possible the undulator field quality.

Recently an additional technique has been developed which consists in placing thin ferromagnetic shims on the surface of the magnet, thus introducing small localized field perturbations. The problem in this case is to find the optimum configuration of shim positions and thicknesses, subject to some practical constraints such as the maximum number of shims to be used and the maximum shim thickness.

Both sorting and shimming can be succesfully managed by stochastic search techniques such as Simulated Annealing [3]; this method has been applied to the optimization of the first two undulators for Elettra.

# **II. BLOCK SORTING**

#### A. Individual Blocks Measurements

The method adopted consists in a Hall Plate scanning performed above and below the magnets over a range of longitudinal and transverse positions. Two components of the magnetic field are measured, giving enough information for predicting (by linear superposition of the individual fields) the on-axis and off-axis transverse field distributions for any configuration of blocks. Although this approach is complicated due to the large quantity of information which has to be handled, it gives the possibility of making accurate predictions both of the main electron trajectory and of the integrated multipole contents of the undulator field.

Taking measurements on two sides of each block is of particular importance because of the high dishomogeneity of magnetization often present in commercially available high field materials such as NdFeB. As an example, fig.1 shows the statistical distributions of the total intensity of the blocks which have been used for the U12.5 undulator. Also shown is the distribution of the difference in field integrals above and below the blocks. It is clear that the effect of non-uniform magnetization can be even larger than that produced by the spread of average intensity values.



Figure 1. Statistics of intensity and dishomogeneity parameters for the U12.5 magnets.

### B. Field Quality Parameters (Cost Function Definition)

The traditional way of assessing the field quality of an undulator is the rms on-axis peak field variation. As recently pointed out by some authors [2] this parameter is not very well correlated to the spectral intensity produced by a particular device. In ref. 2, the requirements that an alternative parameter should satisfy are clearly stated:

- i. It must be well correlated to the undulator performance
- ii. It must be simple to calculate
- iii. It must be general, i.e. independent of the particular device one is considering
- iv. It must be sensitive to small adjustments of the field distribution.

A good candidate to fulfill these requirements is the phase error (or 'phase shake'). This is the difference in phase of photons emitted by an eletcron moving along the real trajectory with respect to the ideal case where no field errors are present. The good correlation between the rms phase error ( $\sigma_{\Phi}$ ) and the on-axis spontaneous emission intensity has been demonstrated by computer simulations [2,3]. As for point ii, above, the evaluation of  $\sigma_{\Phi}$  for any meaured or computed field is straightforward. Points iii, and iv, are also easy to verify. As mentioned before, there is another class of effects that we want to minimize, namely those which affect the circulating beam. Integrated multipoles are responsible for angular deflection of the particles passing through the device. Also the second integrals of the field, describing the final trajectory displacement, should be taken into account if the minimum disturbance condition has to be obtained.

All these considerations lead to the definition of the Cost Function. In the language of stochastic optimization, this is the combination of the various error terms that have to be minimized. In the present work the cost function is simply a weighted sum of three terms, corresponding to:

- a) rms phase error
- b) maximum value of the first field integrals at any transverse position
- c) as in point b) for the second field integrals.

The optimization for the Elettra undulators is in reality complicated by the fact that they are segmented, i.e. build in up to three sections which could be operated individually if necessary. The cost function in this case has also terms corresponding to the single sections, but the general philosophy remains the same.

Having defined a unique cost associated with each possible configuration of blocks, a simulated annealing algorithm is used to find an optimum solution. The algorithm, starting from an arbitrary configuration, works by swapping pairs of randomly chosen blocks. Good and bad swaps are allowed with equal probability at the beginning, but, as the computation proceeds, bad swaps are less and less likely to be accepted. Eventually, when no more bad swaps are allowed, the process terminates. This method has the advantage of being able to approach the global minimum of the cost function without being trapped in a local one (see ref. 4 for details). A complication arises from the fact that the weighting coefficients of the various terms in the cost function must be adjusted empirically in order to achieve an overall satisfactory solution.





Figure 2. Measured (solid line) and predicted (dotted) field variation and phase error under each pole for U12.5, section 1.

The results obtained for the optimized U12.5 undulator are in good agreement with the predictions. As an example fig. 2 shows this comparison for the on-axis field and the phase error in the case of one of the three 12 period sub-sections.

During the process of simulated annealing, a large number of configurations are considered; for each of them the cost function is evaluated, giving all the important field quality parameters. It is therefore possible, with a simple modification of the program, to compute also the corresponding radiated spectrum. In this way it is easy to compare various parameters with the on-axis peak intensity. In fig. 3 the intensities for the first, third and fifth harmonic of the spectrum are plotted as a function of the rms phase error  $\sigma_{\Phi}$ . This calculation refers to the complete 36 period undulator (three sections). The point mentioned before about the good correlation between  $\sigma_{\Phi}$  and spectral intensity is confirmed in this case. It is worth noting that no particular model for the field errors is used in this simulation, suggesting the general validity of this correlation.



Figure 3. Correlation between spectral intensity and rms phase error. Also shown (continuous line) is the result of a model for the effect of random phase errors taken from ref. 3.

Finally, Table 1 and 2 compare predictions and measurements for the three individual sections. The overall agreements is very good, with the exception of the first integral variation which appears to be the most difficult parameter to control.

Table 1. Predicted (measured) rms field error, phase error and relative intensity for the U12.5 undulator. R1, R3 and R5 are expressed in percentage of the ideal case.

	σ <sub>B</sub> (%)	$\sigma_{\Phi}$ (deg)	R1 (%)	R3 (%)	R5 (%)
section 1 section 2	1.0(1.1) 1.2(1.3)	3.8 (4.5) 2.8 (2.9)	98 (99) 98 (99)	95 (95) 96 (98)	88 (85) 93 (93)
section 3	1.4 (1.4)	6.9 (5.4)	97 (97)	84 (89)	71 (85)

Table 2. Predicted (measured) first and second integral variation for the U12.5 undulator over  $\pm 60$ mm in x.

	$\Delta I_1$ (G m)	$\Delta I_2 (G m^2)$
section 1	$\pm 0.9$ (2.0)	± 1.3 (1.9)
section 2	$\pm 0.8$ (1.0)	± 1.0 (1.0)
section 3	$\pm 0.8$ (1.3)	$\pm$ 1.0 (1.3)

#### III. SHIMMING

The shimming technique has been first proposed as a method to reduce the on-axis peak field variation based on the measured effect of shims placed between the poles of a hybrid undulator [5]. It has then been extended to the correction of the transverse field integral in a PPMU [6]. This correction is based on a model of the magnetizing effect of the external undulator field on a thin ferromagnetic shim sticking on its surface [7]. This model allows one to compute a set of 'field integral signatures' describing the effect of a shim placed at a given position in the undulator. The appropriate distribution of thicknesses required to correct the measured integral error is then found by determining the appropriate coefficients in a linear combination of these signature functions.

Combining the two methods, it is simple to extend the calculation of the shim effect in order to include also the perturbation to the longitudinal field distribution. As an example fig. 4 shows the computed and measured 'longitudinal signature' for a 0.4 mm thick shim positioned at the centre of the U12.5 undulator.



Figure 4. Measured and computed effect of one shim ( $88 \times 13 \times 0.4$  mm) on the on-axis field of the U12.5 undulator at 29 mm gap.

With a suitable set of transverse and longitudinal signatures, it is therefore possible to predict the effect that any combination of shims will have on the measured undulator field. This effect includes first and second integrals, on-axis field, trajectory, phase error and spectral intensity.

Similarly to the case of block sorting, a cost function can be evaluated which gives the desired quality parameters for each shim configuration. The simulated annealing algorithm can then be used to find the optimum shim placement in order to improve the field quality of an existing undulator structure. This approach has the advantage that correction of the field integrals and phase optimization can be performed simultaneously. The correction can also be extended to several magnetic gap values, if the appropriate signatures are computed.

This method has been applied to the three U12.5 sections; the purpose was to correct the first and second field integral error to within the specified limits of  $\pm 1$  Gm and

 $\pm 2.5 \text{ Gm}^2$  over a transverse range of  $\pm 25 \text{ mm}$  at all gaps. At the same time we tried to reduce the rms phase error of sections 1 and 3 which showed a larger value compared to the central section 2. This attempt has been succesful, using between 21 and 35 shims of thickness 0.2 to 0.5 mm. Table 3 shows that the measured values of the phase error for the three sections at three different gaps are in very good agreement with prediction. The required field integral correction has also been achieved. More details about the final results can be found in ref. 8.

Table 3. Predicted (measured) rms phase and peak field errors in the three sections of U12.5.  $\sigma_B$  is listed only to show the good agreement with the predictions, but is not used at all in the optimization.

	gap	$\sigma_{\Phi}$ (deg)	σ <sub>B</sub> (%)
section 1	29	3.0 (3.1)	0.9 (0.8)
	50	3.5 (3.5)	0.6 (0.6)
	100	3.7 (3.8)	0.6 (0.7)
section 2	29	2.8 (2.9)	1.2 (1.2)
	50	3.2 (3.2)	0.8 (0.8)
	100	3.5 (3.6)	0.6 (0.6)
section 3	29	2.6 (2.8)	0.9 (0.9)
	50	2.9 (3.0)	0.6 (0.6)
	100	3.4 (3.4)	0.7 (0.7)

# **IV. CONCLUSION**

The simulated annealing method has been succesfully applied to the optimization of the U12.5 undulator, both for block sorting and shimming. The same approach is therefore being used for the shorter period device U5.6, presently under construction.

## V. REFERENCES

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