Measurement of Longitudinal Beam Polarization by Synchrotron Radiation*

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Abstract

A method to measure the longitudinal polarization of an electron beam using synchrotron radiation is proposed. Quantum theory predicts that a longitudinally polarized electron emits a slightly different number of synchrotron photons into the space above and below the orbit plane. The degree of this asymmetry is proportional to the magnetic field strength and the ratio between photon and electron energy. For CEBAF (electron energy $E_e = 4$ GeV and 100 μA beam current), a dedicated bending magnet with the field strength of $B = 6.25$ T permits us to measure a 100% polarized beam with an accuracy of about 1% in one second when the measurement is performed with photons in the range between $E_{\gamma\min} = 6.0$ keV and $E_{\gamma\max} = 600$ keV. The technique for measuring this asymmetry is also discussed.

I. INTRODUCTION

The standard polarimeters in high-energy electron storage rings and colliders are laser polarimeters [1, 2]. A circularly polarized laser beam is directed against the particle beam and the distribution of the backscattered photon beam is measured.

Several years ago the Novosibirsk polarization group used the predictions of the quantum mechanical theory of synchrotron radiation to measure the transverse polarization in a storage ring [3]. They found good agreement between the theory developed by Sokolov and Ternov [4, 5] and the experimental results. The aim of this paper is to show that the longitudinal polarization can be measured as well by using another prediction of the same theory.

II. PRINCIPAL CONSIDERATIONS

The starting point is the famous quantum mechanical description of synchrotron radiation developed by Sokolov and Ternov [4, 5]. According to this theory the angular and spectral distributions of the intensity of the σ and π polarized synchrotron radiation emitted from an electron with arbitrary spin direction can be calculated by the following formula:

$$W_{\sigma,\pi} = \frac{27}{16\pi}\frac{ce_0{}^2}{R^2\epsilon_0^{9/2}}\int_0^\infty \frac{y^2dy}{(1+\xi y)^5}\oint d\Omega\Phi_{\sigma,\pi} \quad (1)$$

*This work was supported by the U.S. Department of Energy under contract DE-AC05-84ER40150.

The indices σ and π refer to the two different planes of synchrotron light polarization: σ refers to light with a polarization vector in the orbit plane, π to light polarized perpendicularly to this plane. $\Phi_{\sigma,\pi}$ describes the angular and spectral distribution of the radiation. For longitudinally polarized electrons these functions (neglecting spin-flips caused by synchrotron radiation) are:

$$\Phi_\sigma = \frac{4}{\gamma^4}\frac{(1+\xi y)}{12\pi^2}(1+\alpha^2)^2K_{2/3}^2(z)$$
$$\left(1+\zeta\xi y\frac{\alpha}{\sqrt{1+\alpha^2}}\frac{K_{1/3}(z)}{K_{2/3}(z)}\right) \quad (2)$$

$$\Phi_\pi = \frac{4}{\gamma^4}\frac{(1+\xi y)}{12\pi^2}\alpha^2(1+\alpha^2)K_{1/3}^2$$
$$\left(1+\zeta\xi y\frac{\sqrt{1+\alpha^2}}{\alpha}\frac{K_{2/3}(z)}{K_{1/3}(z)}\right) \quad (3)$$

where ζ is the electron polarization, $\epsilon_0 = 1-\beta^2$, $z = \frac{\omega}{2\omega_c}(1+\alpha^2)^{3/2}$, $\omega_c = \frac{3}{2}\omega_0\gamma^3$, $\omega_0 = \frac{c}{R}$, c is the speed of light, R is the bending radius, $\gamma = \frac{E}{m_0c^2}$, e_0 is electron charge, $y = \frac{\omega}{\omega_c}$, $\alpha = \gamma\psi$, ψ is the vertical angle between the orbit plane and the direction of the radiated photons, θ is the bend angle, $K_{1/3}(z)$ and $K_{2/3}(z)$ are modified Bessel functions, $\xi = \frac{3}{2}\frac{H}{H_0}\frac{E}{m_0c^2}$, and $H_0 = \frac{m_0^2c^3}{e_0\hbar} = 4.41\times10^{13}$ Oe.

In the case of transverse spin polarization, (2) and (3) must be replaced by:

$$\Phi_\sigma = \frac{1+\xi y}{3\pi^2\gamma^4}(1+\alpha^2)^2$$
$$\left(1+\xi y-\zeta\frac{\xi y}{\sqrt{1+\alpha^2}}\frac{K_{1/3}(z)}{K_{2/3}(z)}\right)K_{2/3}^2(z) \quad (4)$$

$$\Phi_\pi = \frac{2+3\xi y}{6\pi^2\gamma^4}\alpha^2(1+\alpha^2)K_{1/3}^2(z) \quad (5)$$

The dependence of the synchrotron radiation intensity on the transverse polarization described in formulas (4) and (5) was experimentally verified by the Novosibirsk polarization group [3].

Equations (2) and (3) show that the synchrotron light intensity depends slightly on the longitudinal spin polarization $\zeta = \pm1$. This paper discusses the possibility of measuring this asymmetry, which is buried in a highly spin-independent background.

If we combine equations (1), (2), and (3), and assume $\xi y \ll 1$, the angular distribution of the intensity radiated by n_e electrons into the solid angle $d\psi d\theta$ for a given photon frequency y is

$$d^3W = \frac{9n_e}{16\pi^3}\frac{ce_0^2}{R^2}\gamma^5 y^2 dy F(\alpha) d\psi d\theta \quad (6)$$

where

$$F(\alpha) = (1+\alpha^2)^2 \left[K_{2/3}^2(z)\left(1+\zeta\xi y\frac{\alpha}{\sqrt{1+\alpha^2}}\right) + \frac{\alpha^2}{1+\alpha^2}K_{1/3}^2(z)\left(1+\zeta\xi y\frac{\sqrt{1+\alpha^2}}{\alpha}\right)\right] \quad (7)$$

Integrating (6) over $\psi = \pm\pi/2$ (see Fig. 1) and using $F(\alpha)$ from (7) yields the intensity of radiation emitted into the interval $dyd\theta$. This intensity is

$$d^2W = \frac{9n_e}{16\pi^3}\frac{ce_0^2}{R^2}\gamma^5 y^2 dy d\theta \int_{-\pi/2}^{\pi/2}(1+\alpha^2)^2\left[K_{2/3}^2(z) + \frac{\alpha^2}{1+\alpha^2}K_{1/3}^2(z)\right]d\psi \quad (8)$$

and independent of the spin. The intensity of the photons emitted into the space above the orbit plane is obtained by integrating (8) from 0 to $+\pi/2$ and the intensity of the photons emitted into the space below the orbit plane is obtained by integrating (8) from 0 to $-\pi/2$. The difference of these two integrals is spin-dependent,

$$d^2W = \frac{9n_e}{8\pi^3}\frac{ce_0^2}{R^2}\gamma^5 y^2 dy d\theta \zeta\xi y \int_0^{\pi/2}\alpha(1+\alpha^2)^{3/2}\left[K_{1/3}^2(z) + K_{2/3}^2(z)\right]d\psi \quad (9)$$

For further investigations it is convenient to convert the formulas for intensities into formulas for photon numbers by dividing equations (8) and (9) by the photon energy

$$\epsilon_\gamma = \frac{3}{2}\frac{\hbar c}{R}y\gamma^3 \quad (10)$$

Taking into account that $e_0^2/\hbar c = 1/137$ and

$$n_e\frac{e_0}{e_0}\frac{c}{R} = 2\pi\frac{I_0}{e_0}, \quad (11)$$

replacing $d\theta$ by the finite horizontal angle $\Delta\theta$, and $d\alpha = \gamma d\psi$, the total number of emitted photons, N_γ, and the number of photons in the flux difference, $\Delta N_\gamma(\zeta)$, can be presented as follows:

$$N_\gamma = \frac{3}{4\pi^2}\frac{1}{137}\frac{I_e}{e_0}\gamma\Delta\theta \int_{y_1}^{y_2} y dy \int_{-\pi/2}^{+\pi/2} F(\alpha, y) d\alpha \quad (12)$$

and

$$\Delta N_\gamma(\zeta) = \frac{3}{2\pi^2}\frac{1}{137}\frac{I_e}{e_0}\xi\gamma\zeta\Delta\theta \int_{y_1}^{y_2} y^2 dy \int_0^{\pi/2} F(\alpha,\zeta,\xi y) d\alpha \quad (13)$$

where $F(\alpha)$ and $F(\alpha,\zeta,\xi y)$ are the unchanged integrands from (8) and (9).

III. RESULTS OF THE CALCULATIONS AND THE MEASURING TECHNIQUE

Equations (12) and (13) were solved in order to find the resolution in the measurement of the asymmetry $\Delta N_\gamma(\zeta)$ which is mainly limited by the quantum fluctuation of N_γ.

Another limitation comes from the fact that the beam has a finite emittance. This is taken into account by performing the integration in (13) with $\alpha_1 \geq \gamma\sigma_{y'}$ as a lower limit.

The detector can be a transparent differential ionization chamber (DIC) [6] (Fig. 2). It consists of two ionization chambers with electrodes of identical length and identical interelectrode distance. The collecting electrodes are united and connected with an electrometric amplifier. The high voltage for the electrodes is the same but the voltage has opposite sign. The chambers work in the regime of full ion collection so that the dark current is determined by the cable leakage and the variation of the background radiation within the chamber. Systematic errors can be eliminated by measuring with an unpolarized beam and by reversing the electron polarization.

In order to calculate the number of photons absorbed in the ionization chamber the integrands in (12) and (13) have to be multiplied by the absorption function $A(\mu, t)$:

$$A(\mu, t) = 1 - \exp[-\mu(\lambda)\cdot t] \quad (14)$$

where t is the length of the ionization chamber, $\lambda = \lambda_c/y$, and $\mu(\lambda)$ is the linear absorption coefficient described by the empirical expression [7]

$$\mu(\lambda) = 0.023\frac{Z}{A}\rho(Z\lambda)^{2.78} \quad (15)$$

where ρ is the density in g/cm^3, Z is the number of protons and A is the number of nucleons of the ionization gas, and λ is the wavelength in Å.

The results of the calculations are presented in Table 1, and the parameters used for the calculations—magnetic field strength, type of detector, the gas of the ionization chamber and the chamber length, and the spectral and the angular limits—are presented in Table 2. In all these calculations $\Delta\theta$ is 10^{-2} rad and $\sigma_{y'}$ is 2×10^{-5}.

The tables cover the energy range of the existing polarized electron machines: CEBAF (0.8–4.0 GeV, I= 0.1 mA), Bates (MIT) (0.4–0.9 GeV, I= 10 mA) and HERA, LEP, and SLC (27–45 GeV).

In the following the polarization detector proposed for CEBAF is described in more detail (Fig. 2). The synchrotron radiation is created by a superconductive three-pole wiggler with a pole length of 10 cm.

A transparent differential ionization chamber working in the full ion collection regime has a dark current of $I_{dark} \ll 10^{-14}$ A. This corresponds for an air-filled DIC to about 300 photons with an energy of 10 keV. Since the expected current in each part of the differential chamber is higher than 10^{-8} A, the dark current can be neglected.

The ionization chamber can be centered with the help of the visible part of the synchrotron radiation. For visible light $\xi y \sim 0$ and therefore the asymmetry caused by polarization is negligible. The visible light detectors can be silicon photodiodes [8] with a dark current of about 0.1 fA. In order to protect the photodiodes from the radiation damage caused by hard x rays, mirrors can be used as shown in Fig. 2.

IV. ACKNOWLEDGMENT

The authors would like to thank Dr. A. Radyushkin from the CEBAF theory group for valuable discussions.

V. REFERENCES

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Table 1. Results of the Calculations

Energy, current (GeV, mA)	Photon intensity N_γ (photons/s)	Flux asymmetry $\Delta N_\gamma(\zeta)$ (photons/s)	Flux fluctuation $\Delta N_\gamma(fl)$ (photons/s)	Accuracy, time $\Delta\zeta/\zeta$; Δt (%, s)
0.5, 0.1	5.94×10^{10}	4.30×10^{5}	2.44×10^{5}	5.1, 120.
0.5, 10.0	5.94×10^{12}	4.30×10^{7}	2.43×10^{6}	1.0, 32.
1.0, 0.1	2.03×10^{12}	1.98×10^{7}	1.42×10^{6}	1.0, 50.
1.0, 10.0	2.03×10^{14}	1.98×10^{9}	1.42×10^{7}	0.7, 1.0
2.0, 0.1	2.65×10^{13}	2.70×10^{8}	5.14×10^{6}	1.0, 4.0
3.0, 0.1	6.18×10^{13}	6.85×10^{8}	7.86×10^{6}	1.15, 1.0
4.0, 0.1	9.78×10^{13}	1.10×10^{9}	9.88×10^{6}	0.9, 1.0
50.0, 0.001	13.93×10^{12}	5.25×10^{8}	3.37×10^{6}	0.7, 1.0

Table 2. The Parameters Used in Table 1

Energy (GeV)	Type of detector (filling gas, length in m)	Strength of magn. field (T)	Spectral bandwidth $(\lambda_c/\lambda_1)/(\lambda_c/\lambda_2)$	Lower angle of integration $\psi\gamma$
0.5	Air, 0.5	6.25	4.0/14.0	0.02
1.0	Kr, 0.1	6.25	1.7/14	0.04
2.0	Xe, 0.1	6.25	0.4/10.0	0.08
3.0	Xe, 1.0	6.25	0.18/10.0	012
4.0	Xe, 1.0	6.25	0.1/10.0	0.16
50.0	Shower counter	1.5	0.1/10.0	0.16

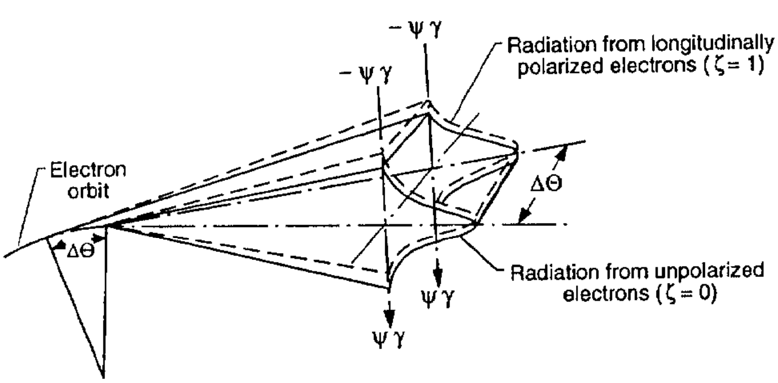

Figure 1: Geometrical definitions.

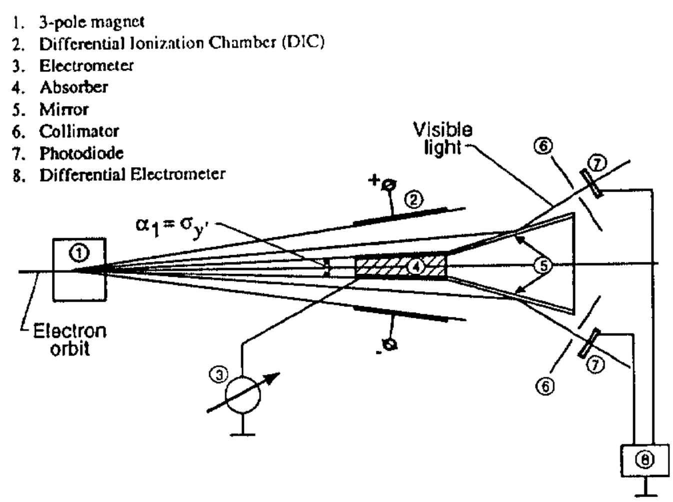

Figure 2: A possible detector design. The synchrotron radiation created by a wiggler is detected by a transparent differential ionization chamber. The chamber measures the asymmetry in the synchrotron radiation distribution. The visible light is used to center the chamber. The detectors for the visible light can be silicon photodiodes with a low dark current.