General Normal Form Procedure to Correct Tune–Shift and Non–Linear Chromaticity for Large Accelerators like the LHC

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Abstract

In future hadron colliders such as the LHC very high fields are needed to reach the design energy. Only superconducting magnets can produce such high fields and only at the cost of strong multipolar errors up to high order. This leads to a large non–linear shift of the tunes (detuning) both in amplitude and momentum, which may forbid a safe operation of the accelerator. The best solution to decrease these effects is to introduce a quasi–local correction via placing a set of non–linear elements in each cell near the source of the errors. A sequence of programs were used to perform this kind of correction. SIXTRACK was used to produce the high order transfer map in five variables using the DA–package of Berz. The tune–shift functions are derived with the Lielib package of E. Forest. Then, based on the approach proposed by A. Bazzani and G. Turchetti and first applied by E. Todesco, we developed a correction procedure to minimize these detuning functions up to fifth order (decapole contribution) considering four–dimensional tune–shift with the momentum deviation as a parameter. For different machine versions we computed correction schemes and compared the results with tracking simulations. In all cases a considerable improvement of the detuning was established.

I. Introduction

In the design of magnets for the new accelerators one can not completely avoid high order multipole errors which limit the dynamic aperture of these new machines. The strategy to tackle this problem is twofold: one has to set reasonable specifications for the magnet errors and propose a proper correction scheme for the residual non–linear content of the magnets. In both cases a profound knowledge of the sources for the aperture limiting effects is needed.
A good indicator of the nonlinearity of the machine is the tune–shift as a function of amplitude and momentum. The correction schemes will deal with minimizing the detuning. The straightforward approach is to carry out the optimization by tracking methods [1]. In that process it is not obvious how the different multipole components enter into the detuning functions especially when there are interfering terms.
There are different analytical methods to derive the tune–shift as a function of multipole errors and phase space coordinates. One of the possible approaches is using normal forms [2, 3]. This choice has the convenient advantage that there are ready–to–use software packages [4, 5] to attack the problem.

II. Correction method

The correction procedure is carried out using an order–by–order approach. We first determine the values of the sextupolar correctors needed to optimize the first order tune–shift. Then the computation of the decapoles is carried out working on the second order tune–shift. The approach used can deal with both amplitude–dependent contribution in the tune–shift and momentum–dependent effects (at the present the correction of the mixed terms is not considered but it is possible as well). In the former case there are $i+2$ coefficients at order i, while in the later one there are always only two coefficients which we denote by $\alpha_{x,i}(K_l), \alpha_{y,i}(K_l)$ where K_l is the integrated gradient of order l. Given a sufficient number of free correctors it is in principle possible to set to zero the tune–shift at order i. If this can not be fulfilled we can always define a norm in the tune–shift space which we can try to minimize. The choice commonly used [3, 6, 7] is:

$$\chi^2_{i,0} = \frac{1}{R^{2i+1}} \int_{\rho_1+\rho_2=R} \left[\delta\nu_{x,i}(\rho_1,\rho_2;K_l)\right]^2 d\rho_1 d\rho_2 + \frac{1}{R^{2i+1}} \int_{\rho_1+\rho_2=R} \left[\delta\nu_{y,i}(\rho_1,\rho_2;K_l)\right]^2 d\rho_1 d\rho_2 \quad (1)$$

for the amplitude–dependent case, where $\delta\nu_{x,i}, \delta\nu_{y,i}$ are the horizontal and vertical tune–shift functions at order i respectively. For the momentum–dependent effect we use:

$$\chi^2_{0,i} = \left[\alpha_{x,i}(K_l)\right]^2 + \left[\alpha_{y,i}(K_l)\right]^2 . \quad (2)$$

Both $\chi_{i,0}, \chi_{0,i}$ are low order polynomial functions of the integrated gradients K_l only. Therefore the correction strategy does not depend on a particular value of ρ_1, ρ_2, δ, but represents a global minimization over the whole phase space.

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The computation of the functions $\chi_{i,0}, \chi_{0,i}$ is carried out using SIXTRACK [8] and the Lielib package of E. Forest [5]. As a first step the one turn transfer map is produced including δ and the corrector strength as additional parameters. The result is a polynomial map in $N_{cor}+5$ variables, where N_{cor} is the number of free correctors. Then the tune–shift is computed using Lie–algebraic techniques. Finally we have added a special routine to minimize $\chi_{i,0}, \chi_{0,i}$.

III. Lattice Model

We consider two realistic models of the LHC including the differences between odd and even octants as well as insertions each having a different purpose. The differences between the models consist mainly in the cell layout. In the first case (LHC version 1) there are eight dipoles per cell and two central correctors consisting of sextupole, octupole and decapole magnets. Near the cell quadrupoles there are additional correctors. In this case the sextupoles are used to set to zero the linear chromaticity.
In the second case (LHC version2) there are only six longer dipoles per cell and each of them has a corrector at both ends: a sextupole and a decapole. Again additional sextupoles are placed near the focusing and defocusing quadrupoles to correct the linear chromaticity.
As far as the errors are concerned only the contributions due to the dipoles have been taken into account. The values of the systematic multipole errors forseen for the LHC dipoles and used in our studies are listed in Table 1.

Table 1
Normal dipole field errors in units of 10^{-4} at $R_r = 10mm$

Order n	Systematic at injection LHC Version 1	Systematic at injection LHC Version 2
0	–	5.00
1	−1.40	±1.15
2	−3.35	−2.19
3	±0.05	±0.11
4	0.45	0.34

The double sign is a feature introduced by the two–in–one geometry of the magnets and it produces a change in the sign of the error from odd to even octants. Therefore in the case of the octupole components there is a sort of self–compensation of the error along the whole ring. Because of this symmetry we decided to introduce in our models only the sextupole and decapole components of the field errors.

IV. Correction Schemes for Dipole Errors

Each correction scheme has to take care of linear chromaticity. This imposes a linear relation between the sextupoles and it fixes the values of the chromatic correctors as a function of the M cell correctors

$$\begin{cases} K_{2,F} &= \alpha_F + \beta_{1,F} K_{2,1} + \cdots \beta_{M,F} K_{2,M} \\ K_{2,D} &= \alpha_D + \beta_{1,D} K_{2,1} + \cdots \beta_{M,D} K_{2,M} \end{cases} \quad (3)$$

Therefore the number of free correctors available to minimize $\chi_{i,0}, \chi_{0,i}$ is reduced. For the LHC version 1 there are three free correctors:

$$K_{2,C}, K_{4,C}, K_{4,D} \quad (4)$$

The decapoles near the cell quadrupoles have been set equal $K_{4,D} = K_{4,F}$ so that the mid–cell symmetry is not broken. In this case we can fix the value of $K_{2,C}$ by minimizing the first order tune–shift (amplitude or momentum). The decapoles can then be used either to correct exactly the second order momentum–dependent tune–shift or to optimize $\chi_{2,0}$. The results are showed in Fig. 1: solid lines represent the detuning computed with normal forms, while dashed lines are obtained by direct tracking. The

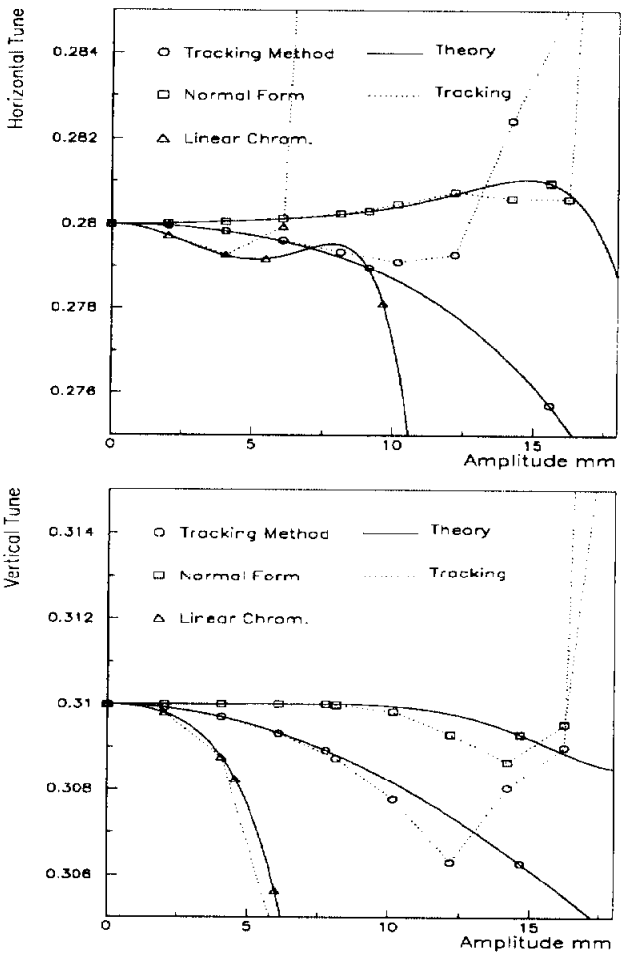

Figure 1: Detuning curves for LHC Version 1

tune–shift of the machine in which only the linear chromaticity is corrected and the results of a tracking–based

correction [1] is shown. Due to the better control on the higher order terms the normal form procedure allows to correct the machine so that a good agreement with the tracking can be found up to the dynamic aperture.
The scheme used for the second lattice, LHC version 2, is completely different. Only two free correctors are used:

$$K_{2,C}, K_{4,C} \tag{5}$$

in order to reduce the number of independent power supplies needed. In this situation the only choice is to minimize one of the functions $\chi_{i,0}, \chi_{0,i}$ using the two correctors. Due to the quasi–locality of the optimization scheme, correcting one of the two tune–shift functions (amplitude or momentum) implies a good correction of the other one (this holds also for LHC version 1). However for the latest version there are cases in which one correction type disturb the other. We found that dropping the concept of having a mid–cell corrector has the price that the effectiveness of the sextupole correction is reduced (compare the corresponding curves in Figs. 1,2).
The situation concerning the correction of the second order tune–shift with the decapoles is different. The correction can be performed almost perfectly. The reason for this difference is the fact that the sextupole strengths enter quadratically in the first order tune–shift, while the second order detuning functions depend linearly on the decapoles. It is interesting to stress that the results of the second order optimization depend on a good correction of the first order detuning because this reduces the interfering terms between sextupoles and decapoles: without this precondition it would be very difficult to achieve an optimal solution using decapoles. The results are shown in Fig. 2 where we compare two different sets of correctors obtained by minimizing $\chi_{1,0}$ and $\chi_{2,0}$ or by using $\chi_{0,1}$ and $\chi_{0,2}$. Besides the problems with the sextupole stated above also in this case a satisfactory correction has been achieved.

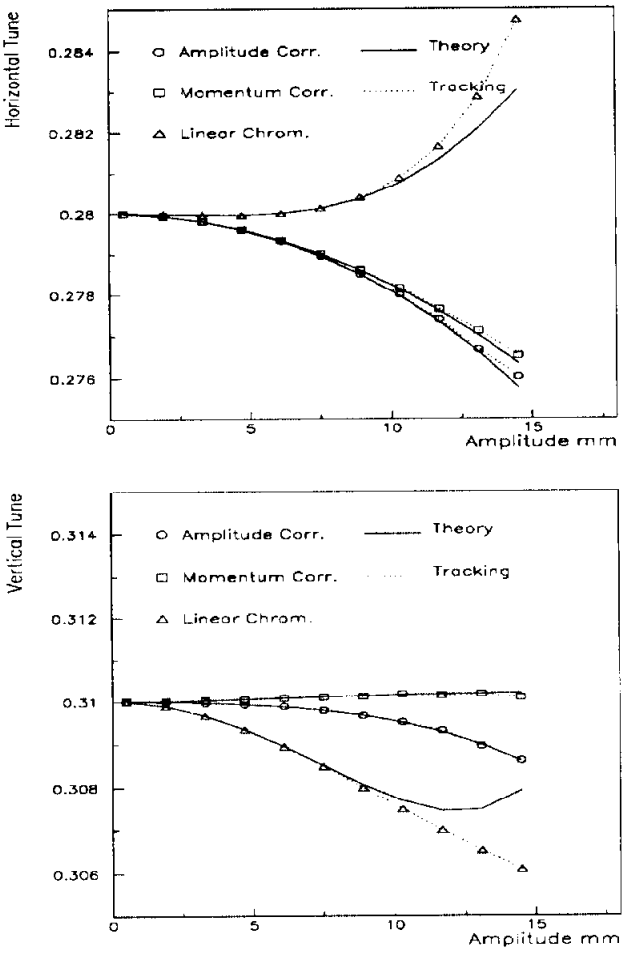

Figure 2: Detuning curves for LHC Version 2

V. Conclusions

We have shown the effectiveness of the normal form approach to correct the tune–shift due to field errors. Both the amplitude-dependent and the momentum–dependent tune–shift can be corrected applying this technique. However, we would like to stress that these correction results have always to be tested against tracking results. This is necessary to avoid solutions which are dynamically not acceptable.
Finally a user–friendly option to perform these corrections in SIXTRACK is in preparation.

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