SPINLIE - new computer code for polarization calculation

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Abstract

The application of the analytical method of spin calculation is described. The calculation of both orbital and spin motion is based on Lie operators technique. The computer code SPINLIE realizing this method is discussed. The input language of SPINLIE is compatible with that of MAD. In SPINLIE elements are described as "thick lenses" for spin motion as well as for orbital calculations. The explicit expressions of Lie operators were found for orbital and spin motion for elements of different type (bending magnets, quadrupoles, sextupoles, RF-cavities, solenoids, kickers). The rules of addition of spin transformations were obtained for the beam passing the collider structure. Good agreement was found for SPINLIE results for the linear spin resonances in comparison with the other codes (SITF [1], SMILE [2]). The first results are presented for the calculation of nonlinear spin resonances.

I. Introduction

Calculations of the spin motion in colliders are interesting in connection with different schemes of experiments with the polarized beams. There are several computer codes for polarization degree calculations. Since there is a necessity to estimate the influence of nonlinear orbital and spin motion on the polarization degree, the attempts have been made to find more reliable and adequate methods for these calculations. This report presents a new computer code SPINLIE for polarization calculations for accelerators and colliders. The terms of sextupole type for orbital and quadratic for spin motion are precisely taken into account due to the usage of the Lie operator technique.

Most of the problems where the nonlinear character of orbital motion should be taken into account (dynamical aperture, lifetime, ...) require taking into consideration large amplitudes as well. Application of analytical methods for these purposes is not always justified. But these methods have some advantages when we are dealing with problems where small amplitudes are under examination (chromatism, polarization, ...). The analytical methods have the best accuracy and, as a rule, higher calculation speed, since all elements (linear and nonlinear) are considered to be thick.

II. The brief comparison of different codes for spin calculation

It is clear, that the main point for comparison in different approaches is the "level" of nonlinearity which is allowed in the codes. As is known, the resonance order is $\mid k \mid + \mid k_x \mid + \mid k_z \mid + \mid k_s \mid$ for a resonance $\nu_{sp} = k + k_x\nu_x + k_z\nu_z + k_s\nu_s$, where ν_{sp}, ν_i are spin, betatron and synchrotron tunes respectively and the values k, $k_i (i = x, z, s)$ are integer.

The connection between the order of the resonance and the solution of the spin motion equation is determined, as is known, by the so-called the "resonance denominator" in the form [3] $(1 - e^{2\pi i(\nu + \sum_i k_i \nu_i)})$ and describes the N-order resonance ($N = \sum_i \mid k_i \mid$) correspondingly. In this context the "spin" codes SLIM [4], SITF and ASPIRRIN [5] are purely linear [1]. The example of the linear calculations is presented in Fig.1 for code SITF and new code SPINLIE. Good agreement is received.

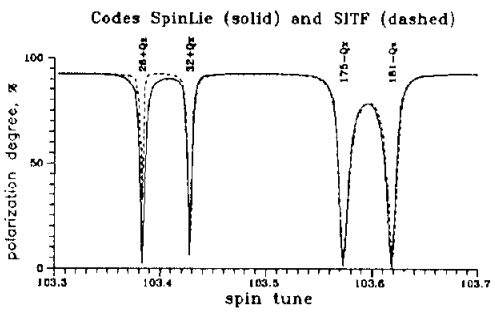

Figure 1: Linear resonanses for LEP structure N21C20.

The nonlinear N-order resonance is caused by three reasons. They are:

[1] The code ASPIRRIN estimates the strength of the synchrotron sideband spin resonances due to the usage of enhancement factors [6].

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a) the correspondent order term $\vec{\omega}^{(N)}$ of the precession frequency $\vec{\Omega}$ expansion in terms of the orbital vector $\vec{Z}$;

b) all kinds of products of the terms $\vec{\omega}^{(k)}$ such, that $\sum k = N$. These products appear from the terms of so-called θ-ordered solution [3] of the spin motion equation and describe noncommutation of the rotation sequences;

c) the terms which take into account the influence of nonlinearity of the orbital motion.

All "linear" codes include the term $\vec{\omega} = \vec{\omega}^{(1)}$ from the item a) because $\vec{\omega}$ has the same (first) order of infinitesimal as orbital vector $\vec{Z}$. The codes SMILE and SODOM [7] take into account item b) only for the high order resonances (for these codes $\vec{\omega}$ is the same as for "linear" codes). The code SITROS [8] involves all items (second order), but the last of them only partially. The terms are omitted, which describe the nonliearity contribution caused by the finite thickness of elements.

All terms a)-c) are taken into account for the second order resonances in the computer code SPINLIE.

III. LIE OPERATORS

There are two completely equivalent methods for nonlinear analytical matrixes calculations: perturbation method [9] and Lie operators technique [10]. As in any correct method, the starting and the final formulae are absolutely the same, but the intermediate steps are different. It seems, that the Lie operator method is shorter but, probably, a more formal way to the result.

A. Operators of the Orbital and Spin Transformations

In accordance with the Lie technique [10], [11] the equation for spin motion has the solution in the following form for the Hamiltonian $\mathcal{H}_{orb} + \vec{\Omega}\vec{n}$, which does not explicitly depend on the azimuth [12]:

$$\vec{n}(\theta) = e^{-\theta:\mathcal{H}_{orb}+\vec{\Omega}\vec{n}:}\vec{n}(0) = \mathcal{M}\vec{n}(0),$$

where the precession frequency vector $\vec{\Omega}$ is defined by BMT's equation [13] and $\vec{n}(0)$ is the initial spin vector for $\theta = 0$. The semicolons emphasize the operator nature of this formula.

In accordance with the Hamilton equations this operator satisfies the equation

$$\frac{d\mathcal{M}}{d\theta} = \mathcal{M} : -(\mathcal{H}_{orb} + \vec{\Omega}\vec{n}) : .$$

Let us separate the total Hamiltonian in two parts: all terms, which describe the linear orbital and spin motion ($\mathcal{H}_0(\vec{Z},\vec{n})$), and all terms of high orders $\mathcal{H}_1(\vec{Z},\vec{n}) + \mathcal{H}_2(\vec{Z},\vec{n})$). The part $\mathcal{H}_0$ includes the polynomial of the 2-nd degree over orbital vector $\vec{Z}$ from $\mathcal{H}_{orb}$ and part $\vec{\Omega}_0 \equiv \vec{\omega^{(0)}}$ of the spin precession frequency $\vec{\Omega}$, which does not depend on $\vec{Z}$. Similarly, $\mathcal{H}_1$ includes the polynomial of the 3-rd degree from $\mathcal{H}_{orb}$ and the linear (over $\vec{Z}$) part $\vec{\omega^{(1)}}$ of $\vec{\Omega}$. At last, as we consider here the sextupole order of the nonlinearity only, therefore $\mathcal{H}_2$ includes only the quadratic part $\omega^{(2)}$ of $\vec{\Omega}$:

$$\begin{aligned}
\mathcal{H}_0 &= h^{(0)}_{ij} Z_i Z_j + \omega^{(0)}_{\alpha} n_\alpha, \\
\mathcal{H}_1 &= h^{(1)}_{ijk} Z_i Z_j Z_k + \omega^{(1)}_{\alpha i} n_\alpha Z_i, \\
\mathcal{H}_2 &= \omega^{(2)}_{\alpha ij} n_\alpha Z_i Z_j,
\end{aligned}$$

where all Roman indexes $i, j, \ldots$ correspond to 1,2,...,6 and all Greek indexes $\alpha, \beta, \ldots$ correspond to x, r, z.

Let us represent the operator $\mathcal{M}$ as a product of two exponential operators [11], [12]: $\mathcal{M} = \mathcal{M}_r \cdot \mathcal{M}_0$. As is known, operator $\mathcal{M}_0$ is simply matrixes for the linear transformation from the initial to the current azimuth θ: $\mathcal{A}(\theta)$ for orbital or $\mathcal{S}(\theta)$ for spin motion. Using the equation for the operator $\mathcal{M}$, the Lie technique and restricting the sextupole terms only, one can find:

$$\mathcal{M}_r = e^{-:f_2:}e^{-:f_1:} \simeq E - : f_1 : - : f_2 : + \frac{: f_1 :^2}{2},$$

where E is a unit operator and

$$\begin{aligned}
: f_1 : &= -\int_0^\theta d\theta' : (-\mathcal{H}_1(\mathcal{A}\vec{Z}, \mathcal{S}\vec{n})\,|_{\theta'}) :, \\
: f_2 : &= -\int_0^\theta d\theta' : (-\mathcal{H}_2(\mathcal{A}\vec{Z}, \mathcal{S}\vec{n})\,|_{\theta'}) : + \\
&+ \frac{1}{2}\int_0^\theta d\theta' : \Big(: f_1 \,|_{\theta'} : \Big(- \mathcal{H}_1(\mathcal{A}\vec{Z}, \mathcal{S}\vec{n})\,|_{\theta'} \Big)\Big) : \quad .
\end{aligned}$$

Thus, we obtain the following rules for the calculation of the Lie operators $\mathcal{F}$, $\mathcal{W}^{(1)}$ and $\mathcal{W}^{(2)}$:

$$\begin{aligned}
: f_1 : &= \mathcal{F}_{ijk} : Z_i Z_j Z_k : + \mathcal{W}^{(1)}_{\alpha i} : n_\alpha Z_i :, \\
: f_2 : &= \mathcal{W}^{(2)}_{\alpha ij} : n_\alpha Z_i Z_j :; \\
\mathcal{F}_{ijk}(\theta) &= \int_0^\theta d\theta' h^{(1)}_{lmn} \mathcal{A}_{li}(\theta')\mathcal{A}_{mj}(\theta')\mathcal{A}_{nk}(\theta'), \\
\mathcal{W}^{(1)}_{\alpha i}(\theta) &= \int_0^\theta d\theta' \mathcal{S}^{-1}_{\alpha\beta}(\theta')\omega^{(1)}_{\beta j}\mathcal{A}_{ji}(\theta'), \\
\mathcal{W}^{(2)}_{\alpha ij}(\theta) &= \int_0^\theta d\theta' \Big(\mathcal{S}^{-1}_{\alpha\beta}(\theta')\omega^{(2)}_{\beta kl}\mathcal{A}_{ki}(\theta')\mathcal{A}_{lj}(\theta') + \\
&+ \frac{1}{2} e_{\alpha\beta\gamma}\mathcal{W}^{(1)}_{\beta i}(\theta')\mathcal{S}^{-1}_{\gamma\lambda}(\theta')\omega^{(1)}_{\lambda k}\mathcal{A}_{kj}(\theta') + \\
&+ \mathcal{S}^{-1}_{\alpha\beta}(\theta')\omega^{(1)}_{\beta k}\mathcal{A}_{kl}(\theta')J_{lm}\mathcal{F}_{mij}(\theta')\Big).
\end{aligned}$$

B. The rules for transformation of the operators

Let us denote all constructions for the transformation from the azimuth θ_0 to θ' as: $\mathcal{A}\,|_{\theta_0\to\theta'} \equiv \mathcal{A}^{(u)}$, $\mathcal{S}\,|_{\theta_0\to\theta'} \equiv \mathcal{S}^{(u)}$, $\mathcal{F}\,|_{\theta_0\to\theta'} \equiv \mathcal{F}^{(u)}$, $\mathcal{W}^{(1)}\,|_{\theta_0\to\theta'} \equiv \mathcal{U}^{(1)}$, $\mathcal{W}^{(2)}\,|_{\theta_0\to\theta'} \equiv \mathcal{U}^{(2)}$. Then

$$Z_i(\theta') = A^{(u)}_{ij} Z_j(\theta_0) + A^{(u)}_{ij} J_{jk} \mathcal{F}^{(u)}_{klm} Z_l(\theta_0) Z_m(\theta_0$$

$$\vec{n}(\theta') = \mathcal{S}^{(u)} e^{-\mathcal{U}^{(1)}_{\alpha i} Z_i(\theta_0) n_\alpha(\theta_0):} \cdot e^{-\mathcal{U}^{(2)}_{\beta jk} Z_j(\theta_0) Z_k(\theta_0) n_\beta(\theta_0):} \cdot \vec{n}(\theta_0$$

Similarly, let us use for the transformation from θ' to θ the values $A^{(v)}_{ij}$, $\mathcal{S}^{(v)}$, $\mathcal{F}^{(v)}$, $\mathcal{V}^{(1)}$, $\mathcal{V}^{(2)}$ and for "merging"

transformation from θ_0 to θ the values $A_{ij}^{(w)}$, $\mathcal{S}^{(w)}$, $\mathcal{F}^{(w)}$, $\mathcal{W}^{(1)}$, $\mathcal{W}^{(2)}$. Then, the rules of merging of elements to a unified element from θ_0 to θ are:

$$\begin{aligned}
\mathcal{A}_{ij}^{(w)} &= \mathcal{A}_{ik}^{(v)}\mathcal{A}_{kj}^{(u)},\\
\mathcal{F}_{ijk}^{(w)} &= \mathcal{F}_{ijk}^{(u)} + \mathcal{F}_{lmn}^{(v)}\mathcal{A}_{li}^{(u)}\mathcal{A}_{mj}^{(u)}\mathcal{A}_{nk}^{(u)},\\
\mathcal{S}_{\alpha\beta}^{(w)} &= \mathcal{S}_{\alpha\gamma}^{(v)}\mathcal{S}_{\gamma\beta}^{(u)},\\
\mathcal{W}_{\alpha i}^{(1)} &= \mathcal{U}_{\alpha i}^{(1)} + \mathcal{S}^{(u)-1}_{\alpha\beta}\mathcal{V}_{\beta j}^{(1)}\mathcal{A}_{ji}^{(u)},\\
\mathcal{W}_{\alpha ij}^{(2)} &= \mathcal{U}_{\alpha ij}^{(2)} + \mathcal{S}^{(u)-1}_{\alpha\beta}\mathcal{V}_{\beta kl}^{(2)}\mathcal{A}_{ki}^{(u)}\mathcal{A}_{lj}^{(u)} +\\
&+ \frac{1}{2}e_{\alpha\beta\gamma}\mathcal{U}_{\beta i}^{(1)}\mathcal{S}^{(u)-1}_{\gamma\lambda}\mathcal{V}_{\lambda k}^{(1)}\mathcal{A}_{kj}^{(u)} +\\
&+ \mathcal{S}^{(u)-1}_{\alpha\beta}\mathcal{V}_{\beta k}^{(1)}\mathcal{A}_{kl}^{(u)}J_{lm}\mathcal{F}_{mij}^{(u)}.
\end{aligned}$$

IV. Spin polarization calculation

In SPINLIE code the calculation of the equilibrium level polarization is based on the DK - formula [14]. Analytical determination of the spin transformation (the sextupole order is taken into account for both orbital and spin motions) is used. The components of these transformations are the polynoms of orbital variables $\vec{Z}$. These polynoms have been determined for each element type of the collider magnetic structure. The special rules of explicit summing of these polynoms are used for the calculation of one-turn spin transformation (linear and sextupol orders). The periodical solution is found for the equilibrium spin vector from the one-turn transformation. This solution explicitly depends on the first and second powers of $\vec{Z}$ and so one can find the spin chromaticity vector $\vec{d}$ and the degree of the equilibrium polarization.

A. Periodical solution

Let us denote the one turn transformation from θ to $\theta+2\pi$ as $\mathcal{A}\mid_{\theta\to\theta+2\pi}\equiv\mathcal{A}^{(w)}$, $\mathcal{S}\mid_{\theta\to\theta+2\pi}\equiv\mathcal{S}^{(w)}$, $\mathcal{F}\mid_{\theta\to\theta+2\pi}\equiv\mathcal{F}^{(w)}$, $\mathcal{W}^{(1)}\mid_{\theta\to\theta+2\pi}\equiv\mathcal{W}^{(1)}$, $\mathcal{W}^{(2)}\mid_{\theta\to\theta+2\pi}\equiv\mathcal{W}^{(2)}$. To find the periodical solution $\tilde{\vec{n}}(\theta;\vec{Z}(\theta)) = \tilde{\vec{n}}(\theta+2\pi;\vec{Z}(\theta))$ we use the "gradient" method. It is based on the introduction of small dissipation into transformation and calculation of relaxative solution. Practically, it means introducing synchrotron damping decrements $\alpha^{(\nu)}$ [15] into the one turn orbital matrix $\mathcal{A}$ and summing total transformation so many times N that the $\mathcal{A}^N$ becomes negligible with computer accurancy. Physically, it corresponds to synchrotron damping of orbital imperfections on azimuth θ (due to photon emission) and summing of spin distortions. The resulting transformation from θ to $\theta+2\pi N$ will be periodical and is described by the following expressions:

$$\begin{aligned}
\mathcal{A}\mid_{\theta\to\theta+2\pi N} &= \mathcal{A}^{(w)^N} \overset{N\to\infty}{\Longrightarrow} 0,\\
\mathcal{S}\mid_{\theta\to\theta+2\pi N} &= \mathcal{S}^{(w)^N},\\
\mathcal{F}\mid_{\theta\to\theta+2\pi N} &= \sum_{n=0}^{N}\mathcal{F}^{(w)}\mathcal{A}^{(w)^n}\mathcal{A}^{(w)^n}\mathcal{A}^{(w)^n} \overset{N\to\infty}{\Longrightarrow} \tilde{\mathcal{F}};
\end{aligned}$$

$$\begin{aligned}
\vec{\mathcal{W}}^{(1)}\mid_{\theta\to\theta+2\pi N} &= \sum_{n=0}^{N}\left(\mathcal{S}^{(w)^{-1}}\right)^n\vec{\mathcal{W}}^{(1)}\mathcal{A}^{(w)^n} \overset{N\to\infty}{\Longrightarrow} \widetilde{\vec{\mathcal{W}}^{(1)}},\\
\vec{\mathcal{W}}^{(2)}\mid_{\theta\to\theta+2\pi N} &= \sum_{n=0}^{N}\Big\{\left(\mathcal{S}^{(w)^{-1}}\right)^n\vec{\mathcal{W}}^{(2)}\mathcal{A}^{(w)^n}\mathcal{A}^{(w)^n} +\\
&+ \frac{1}{2}\sum_{k=0}^{n-1}\left(\mathcal{S}^{(w)^{-1}}\right)^k\Big[\vec{\mathcal{W}}^{(1)},\left(\mathcal{S}^{(w)^{-1}}\right)^{n-k}\cdot\\
&\cdot\ \vec{\mathcal{W}}^{(1)}\mathcal{A}^{(w)^{n-k}}\Big]\mathcal{A}^{(w)^k}\mathcal{A}^{(w)^k} +\\
&+ \left(\mathcal{S}^{(w)^{-1}}\right)^n\vec{\mathcal{W}}^{(1)}\Big[\sum_{k=0}^{n-1}\mathcal{A}^{(w)^{n-k}}\cdot\\
&\cdot\ J\mathcal{F}^{(w)}\mathcal{A}^{(w)^k}\mathcal{A}^{(w)^k}\Big]\Big\} \overset{N\to\infty}{\Longrightarrow} \widetilde{\vec{\mathcal{W}}^{(2)}}.
\end{aligned}$$

The summing can be made by two one-turn transformation, two two-turns, two four-turn, ...and so on in powers of 2 this procedure is not too long and 20-25 summings are usually enough. This multiturn transformation corresponds to the condition of periodicity:

$$\tilde{\vec{n}}(\theta) = \vec{n}_0(\theta) + \widetilde{\vec{\mathcal{W}}^{(1)}}(\theta,\vec{Z}(\theta)) + \widetilde{\vec{\mathcal{W}}^{(2)}}(\theta,\vec{Z}(\theta),\vec{Z}(\theta)).$$

B. Spin Chromaticity and its transformaton

Now we can find the periodical spin vector $\tilde{\vec{n}}(\theta)$. The spin chromaticity can be determined from it as a derivative over the sixth component of the orbital vector $\vec{Z}$. It is necessary to integrate the spin chromaticity over the ring in order to calculate the polarization degree. There are two possibilities: calculate $\tilde{\vec{n}}(\theta)$ for each current azimuth θ or calculate $\tilde{\vec{n}}(\theta_0)$ for initial azimuth and after that "pull" this vector from θ_0 to θ. We found the following rule for the "pulling":

$$\begin{aligned}
\mathcal{W}_{\alpha i}^{(1)}\mid_{\theta\to\theta+2\pi N} &= \mathcal{S}_{\alpha\beta}\mid_{\theta_0\to\theta}\Big\{\mathcal{W}_{\beta j}^{(1)}\mid_{\theta_0\to\theta_0+2\pi N} -\\
&- \mathcal{W}_{\beta j}^{(1)}\mid_{\theta_0\to\theta}\Big\}\mathcal{A}_{ji}^{-1}\mid_{\theta_0\to\theta},\\
\mathcal{W}_{\alpha ij}^{(2)}\mid_{\theta\to\theta+2\pi N} &= \mathcal{S}_{\alpha\beta}\mid_{\theta_0\to\theta}\cdot\\
&\cdot\ \Big\{\Big(\mathcal{W}_{\beta kl}^{(2)}\mid_{\theta_0\to\theta_0+2\pi N} - \mathcal{W}_{\beta kl}^{(2)}\mid_{\theta_0\to\theta}\Big) -\\
&- \frac{1}{2}e_{\beta\gamma\lambda}\mathcal{W}_{\gamma k}^{(1)}\mid_{\theta_0\to\theta}\mathcal{W}_{\lambda l}^{(1)}\mid_{\theta_0\to\theta_0+2\pi N} -\\
&- \Big(\mathcal{W}_{\beta m}^{(1)}\mid_{\theta_0\to\theta_0+2\pi N} - \mathcal{W}_{\beta m}^{(1)}\mid_{\theta_0\to\theta}\Big)\cdot\\
&\cdot\ J_{np}\mathcal{F}_{pkl}\mid_{\theta_0\to\theta}\Big\}\mathcal{A}_{ki}^{-1}\mid_{\theta_0\to\theta}\mathcal{A}_{lj}^{-1}\mid_{\theta_0\to\theta}.
\end{aligned}$$

V. Some SPINLIE calculation results

All presented calculations was performed for the VEPP2-M collider structure. Imperfection was introduced by special skew. Its strength corresponded to the measured width of the coupling resonance. Fig.2 shows the suppression of the some resonances for the different kind of the structure symmetry. As was shown earlier the strength of the nonlinear

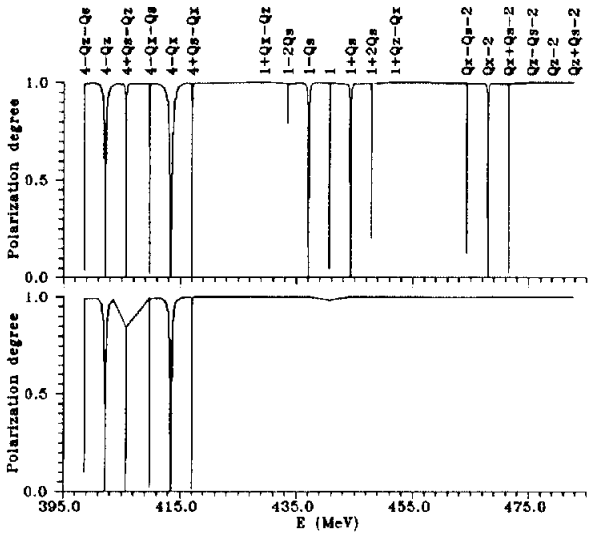

Figure 2: The spin resonances for different kinds of the symmetry of the magnetic structure.

resonance is determined by the contribitions of the different terms. We compare them and the result is presented in Fig.3. One can see that all terms have approximately the same values for VEPP2-M. We assume that the relation between the different terms can been changed for different energies and symmetry of collider structures and this is the matter for special investigation.

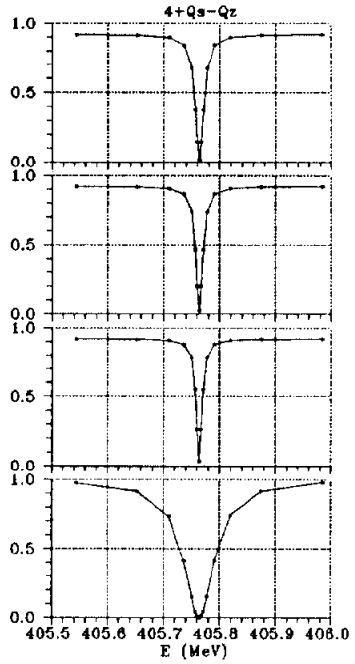

Figure 3: The comparison of each term contributions to the nonlinear resonance. a - $\left[\vec{\omega}^{(1)},\vec{\omega}^{(1)}\right]$ term; b - term caused by the orbital motion nonlinearity; c - the sextupole order term $\vec{\omega}^{(2)}$ in the expansion of the spin precession frequency; d - all terms together.

REFERENCES

[1] A.Ackerman, J.Kewisch, T. Limberg, "The linear polarization calculation program SITF", LEP Note 9999, 1988.

[2] S.Mane, Proc. 8th Int. Symp. on High Energy Spin Physics, Minneapolis, 1988 (Am. Inst. Phys. Conf. no. 187, vol. 2 (1989) p.959).

[3] S.Mane, Phys. Rev. A36 (1987) 120.

[4] A.Chao, Nucl. Instr. Meth. 29 (1981) 180.

[5] V.Ptitshyn, Yu.Shatunov, will be published.

[6] S.Mane, NIM A292 (1990) 52.

[7] K.Yokoya, "Non perturbative Calculation of Equilibrium Polarization of Stored Electron Beams", preprint KEK 92-6, 1992.

[8] J.Kewisch, "Simulation of Electron Spin Depolarization with the Computer Code SITROS, reprint DESY 83-032, 1983.

[9] Braun Karl L., Advances in Particle Physics, vol.1, pp.71-134, 1968.

[10] J.Dragt, "Lecture on Nonlinear Orbit Dynamics", Am. Inst. Phys. Conf. no. 87, (1982) p.147.

[11] J.Dragt, E.Forest, J.Math.Phys. 24(12), pp. 2734-2744, 1983.

[12] Yu.Eidelman, V.Yakimenko, "The Spin Motion Calculation with Using Lie Method in Collider Nonlinear Magnetic Field", preprint INP 90-127 Novosibirk, 1990 (Submited to Particle Accelerators).

[13] V.Bargmann, L.Michel, V.Telegdi, Phys.Rev.Lett. 2(10), pp.435-437, 1959.

[14] Ya.S.Derbenev, A.M.Kondratenko, Zh. Eksp. Teor. Fiz. 64 (1973) 1918 (Sov. Phys. JETF 37 (1973) 966).

[15] H.Mais, G.Ripken, "Theory of Spin-orbit Motion in Electron-positron Storage Ring", preprint DESY 83-062, 1983.