SKEW QUADRUPOLE EFFECTS IN THE IBM COMPACT SYNCHROTRON

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Abstract

The normal operating condition for the IBM x-ray source is to have a skew quadrupole magnet energized to a focal length of -20 m. This paper details the effects seen by synchrotron light monitors as a consequence of varying the strength of this magnet. In particular, the cross section of the electron beam appears vertically expanded and *tilted*. Furthermore, these effects are slightly different in the two dipole magnets. These effects can be understood by deriving the normal modes for the linearly coupled betatron motions for the case of operation far from a coupling resonance line.

INTRODUCTION

The IBM compact synchrotron storage ring Helios, manufactured by Oxford Instruments, has two 180° bending angle superconducting dipole magnets separated by straight sections containing quadrupole magnets, injection magnets, rf cavity, diagnostics, and specialty magnets. One of these specialty magnets is a skew quadrupole designed to couple horizontal and vertical betatron motions if need be. During commissioning it was determined that the beam lifetime at full energy for typical stored currents could be increased significantly by energizing this magnet to 20 A corresponding to a thin lens focal length of -20 m. This effect agrees with Touschek scattering estimates. Since this lifetime enhancement is very desirable, this has become the normal operation condition.

Located at synchrotron light ports looking at the dipoles' centers are two synchrotron light monitors (SLM's). These devices focus the visible synchrotron light to form images of the electron beam cross section on CCD cameras which can then be frame grabbed for electronic processing and storage. Figure 1 shows a collection of images from the two SLM's illustrating the effect of the skew quadrupole. The vertical size at zero skew quadrupole current suggests there are some residual coupling fields in the storage ring. It is speculated that slight misalignment of the dipole coils could account for this residual since there is significant normal quadrupole field through the body of the dipole magnets.

Though the images are qualitatively similar in the two dipoles there are differences. Supported by detailed measurement is the observation that at any given

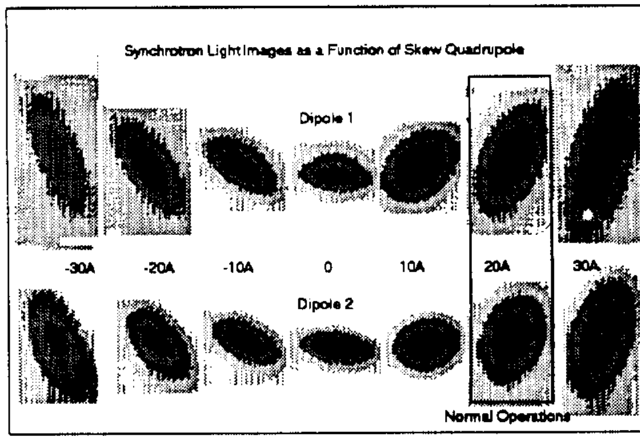

Figure 1. Composite SLM Images from Dipoles. The effect of the skew quadrupole magnet is to produce both vertical extension to the beam and an apparent tilt to the electron beam cross section. Relative image intensities have been adjusted to improve this presentation.

skew quadrupole strength the electron beam is more vertically extended in Dipole 1 than in Dipole 2. Furthermore, the apparent *tilt* of the image is more for Dipole 1.

NONRESONANT OPERATION OF A SINGLE THIN LENS SKEW QUADRUPOLE

The effect of a single thin lens skew quadrupole in a ring can be solved in the absence of higher order magnetic fields. One way to proceed would be to introduce the 4 by 4 transfer matrices of combined horizontal and vertical betatron motion and solve for the normal modes[1]. In the simple case of a single thin lens skew quadrupole, however, there is a 2 by 2 solution that more clearly illustrates several interesting features directly applicable to the Helios observations. The latter treatment is given here.

If we are explicitly seeking the normal modes of this problem then the horizontal (x) and vertical (y) motions should be synchronized and the thin lens skew quadrupole transfer matrix for the tranverse horizontal direction can be expressed as

$$S = \begin{pmatrix} 1 & 0 \\ kR & 1 \end{pmatrix} \quad \text{where} \quad R = \left(\frac{y}{x}\right)_{s=0} . \qquad (1)$$

The ratio R is a constant (to be determined) and k is the skew quadrupole strength. The azimuth coordinate s is measured from the skew quadrupole position.

Let M_{x0} denote the transfer matrix in the horizontal direction for one time around the ring in the absence of a skew quadrupole and let M_x be the transfer matrix including the skew quadrupole. The full ring transfer matrix has an explicit form in terms of betatron tune, ν for coupled and ν_x for uncoupled, and the α, β, and γ lattice parameters. This applies to both M_{x0} and M_x but for uncoupled and coupled values accordingly. Therefore, we can write $M_x = M_{x0}S$ or explicitly

$$\begin{pmatrix} \cos 2\pi\nu + \alpha_x \sin 2\pi\nu & \beta_x \sin 2\pi\nu \\ -\gamma_x \sin 2\pi\nu & \cos 2\pi\nu - \alpha_x \sin 2\pi\nu \end{pmatrix} = \begin{pmatrix} \cos 2\pi\nu_x + \alpha_{x0} \sin 2\pi\nu_x & \beta_{x0} \sin 2\pi\nu_x \\ -\gamma_{x0} \sin 2\pi\nu_x & \cos 2\pi\nu_x - \alpha_{x0} \sin 2\pi\nu_x \end{pmatrix} \begin{pmatrix} 1 & 0 \\ kR & 1 \end{pmatrix} . \quad (2)$$

The equation for the vertical direction is the same except that the x's and y's are interchanged and $R \rightarrow R^{-1}$. Equating the traces gives

$$\cos 2\pi\nu = \cos 2\pi\nu_x + \tfrac{1}{2} kR\beta_{x0} \sin 2\pi\nu_x \quad (3)$$

and from the vertical direction,

$$\cos 2\pi\nu = \cos 2\pi\nu_y + \tfrac{1}{2} k \frac{1}{R} \beta_{y0} \sin 2\pi\nu_y . \quad (4)$$

Eliminating R from these two equations gives

$$\cos 2\pi\nu = \tfrac{1}{2}(\cos 2\pi\nu_x + \cos 2\pi\nu_y) \pm \quad (5)$$
$$\tfrac{1}{2}\sqrt{(\cos 2\pi\nu_x - \cos 2\pi\nu_y)^2 + k^2\beta_{x0}\beta_{y0} \sin 2\pi\nu_x \sin 2\pi\nu_y} .$$

There are three distinct regions of betatron tune space depending on whether the uncoupled tunes are far from or near to a sum or difference resonance line, $\nu_x \pm \nu_y = integer$. The radical of Eqn. (5) becomes imaginary in the stop band of the sum line. On the other hand, a common practice for studying coupling effects in storage rings is to bring the betatron tune operating point close to a difference resonance line [2], thereby amplifying coupling effects near degeneracy. However, the normal betatron tune operating point for Helios, with ν_x, ν_y being 1.43,0.64, is far from such a condition. Expanding the square root to second order in k in this case gives

$$\cos 2\pi\nu_\pm = \cos 2\pi\nu_{x,y} \pm \frac{\tfrac{1}{4} k^2\beta_{x0}\beta_{y0} \sin 2\pi\nu_x \sin 2\pi\nu_y}{\cos 2\pi\nu_x - \cos 2\pi\nu_y} . \quad (6)$$

Corresponding to these two solutions are the amplitude ratios

$$R_+ = \frac{\tfrac{1}{2} k\beta_{y0} \sin 2\pi\nu_y}{\cos 2\pi\nu_x - \cos 2\pi\nu_y}$$
$$\frac{1}{R_-} = \frac{-\tfrac{1}{2} k\beta_{x0} \sin 2\pi\nu_x}{\cos 2\pi\nu_x - \cos 2\pi\nu_y} . \quad (7)$$

These expressions were used to determine starting coordinates for tracking results shown in Figure 2.

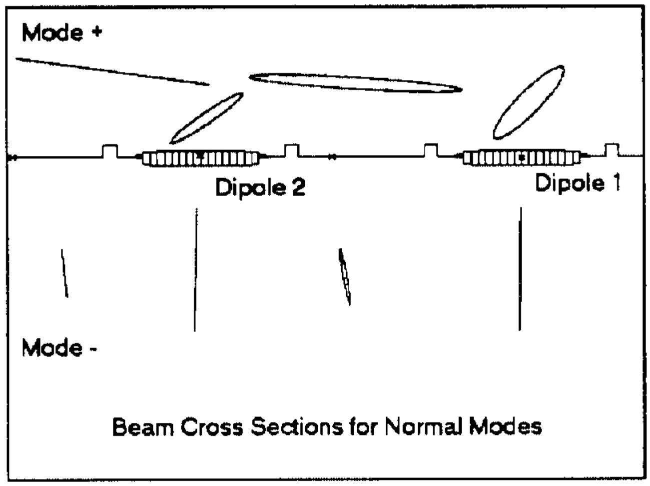

Figure 2. X-Y Tracking Maps for the Normal Modes. Maps for the two normal modes at standard Helios operating conditions are shown for azimuth locations depicted by x's in the beamline schematic.

The other matrix elements of Eqn. (2) provide expressions for modifying the lattice parameters at the skew quadrupole position. In particular,

$$\beta_y = \beta_{y0} \frac{\sin 2\pi\nu_y}{\sin 2\pi\nu} \quad \text{and}$$
$$\alpha_y = \alpha_{y0} \frac{\sin 2\pi\nu_y}{\sin 2\pi\nu} + \frac{\cos 2\pi\nu - \cos 2\pi\nu_y}{\sin 2\pi\nu} \quad (8)$$

with similar expressions for the x direction.

For each normal mode there is a major member and a minor member in terms of contributions from the uncoupled vertical and horizontal motions. In the weak coupling case one such mode ν_+ is close to the uncoupled horizontal tune ν_x and the y amplitude is much smaller than the x amplitude. The expressions for β and α (Eqns. (8)) then show that the new Twiss parameters in the x direction are little changed by the

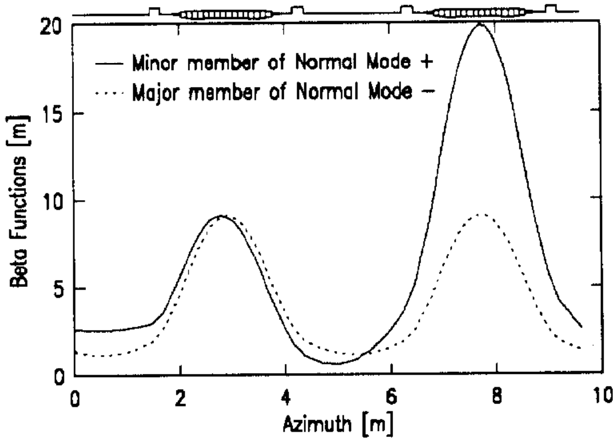

Figure 3. Vertical Beta Functions for Normal Modes

coupling, however, the parameters in the y direction are very much altered.

Figure 3 shows the vertical betatron functions for each of the normal modes. The strongly perturbed function for the minor member of the + normal mode has been calculated in a standard beamline lattice code using a representation with initial values for the Twiss parameters from Eqns. (8). As illustrated by the beamline schematic, the Helios skew quadrupole is not at a high symmetry point. This helps explain the asymmetry in the beta function maxima for the minor member of the + normal mode.

It is a good approximation that the energy change due to emission of synchrotron radiation initially results in a pure horizontal excitation. Expressed in terms of the normal modes, the vertical components of such an excitation must initially cancel. As a consequence, the coupling results in two contributions to

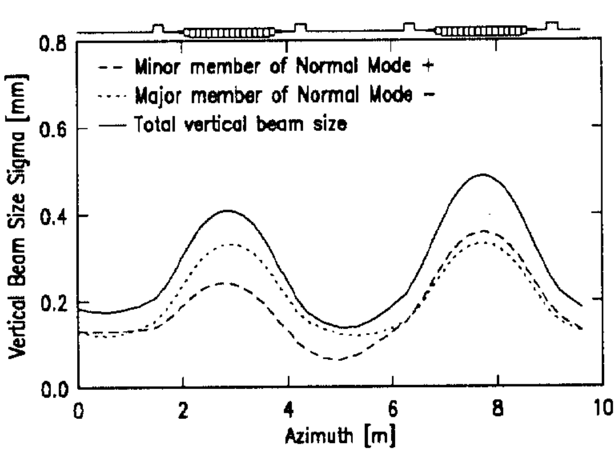

Figure 4. Contributions to the Vertical Beam Size

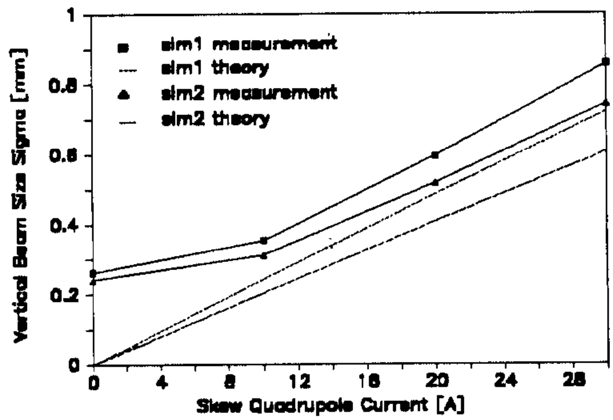

Figure 5. Vertical Beam Size

the vertical beam size. One comes from the y component of the + normal mode:

$$\sigma_{y+}^2(s) = \frac{\beta_{y+}(s)}{\beta_{y+}(0)} \frac{\varepsilon_{x0}\beta_{x0}(0)}{(R_+^{-1} - R_-^{-1})^2} \qquad (9)$$

and the other comes from the y component of the - normal mode:

$$\sigma_{y-}^2(s) = \frac{\beta_{y-}(s)}{\beta_{y-}(0)} \frac{\varepsilon_{x0}\beta_{x0}(0)}{(R_+^{-1} - R_-^{-1})^2} \qquad (10)$$

where ε_{x0} is the total emittance. Over time with incommensurate phase advances in general, the vertical components of the two normal modes will have random relative phase. They will, therefore, add as uncorrelated contributions: $\sigma_y^2(s) = \sigma_{y+}^2(s) + \sigma_{y-}^2(s)$. Figure 4 shows these contributions to the vertical beam size for the case of normal Helios operating conditions.

In Figure 5 the measured vertical beam sizes at the two SLM's as a function of skew quadrupole strength are compared with calculation. The discrepancy at zero skew quadrupole strength due to residual coupling effects is not addressed by this calculation.

REFERENCES

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