Numerical Studies of Strong Focusing in Planar Undulators

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Abstract

The present trend towards short wavelength operation with long undulators places tight requirements on the electron beam quality and hence the need to maintain a well focused beam. This paper examines the performance of alternating gradient (AG) sextupole focusing in planar undulators [1]. Numerical simulation results of Free Electron Laser (FEL) performance using AG sextupole focusing are compared to results using only natural focusing and to those using quadrupole focusing.

Introduction

Free Electron Laser performance is affected by the overlap (in six dimensional phase space) between the electron beam and the optical beam. The extent of overlap (assuming perfect alignment) is dependent on electron beam size, emittance and energy spread and optical beam size and Rayleigh range. In general, the brighter the electron beam, the better the FEL performance. The electron and optical beam overlap is maintained by the well known optical guiding phenomenon. However, without focusing the electron beam diverges. When the electron beam density is reduced, FEL performance is degraded. Hence, maintaining a focused electron beam over the distance of the FEL undulator is paramount to high FEL performance. Obviously, for a given beam, the longer the undulator the more significant focusing becomes. Recent proposals for short wavelength devices have called for long undulators from 25 to over 60 meters long.

It is possible to make some simple quantitative statements about how electron beam focusing affects FEL performance. For a high gain (exponential regime) amplifier, the power output can be written as

$$P(z) = P_0 e^{z/L_g} \tag{1}$$

where P_0 is the input power, z is the distance along the device and L_g is the power gain (e-folding) length. The gain length can be characterized by the fundamental FEL parameter, ρ, [2]

$$L_g = \frac{\lambda_u}{4\sqrt{3}\pi\rho} \tag{2}$$

where λ_u is the undulator period. The fundamental FEL parameter scales with the third power of peak beam current density, $J^{1/3}$.

A general feature of free electron lasers is that if variations occur on a scales shorter than a gain length, performance is affected. For focusing, this rule of thumb implies a limit on the beta function, $\beta > L_g$. A limit on the focusing strength as a function of the emittance is given by

$$\beta \geq \frac{2\gamma\varepsilon_n}{\rho\left(1 + a_u^2/2\right)} \tag{3}$$

where γ is the beam Lorentz factor, ε_n is the normalized rms emittance and a_u is the usual normalized (unitless) undulator parameter. This limit is derived in the one dimensional limit assuming no energy spread. It is useful for setting an approximate limit on how strong a given FEL's focusing channel can be. As is discussed in the following sections, it is not merely the strength but also the type of focusing that affects FEL performance.

Sextupole Focusing

Free electron lasers provide weak natural focusing; in both planes for helical devices, but only in one plane for planar undulators. Fortunately, this problem can be solved by using Scharlemann's curved pole faces [3]. Constant gradient sextupole focusing (via curved pole faces or any other means), like natural focusing, has a constant transverse velocity for each electron. For a distribution of electrons there is a corresponding distribution of velocities. Natural focusing does not perturb the transverse phase space; however, it can not provide sufficient focusing for longer devices.

A relation between the focusing betatron wavenumbers is required by the Maxwell equations:

$$k_{\beta_x}^2 + k_{\beta_y}^2 = \frac{e^2}{2E_b^2} B_u \tag{4}$$

where x and y are the two transverse directions, B_u is the undulator magnetic field, e is the electron charge and E_b is the beam energy. For a planar undulator $k_{\beta x}=0$ or $k_{\beta y}=0$; typically for a helical undulator or curved pole faces, $k_{\beta x}=k_{\beta y}$. Relation (4) sets a limit on the weak (constant gradient)

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focusing strength. A large k_β^2, and hence a strong focus in one direction implies a negative k_β^2, and hence a defocus in the other direction. Consequently, strong focusing implies alternating gradients.

External quadrupoles have been proposed and used [4] to produce AG focusing in an FEL. An external FODO lattice can provide strong focusing for undulators. Unfortunately, only iron free undulators can be used with conventional schemes. Various novel schemes have been devised in an attempt to overcome this limit: canted poles, Panofsky quads, edge field permanent magnet arrays [5], etc.. Regardless, it has long been stated that quadrupole focusing can degrade FEL performance because it modulates the transverse velocity of the electrons.

Recently the idea of alternating gradient sextupole focusing in an FEL was studied. The original scheme called for alternating the curvature of the undulator poles, thus producing a strong focusing lattice. Other methods to achieve this type of lattice have also been considered: external sextupoles, side mounted magnet arrays, etc.. The beam dynamics and hence the FEL action are the same (in the limit of averaging over an undulator period) and so the exact form of the sextupole focusing is unimportant. Here the wavenumbers are allowed to be imaginary so that relation (4) does not limit the focusing strength. This is analogous to quadrupole strong focusing and to the feed down effect in circular machines.

The transverse electron velocity is, in general, different in a focusing section from that in a defocusing section. Of course a given electron can be matched between the two sections, but a distribution of velocities precludes matching.

This leads to the central question: Is the continuos periodic oscillation of the transverse beam velocity caused by a quadrupole lattice better or worse than the discrete changes caused by sextupoles? The answer is that quadrupoles are better. Upon reflection this seems to be intuitively correct. Abrupt disruptions of the beam phase space will tend to degrade the FEL action [6]. Smooth changes which occur on scales greater than a gain length are less deleterious. It is perhaps easiest to show this by performing a complete 3D simulation.

Simulations

The code TDA3D [7] was modified to allow for sextupole focusing. This code solves the averaged FEL equations in 3D and takes into account known phenomenon for the regime studied here. The sextupole focusing is accounted for in the simulation by modifying the vector potential of the undulator (a_u). Quadrupole focusing is simulated by adding a term to the particle equations of motion.

The example parameter set discussed here is the SLAC based X ray FEL [8,9]. The parameters are given in Table 1. It serves as a good test case due to the long length of the undulator and low beam emittance. Notice that applying equation (3) yields a beta function of 5 meter for peak FEL performance.

Table 1: SLAC X ray FEL parameters used in the simulations for this paper.

γ	Energy (mc^2)	14000
ε_n	Emittance normalized (mm-mrad)	3×10^{-6}
	Peak Current (A)	2500
	Pulse Length (fS)	160
a_u	Undulator parameter	6
λ_u	Undulator period (cm)	8.3
λr	Optical wavelength (nm)	4
ρ	FEL parameter	1.7×10^{-3}

In order to reliably compare sextupole and quadrupole focusing, identical lattices were calculated (same period, beta function and phase advance per cell). Monoenergetic beams where used (no energy spread). in a focus/ defocus (FD) lattice (no drifts). A study of the effect of the phase advance per cell was first done. Typically, a phase advance per cell of 90 degrees is used to minimize the average beam envelope. However, this creates large fluctuations in the beam size. As expected, simulations confirm that when the phase advance is large and hence the beam is modulated a great deal, then the FEL action is degraded. To avoid this added effect, subsequent comparisons where performed with a phase advance per cell of about ten degrees.

Figure 1 shows the results of a series of simulations. Three sets of data points are plotted: quadrupole lattice, sextupole lattice and 3D semi-analytic calculations10. The quadrupole set is clearly the best. As expected, there is an optimal focusing strength. Peak quadrupole performance occurs close to, but not precisely at the theoretically predicted 5 meter beta function. A figure of merit for the effect of focusing on an FEL is given by the variation of the phase over a betatron period. This is related to the extent of detrapping of electrons from the pondermotive well. For quadrupole cases, this effect is small. For the sextupoles used in this example, detrapping becomes significant for a beta function ≤ 5 meters.

Conclusions

Strong (alternating gradient) sextupole focusing does not work as well as comparable quadrupole focusing. Strong quadrupole focusing performs very well in an FEL. Simulations indicate that the 1D emittance limit on focusing can even be exceeded provided that a small phase advance per cell is used. This implies a need for a high field gradient focusing systems. Such systems would allow for the construction of shorter, higher gain free electron lasers.

There may be occasions when sextupole focusing is advantageous, but this is unknown at this time. An independent numerical confirmation of this work would be a useful endeavor.

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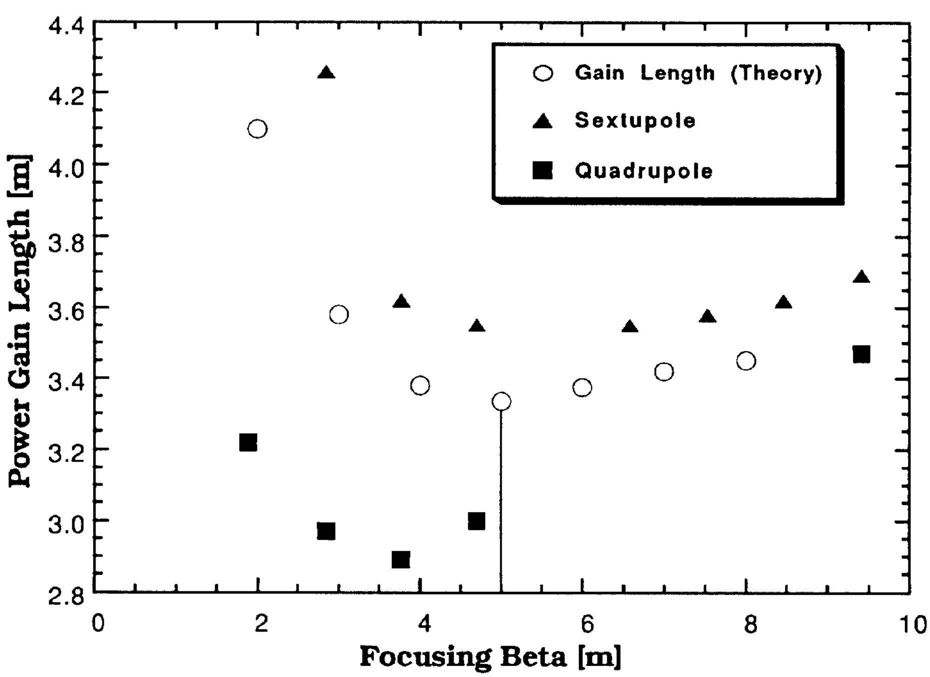

Figure 1: A comparison of quadrupole and sextupole focusing in an FEL. Analytic results are also plotted for comparison. The emittance limited optimal focusing is indicated by the vertical line (at 5 meters).