Disruption Effects from the Collision of Quasi-Flat Beams*

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Abstract

The disruption effects from the collision of round beams and flat beams in linear colliders have been studied in the past, and has by now been well understood. In practice, however, in the current SLC running condition and in several designs of the next generation linear colliders, the *quasi-flat* beam geometries are expected. Namely, the beam aspect ratio $R \equiv \sigma_x/\sigma_y > 1$, but not infinitely large. In this regime the disruption effects in both x and y dimensions should be carefully included in order to properly describe the beam-beam interaction phenomena. In this paper we investigate two major disruption effects for the the quasi-flat beam regime: The luminosity enhancement factor and the effective beamstrahlung. Computer simulations are employed and simple scaling laws are deduced.

I. INTRODUCTION

One of the most important issues in the design and operation of e^+e^- linear colliders is the effect of the beam-beam interaction. The single-pass nature of linear colliders demands that a high luminosity can only be achieved by colliding tiny, intense bunches of electrons and positrons. In this circumstance, these bunches interact strongly with one another, inducing large *disruption*, or pinch, effect between the colliding beams, and producing intense radiation called *beamstrahlung*.

In the case of the disruption effects, there have been detailed studies for the *round* beam, i.e., $R = \sigma_x/\sigma_y = 1$, and for the *flat* beam collisions[1][2]. Typically, in the flat beam limit where $R \gg 1$, the horizontal motion of beam particles is nigligible, and the problem has been studied in the one-dimensional approximation. However the current SLC running condition lies in the regime where R is larger, but not so much larger than one. As a result the horizontal motion of particles cannot be ignored. It happens that several of the next generation linear colliders, i.e., CLIC, DLC and TESLA, call for beam dimensions which also fall into this catagory. There is thus a need for a scaling law which can help estimate the disruption effect in the quasi-flat beam regime. In addition, in this regime it is also important that the calculation on beamstrahlung has the disruption effect properly included.

II. LUMINOSITY ENHANCEMNET FACTOR

The collective fields in one beam deform the other beam during collision, by an amount controlled by global

disruption parameters, which may be different in the two transverse directions[1][2]:

$$D_{x,y} = \frac{2Nr_e\sigma_z}{\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \quad . \qquad (1)$$

The deformation of the colliding bams results in effective beam sizes, $\bar{\sigma}_x$ and $\bar{\sigma}_y$, which are different from their nominal values. This in turn gives an effective luminosity different from the nominal one. The luminosity enhancement factor is defined as the ratio of the effective luminosity to the nominal luminosity due to the change of beam size:

$$H_D \equiv \frac{\bar{\mathcal{L}}}{\mathcal{L}} = \frac{\sigma_x\sigma_y}{\bar{\sigma}_x\bar{\sigma}_y} \quad . \qquad (2)$$

The luminosity enhancement factor is calculable analytically only in the $D_{x,y} \ll 1$ limit. Beyond this limit the dynamics of beam-beam interaction becomes nonlinear, and one must use simulations. For the case of round beams, simulations produce the behavior[2]:

$$H_D = 1 + D^{1/4}\left(\frac{D^3}{1+D^3}\right)\left\{\ln(\sqrt{D}+1)+2\ln(0.8/A)\right\}, \quad (3)$$

where for round beam $D \equiv D_x = D_y$ and $A \equiv A_x = A_y = \sigma_z/\beta^*$, and β^* is the β-function at the interaction point. This scaling law is valid to about 10% accuracy. The largeness of H_D in the $D_{x,y} \gg 1$ limit was recognized to be associated with the near equilibrium pinch-confined transverse beam profiles[2]. In this regime the beam particles undergo multiple betatron oscillations during the collision, and tend to be traped in a much narrower focusing potential of the opposing beam.

In the flat beam limit where one-dimensional approximation is employed, simulation gives the following scaling law[2]:

$$H_D(R \gg 1) \simeq H_D(R = 1)^{1/3}, \qquad (4)$$

when D_y and A_y are fixed and $D_x, A_x \to 0$.

It was later shown that there is actually a theoretical basis for such a cubic relationship[3]. The near equilibrium pinch-confined states are approached through collisionless damping due to mixing and filamentation in phase space. It was already pointed out[1] that the disruption parameter D is related to the square of the wave-number (of the betatron oscillation), κ_β. The emittance growth due to the disruption effect occurs in a length scale of κ_β^{-1}, but the beam rethermalizes in a length β^* due to the nonlaminar effects of the finite emittance. Thus the

fundamental quantity which governs the luminosity enhancement is evdently $\sqrt{D_y}/A_y \simeq \kappa_\beta \beta_y^*$ in the 1-D calculation. Indeed, this is precisely the leading logarithmic behavior in (3) for H_D. When the same prescription is applied to round beams, it was shown that the cubic relationship between the two limits can be deduced.

The less than quadratic dependence, which one might naively assume, can also be appreciated intuitively. In the round beam case the change of beam size in either x or y direction will enhance the pinching of the other dimension, i.e., the focusing in the two dimensions are fully coupled. On the other hand, it is well-known that the field strength in a non-round, i.e., $R > 1$, charge distribution is mainly determined by its major dimension, σ_x. This means that the lack of horizontal disruption renders a milder pinch effect for the flat beams.

From (2) we see that for round beams the effective beam size is given by

$$\bar{\sigma} = \sigma H_D^{-1/2} \quad , \qquad R = 1 \quad . \tag{5}$$

On the other hand, since in the flat beam limit the horizontal beam size is assumed to be fixed, the cubic relationship (4) suggests that

$$\bar{\sigma}_y = \sigma_y H_{D_y}^{-1/3} \quad , \qquad R \gg 1 \quad . \tag{6}$$

It was therefore proposed recently that the luminosity enhancement factor for quasi-flat beams scales as[4]:

$$H_D = H_{D_x}^{1/2} H_{D_y}^{1/3} \quad , \tag{7}$$

Noteice, however, that although this scaling law approaches the right flat beam limit of (4), it does not converge to the correct round beam scaling of (3). It is evident that the power law of the H_{D_y} dependence should be more complex than the simple cubic scaling when $R \to 1$.
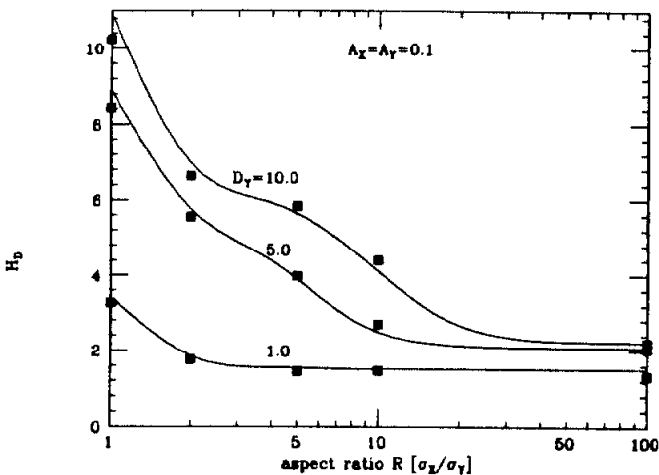

Fig.1 Evolution of H_D as a function of R

Computer simulation using ABEL[5] was performed to study the evolution of H_D as a function of R. This is done by fixing D_y, A_x, and A_y in the calculations,

while varying D_x. Figure 1 shows the simulation results of H_D as a function of R, with $A_x = A_y = 0.1$ and three choices of D_y. We find that these results (shown in squares) agrees very well with the following scaling behavior (shown in solid curves):

$$H_D = H_{D_x}^{1/2} H_{D_y}^{f(R)} \quad ,$$

$$f(R) = \frac{1 + 2R^3}{6R^3} = \begin{cases} 1/2 & , \quad R \to 1 \\ 1/3 & , \quad R \to \infty \end{cases} \quad . \tag{8}$$

This new scaling law now applies to all values of R.

III. EFFECTIVE BEAMSTRAHLUNG

High energy $e^+ e^-$ beams generally follow Gaussian distributions in the three spatial dimensions, and their local field strength varies inside the beam volume. In the weak disruption limit, where particle motions have small deviations from the z direction, it is possible to integrate the radiation process over this volume and derive relations which depend only on averaged, global beam parameters. It is found in such para-axial, or fixed impact parameter, approximation, that the beamstrahlung intensity is controlled by a global *beamstrahlung parameter*[6][7],

$$\Upsilon_0 = \gamma \frac{\langle B \rangle}{B_c} = \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\sigma_x + \sigma_y)} \quad , \tag{9}$$

where $\langle B \rangle$ is the mean electromagnetic field strength of the beam, $B_c = m_e^2/e \simeq 4.4 \times 10^{13}$ Gauss is the Schwinger critical field, N is the total number of particles in a bunch, γ is the Lorentz factor of the beam, r_e is the classical electron radius, and α is the fine structure constant.

In the most general designs for linear colliders, the photon spectrum due to beamstrahlung is not a factorized function of the electron and positron sources and depends on the detailed evolution of the bunches in the collision process. In general, then, the spectrum of radiation depends on the disruption process and must be computed by detailed simulation.[5] However, typical beams in linear colliders are very long and narrow. Since all particles oscillate within the focusing potential that is defined by the geometry of the oncoming beam, the oscillation amplitudes are small compared with their periodicity in z. Then the assumption of small deviations from the z direction remains approximately valid. The main effect of disruption on beamstrahlung is therefore the change of effective EM fields in the bunch due to the deformation of the transverse beam sizes. Thus, beamstrahlung is in practice still factorizable even under a non-negligible disruption effect, if one computes its magnitude using an effective beam size which takes the global disruption into account. This means one shall only replace the nominal beam size σ_x, σ_y in (9) by the corresponding effective size $\bar{\sigma}_x$ and $\bar{\sigma}_y$ following the prescription in (8):

$$\bar{\sigma}_x = \sigma_x H_{D_x}^{-1/2} \quad , \qquad \bar{\sigma}_y = \sigma_y H_{D_y}^{-f(R)} \quad . \tag{10}$$

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Then the effective beamstrahlung parameter is given by

$$\Upsilon = \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\bar{\sigma}_x + \bar{\sigma}_y)} \quad . \tag{11}$$

As long as the effect of disruption on beamstrahlung can be grouped under the globle beamstrahlung parameter, the recently derived beamstrahlung photon spectrum[8], which invokes the mean-field approximation, is readily applcable. The number of soft photons radiated per unit time, calculated by the classical theory of radiation, is

$$\nu_{cl} = \frac{5}{2\sqrt{3}} \frac{\alpha^2}{r_e \gamma} \Upsilon \quad . \tag{12}$$

Note that for a given field strength ν_{cl} is independent of the particle energy. This expression applies to the infrared limit of the spectrum where photon energies approach zero. For a hard photon, up to the initial energy of the electron, the quantum mechanical calculation gives a more general formula:

$$\nu_\gamma = \nu_{cl}[1 + \Upsilon^{2/3}]^{-1/2} \quad . \tag{13}$$

In a multi-photon radiation process, it was found useful to introduce a linear interpolation between these two values. Let x be the energy fraction of the initial electron carried by the photon. Then define

$$\bar{\nu}(x) = \frac{1}{1-x} \int_x^1 dx'[x'\nu_{cl} + (1-x')\nu_\gamma]$$
$$= \frac{1}{2}\Big[(1+x)\nu_{cl} + (1-x)\nu_\gamma\Big] \quad . \tag{14}$$

With these basic parameters introduced, $f_r(x)$ is given by[8]

$$f_r(x) = \frac{1}{\Gamma(1/3)} \Big(\frac{2}{3\Upsilon}\Big)^{1/3} x^{-2/3}(1-x)^{-1/3}$$
$$\times \exp\Big[-\frac{2x}{3\Upsilon(1-x)}\Big] \cdot G(x) \quad , \tag{15}$$

where Υ is given by (11),

$$G(x) = \frac{1-w}{g(x)}\Big\{1 - \frac{1}{g(x)n_\gamma}\big[1 - e^{-g(x)n_\gamma}\big]\Big\}$$
$$+ w\Big\{1 - \frac{1}{n_\gamma}\big[1 - e^{-n_\gamma}\big]\Big\} \quad , \tag{16}$$

$$g(x) = 1 - \frac{\bar{\nu}}{\nu_\gamma}(1-x)^{2/3} \quad ,$$

and

$$w = \frac{1}{6}\sqrt{\frac{3\Upsilon}{2}} \quad , \qquad n_\gamma = \sqrt{3}\sigma_z \nu_\gamma . \tag{17}$$

n_γ is the mean number of photons radiated per electron throughout the collision. The approximations are valid for $\Upsilon \leq 5$.

IV. EXAMPLE

To varify the validity of our handling of the disruption effect in beamstrahlung, we calculate the beamstrahlung spectrum in TESLA with center-of-mass energy at 1 TeV[9]. In this design, $N = 5.8 \times 10^{10}$, $\sigma_x = 404$nm, $\sigma_y = 50.5$nm, $\sigma_z = 1100\mu$m, $\beta_x^* = 8$mm, and $\beta_y^* = 2.5$mm. Therefore $D_x = 1.95$, $D_y = 15.6$ ($R = 8$), and $A_x = 0.14$, $A_y = 0.44$. This gives $\bar{\sigma}_x = 172$nm and $\bar{\sigma}_y = 27.0$nm form (10). In turn, we find $H_D = 4.4$ from (8). According to our prescription the disruption effect changes the beamstrahlung parameter from $\Upsilon_0 = 0.10$ to $\Upsilon = 0.24$. With this effective beamstrahlung parameter, we calculate the beamstrahlung spectrum using (15). This is then compared with the simulation result, shown in Fig. 2. We see that our prescription indeed agrees very well with the simulation.
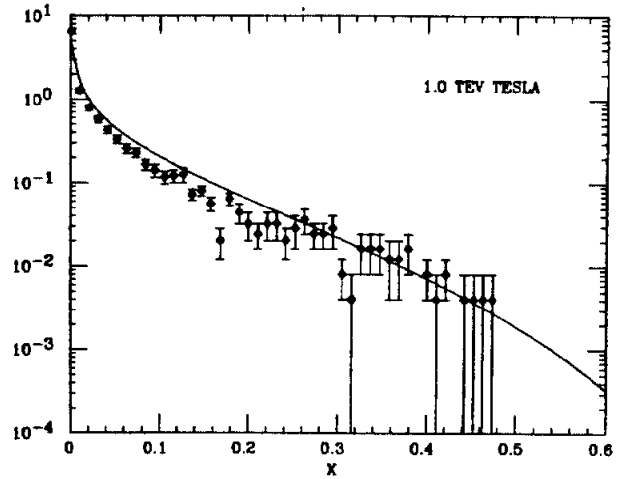
Fig.2 Beamstrahlung spectrum in a 1 TeV TESLA.

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