Wiggler as Spin Rotators for RHIC*

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Abstract

The spin of a polarized particle in a circular accelerator can be rotated with an arrangement of dipoles with field mutually perpendicular and perpendicular to the orbit. To achieve spin rotation, a given field integral value is required. The device must be designed in a way that the particle orbit is distorted as little as possible. It is shown that wigglers with many periods are suitable to achieve spin rotation with minimum orbit distortions. Wigglers are also more compact than more established structures [1] and will use less electric power. Additional advantages include their use for non distructive beam diagnostics. Results are igiven for the Relativistic Heavy Ion Collider (RHIC) in the polarized proton mode.

I. INTRODUCTION

Devices to rotate the spin of the proton from vertical to radial and conversely can be built with two series of magnets, each series with magnetic fields directed along the vertical and radial axes respectively. The two series are longitudinally shifted with respect to each other, as schematically shown in Fig. 1. In each configuration the magnetic field integral must be zero, so that a proton entering the structure on axis would also emerge on axis.

Advantages of configurations with more than six dipoles (wiggler rotators) are: (i) smaller overall magnet volumes, (ii) smaller beam displacement from the equilibrium orbit, especially important at lower proton energies (injection), (iii) lower electric power, (iv) possibility of using permanent magnets, (v) use of proton undulator radiation (synchrotron) for non-destructive beam diagnostics. The main disadvantage is the greater rotator length.

We have studied the behavior of a series of wiggler spin rotators, both with conventional and superconducting magnets.

II. EQUATIONS FOR SPIN AND ORBIT

The proton beam propagates along the z-axis (Fig.1). y is the vertical and x the transverse axis. Spin rotation is given by

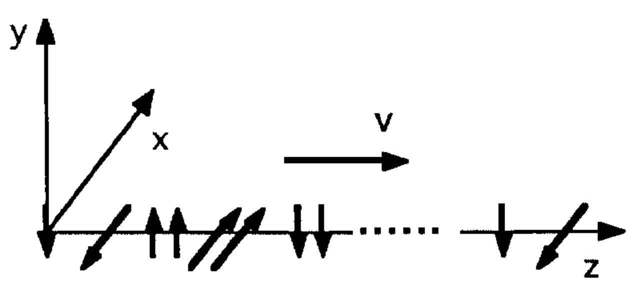

Fig.1 Spin rotator with transverse magnets.

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the vector equation for the unitary spin vector in a transverse magnetic field

$$\frac{d\hat{s}}{dt} = \Omega \times \hat{s} = -\frac{e}{m\gamma}(1 + G\gamma)\boldsymbol{B}_{\perp} \times \hat{s} \qquad (1)$$

with γ the relativistic energy of the protons and

$$G = 1.7928\,;\ \frac{e}{m} = 9.58\ 10^7\ \text{sec}^{-1}\ \text{T}^{-1}\ . \qquad (2)$$

The equation for the trajectory and the first field integral are

$$x, y(z) = \int \frac{I_{x,y}(z)}{\sqrt{1 - I_{x,y}^2(z)}}\,dz \qquad (3)$$

$$I_{x,y}(z) = \frac{e}{\beta\gamma mc}\int B_{x,y}(z)\,dz\ . \qquad (4)$$

A proton on axis at the entrance will emerge on axis if integral (4) is zero for the whole rotator.

The synchrotron radiation generated by the high energy protons in the wiggler exhibits a line spectrum with fundamental wavelength given by

$$\lambda_1 = \lambda_0 \frac{1 + k^2/2}{2\gamma^2}\ ;\ k = \frac{e}{2\pi mc}\lambda_0 B \qquad (5)$$

where k is a quantity defining the maximum bending angle of the trajectory in units of $1/\gamma$. $\lambda 0$ is the period of the wiggler, i.e. the length of a full field spatial oscillation along the z axis.

III. RESULTS FOR RHIC

The integration of the six equations (3,4) using z as the independent variable was numerically performed by a Runge-Kutta plus Predictor Corrector routine. Results of the calculations for wiggler rotators capable of rotating the polarization between x and y are shown in Table 1. The values are calculated for γ=200; however, they are in very good approximation valid for a very wide range of proton energies.

Entries in Table 1 are for normal conducting and superconducting magnets. The total length L of a rotator including the two end poles of half strength, and the total relative snake volume V and the relative magnetic field energy W can be estimated as follows

$$L = (n+1)\frac{\lambda_0}{2}\,;\ V = L\lambda_0^2\,;\ W = (n+1)\lambda_0^2 B^2\ .$$

The expression for the volume relies on the observation that the transverse dimensions of each component dipole (in the normal conducting case) are of the order of the period, and for the energy stored in the field, on the observation that the energy is roughly proportional to the volume of the magnetic gap.

Column 4 is the (odd) number of full field dipoles of half period length. Column 7 is the fundamental wavelength of the radiation. It lies in the infrared; however its harmonics, say

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the 5th, 7th and 9th, are in the near infrared or in the visible, then easily detectable.

The product $\lambda_0 B$ needed to appropriately rotate the spin and, accordingly, also the length of a wiggler rotator slowly grows with n . Thus a 15-pole is about twice as long than a 1-pole. However the volume of the magnet decreases, and with it the magnet cost, in spite of the greater complication of fabrication. The energy stored in the magnetic field also decreases for normal conducting snakes. In the SC case, the values given for the volume and energy are indicative.

Figures 2 through 5 show some results (γ=200) of the integration of the spin and orbit equations for a SC 15-pole compared with a NC 1-pole rotator.

IV. REFERENCES

[1] J.Collins, S.F.Heppelman, R.W.Robinett (Eds.) "Polarized Collider Workshop", *AIP Proc* **223** (1990)

Table 1
Comparison of wiggler spin rotators

B [T]	λ_0 [m]	L [m]	n	V	W	λ_1 [μm]
Normal Conducting						
1.728	6.0	6.0	1	216	215	85.40
	3.6	7.2	3	93	155	47.25
	2.1	10.2	9	45	132	26.70
	1.63	13.0	15	35	127	20.58
1.85	1.52	12.2	15	28	127	19.19
SuperConducting						
3.2	0.88	7.04	15	6	126	11.11
4.0	0.70	5.60	15	3	125	8.84
6.0	0.47	3.76	15	1	127	5.94

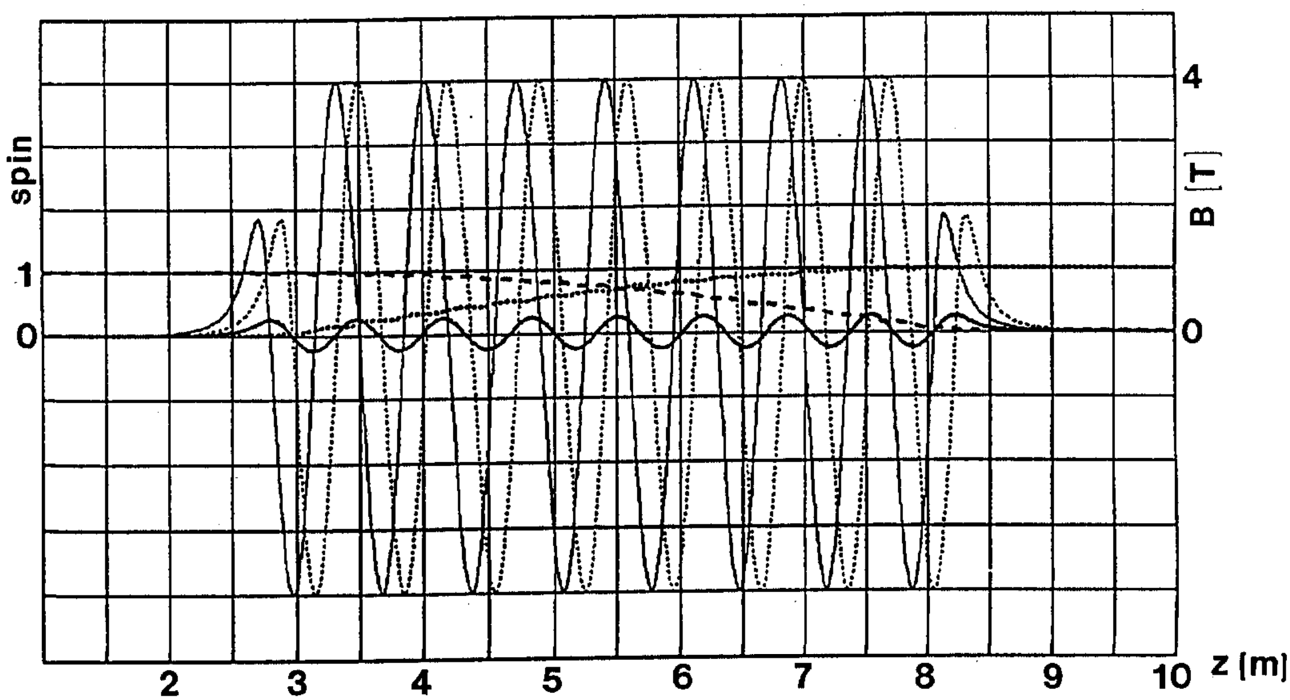

Fig.2. Transition for a 15-pole 4 T SC rotator. Spin from vertical to radial. Thin lines are the field components: B_x (solid), B_y (dotted), thick lines are the spin components: S_x (solid), S_y (dotted), S_z (dashed).

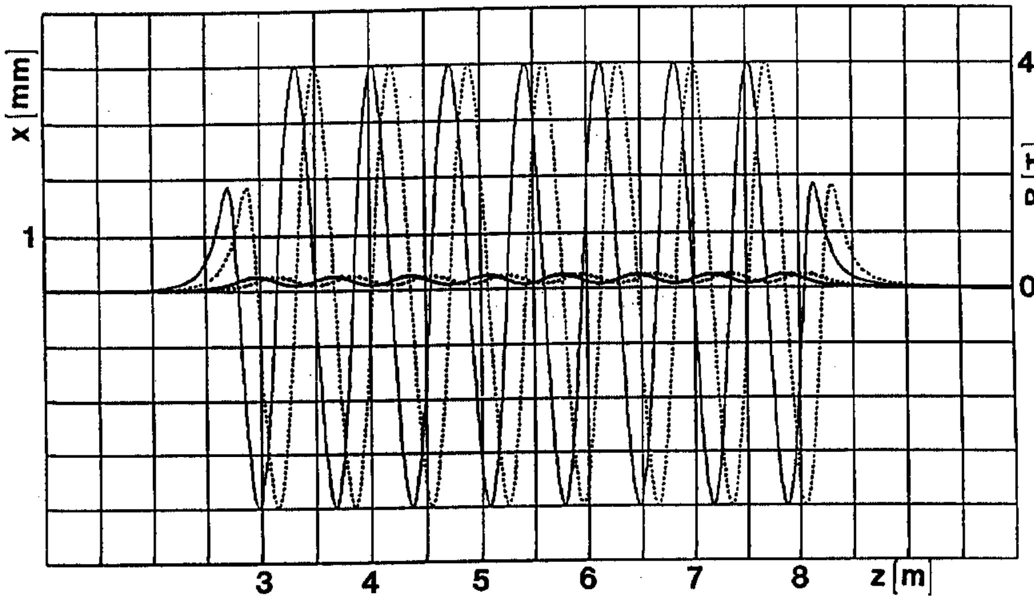

Fig.3. Trajectory in the 15-pole SC rotator of Fig.2. Thin lines are the field components: B_x (solid), B_y (dotted), thick lines are the trajectory: y (vertical, solid), x (radial, dotted).

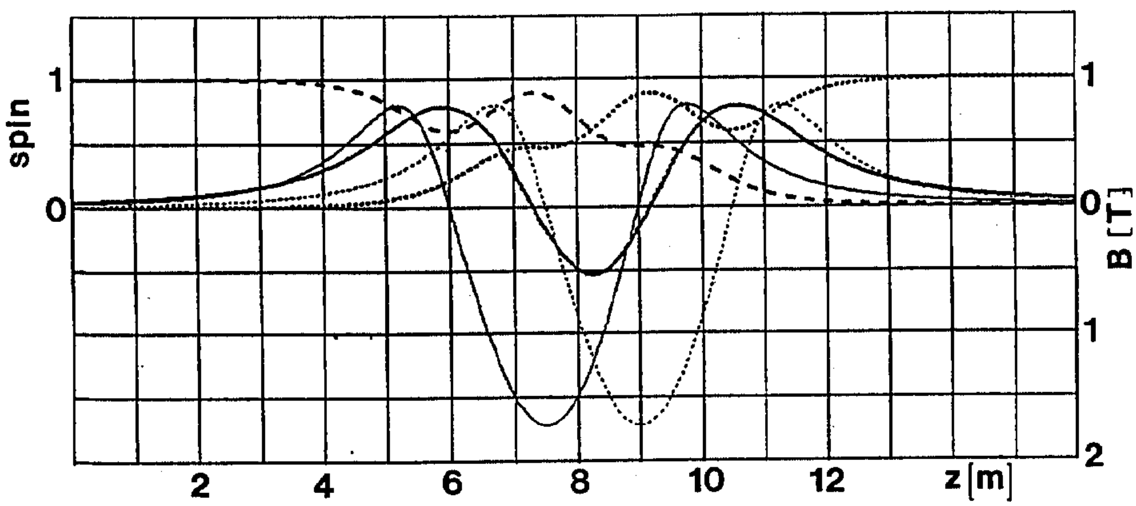

Fig.4. Transition for a 1-pole rotator. Spin from vertical to radial..

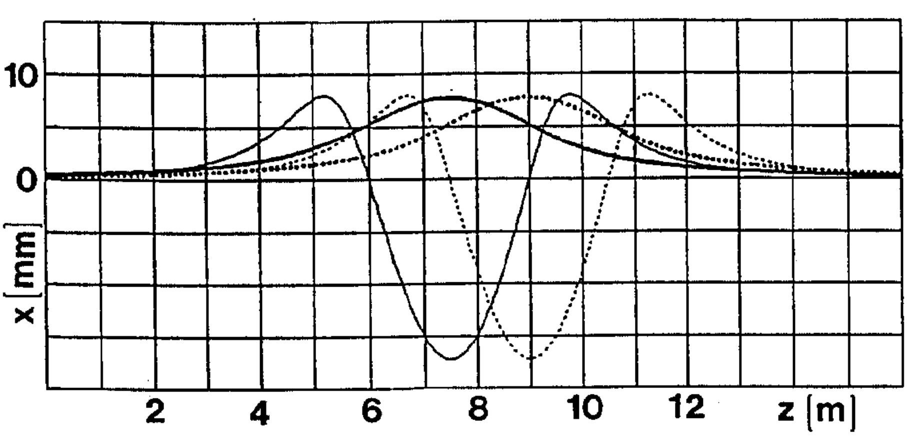

Fig 5. Trajectory in a 1-pole rotator.