NEW METHOD FOR CONTROL OF LONGITUDINAL EMITTANCE DURING TRANSITION IN PROTON SYNCHROTRONS

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Abstract

A new method for controlling longitudinal emittance growth during transition crossing in proton or heavy ion synchrotrons is described. Longitudinal focusing forces are eliminated near transition through the use of rf harmonics. The rf system provides only the required accelerating voltage to each particle in each bunch during the non-adiabatic period near transition, hence momentum growth is minimized. Bunch length is maximized just at transition, minimizing space charge forces and associated instabilities.

I. INTRODUCTION

A bit of history. All synchrotrons operate on a principle of phase stability [1,2] whereby ensembles (bunches) of particles with momenta slightly removed from a 'synchronous' momentum receive, on average, the same acceleration by engaging in small oscillations about the 'synchronous' phase on an rf accelerating voltage wave. Deviations in the orbit lengths of off-momentum particles can be related to a synchronous orbit length C_o by

$$\frac{\Delta C}{C_o} = \alpha_o\left(\frac{\delta p}{p_o}\right) + \alpha_1\left(\frac{\Delta p}{p_o}\right)^2 + \cdots. \quad (1)$$

Similarly, with the advent of the strong focusing lattice [3,4], to first order in α The deviation in rotation period of off momentum particles has been expressed

$$\frac{\Delta T}{T} = \left(\alpha_o - \frac{1}{\gamma^2}\right)\frac{\Delta P}{p_o} = \left(\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p_o} = \eta\,\frac{\Delta p}{p_o}\;. \quad (2)$$

γ is the particle total energy divided by its rest energy, E/E_o. The 'momentum compaction factor' α has been related to an 'energy' γ_t; $\alpha = \gamma^{-2}$, and a 'momentum slip factor' η has been defined in terms of γ and γ_t.

The average accelerating voltage per turn for a synchronous particle with fixed orbit length is,

$$V_{acc} = eV\sin(\phi_s) = \frac{C}{c}\frac{d(pc)}{dt} = \beta^2 E_o T_o\,\dot{\gamma}\;. \quad (3)$$

ϕ_s is the phase angle at which synchronous particles cross the rf accelerating gap(s). An off-momentum particle, arriving at the rf accelerating gap(s) at some angle ϕ will see accelerating voltage $V\sin(\phi)$.

These considerations lead to derivation of a harmonic oscillator equation for small amplitude motion of non-synchronous particles. The period of oscillation, expressed in terms of the rotation period of a synchronous particle, is;

$$T_\phi = T_o\left[\frac{2\pi E_o\gamma}{eVh\eta}\right]^{1/2}. \quad (4)$$

h is the harmonic number of the rf, i.e. $F_{rf} = h\,F_o$. In order for the phase oscillations to average effectively the energy delivered to all off-momentum particles the period of oscillation must be short with respect to the time over which the synchronous energy changes appreciably. Indeed the phase oscillation equation was derived under the assumption that the parameters V, γ, and η remained sufficiently constant during a phase oscillation so that the change in frequencu during one period is a small fraction of the frequency. This is the condition of 'adiabaticity'.

The very small values of α available in a strong focusing lattice reduce drastically the amplitude of radial motion caused by the momentum deviations inherent in synchrotron phase oscillations. The same small value introduces the possibility that η may pass through zero at some point within the momentum range of the accelerator. If this happens the period of phase oscillation appears to become infinite and the condition of adiabaticity is clearly lost. Stable phase oscillations can be established for either sign of η by changing the sign of $\cos(\phi_s)$, (i.e. by changing the operating point from ϕ_s to π-ϕ_s, giving the same accelerating voltage). But the period of interest is the period when adiabaticity cannot exist, i.e. the 'transition' period.

Courant and Snyder pointed out in an early description of the Alternating Gradient (strong focusing) synchrotron that the condition of non-adiabaticy exists when

$$\frac{|\gamma - \gamma_t|}{\gamma_t} < \left[\frac{\gamma_t(eV\sin(\phi_s))^2}{4\pi h E_o eV|\cos(\phi_s)|}\right]^{1/3}. \quad (5)$$

It is useful to express $\gamma(t)$ near transition in terms of time measured from the time at which synchronous particles pass through transition, (assuming that γ is increasing linearly),

$$\gamma(t) = \gamma_t + \dot{\gamma}t. \quad (6)$$

By combining eqns. 3, 5, and 6, an expression for the 'non-adiabatic time period' around transition becomes,

$$T_{na} = \pm T_o \left[\frac{\gamma_t^4 \beta^4 F_o E_o}{4\pi h \dot{\gamma} e V |\cos(\phi_s)|} \right]^{1/3} . \qquad (7)$$

The non-adiabatic time is affected by the synchronous phase angle which exists just prior to transition. While the accelerating voltage is set by the requirements in eq. 4, ϕ_s is determined by the requirement that adequate bucket area to contain the beam longitudinal emittance be maintained [5].

Adiabaticity exists at times removed from transition by $\pm T_{na}$. But transition is a local effect, usually occurring at different times for different increments of momenta within the distribution. In the moving reference frame describing the particle phase space distribution γ_t moves from higher to lower momentum, i.e. higher momentum particles pass through transition before lower momentum particles. In order to ensure adiabaticity for all particles within a distribution it is necessary to determine the period required for the entire ensemble to cross transition. This 'non-linear' time has been shown to be [12,13],

$$T_{nl} = \frac{\gamma_t (\beta^2 + \alpha_1/\alpha_2 + 1/2)}{\dot{\gamma}} . \qquad (8)$$

II. TRANSITION PROBLEMS

Problems associated with transition crossing have been extensively discussed in tha literature [6-12]. In many cases it was assumed that the rf voltage would remain constant with fixed ϕ_s until 'transition' at which time the phase would be jumped to $\pi - \phi_s$ and possibly the voltage changed to account for a change in sign of the effective space charge force. Dynamic analyses for this situation indicate that the bunch momentum spread becomes very large while the phase extent of the bunch becomes minimum at transition. During this period the space charge forces within the bunch have been calculated to be sufficient to cause a shift in betatron tune, "Umstätter effect", with possible transverse emittance dilution.

In other cases the rf voltage is allowed to fall toward the minimum required for acceleration while the phase angle moves smoothly through $\pi/2$ toward a new value between $\pi/2$ and π. This has been termed "duck under"[14].

In all cases problems associated with transition can be traced to the fact that for very small values of η there is very little dispersion in the rotation periods regardless of momentum. Particles arriving at the accelerating gap at phases other than the synchronous phase receive either too much or too little acceleration for long periods of time because they are essentially 'frozen' in time. This causes long tail of particles in the distribution which cannot be matched to the accelerating bucket above transition. The result is loss of some particles followed by frequently observed quadrupole oscillation of the remainder of mismatched particles, or 'tumbling', which results in substantial longitudinal dilution.

III. PROPOSED SOLUTION

Many solutions to the problems described here have been proposed. Lattices can be designed with imaginary or infinite γ_t [15], or quadrupoles can be pulsed near transition to force a quick transition (so called γ_t-jump). Although convincing computations have been done, the first alternative needs an existence proof. The second alternative has the possibility for introducing transverse dilution if chromaticity discontinuities are introduced by the jump.

We propose here a solution involving only the rf accelerating system. It is proposed to remove, to the extent possible, all longitudinal focusing during the transition period by moving the phase of the rf wave to near $\pi/2$ and adding of a second or third harmonic component to the rf wave such that the voltage is constant over a substantial phase range around the (approximate) centroid of the charge distribution. In this way, with proper adjustment of the rf amplitude, all particles can receive the necessary accelerating voltage while moving only slightly in phase.
a precedent for this exists in isochronous cyclotrons, which operate essentially at transition throughout their accelerating cycle [16,17]. By the addition of 27.5% second harmonic at the proper phase the rf amplitude can be held constant to ±0.25% over a phase range of ±35 degrees.

Fig. 1 is a graphical picture of what can be expected of this procedure. Part (a) shows the outline of a charge distribution matched to an accelerating bucket prior to the start of the non-adiabatic period. γ_t is represented by a cross-hatched region above the bucket. In part (b) γ_t has moved to the synchronous energy. Particles above the synchronous energy moved initially to earlier time but reversed direction as γ_t moved downward through them. At this time all particles are moving toward later time, the particles with positive energy being above transition, particles with negative energy being below. In part (c) transition has moved below the entire distribution. The harmonic voltage is removed and the distribution can be matched to an accelerating bucket above transition. This procedure is described in detail in [13].

Extensive computer simulations of this procedure, including the effects of space charge forces and ring impedance have been completed [18,19].

At the present time a third harmonic rf system has been installed in the Fermilab Main Ring and beam studies are in process. The apparently simple scheme is difficult to implement because of the necessity for compensation for beam loading of the harmonic cavity and problems associated with verification of correct phases and amplitudes of the combined rf systems. Efforts to resolve these difficulties are reported in [20,21].

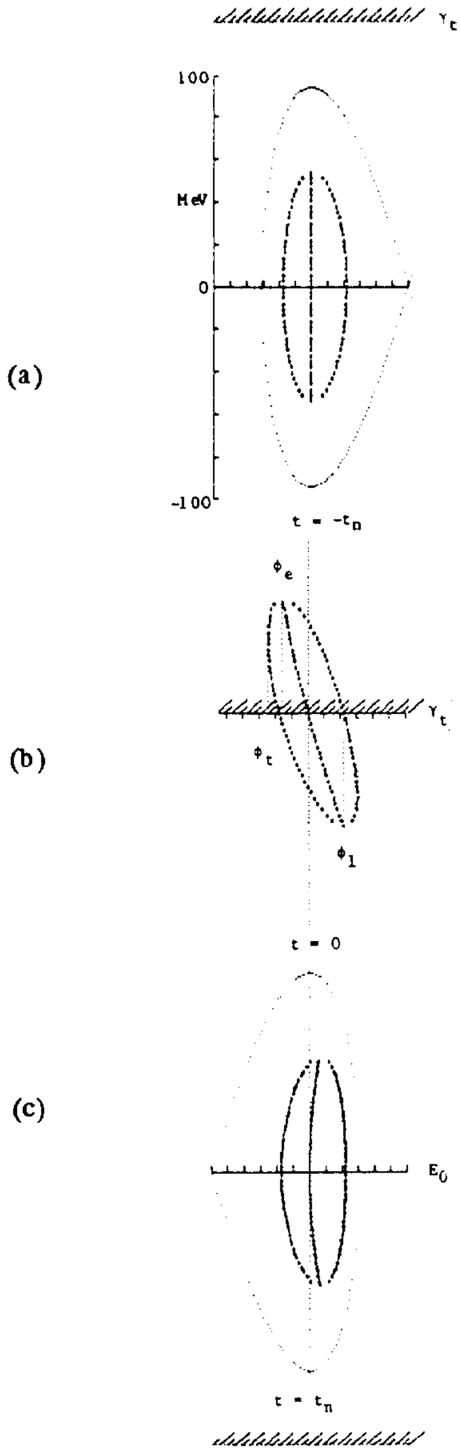

Figure 1. Schematic Representation of Non-Focusing Transition Crossing

IV. REFERENCES

[1] V.Veksler, A New Method of Acceleration of Relativistic Particles, Jour. Phys. U.S.S.R. 9, 153 (1945)

[2] E. M. McMillan, The Synchrotron- A Proposed High Energy Accelerator, The Physical Review, 68: 143-144 (1945)

[3] Nicholas Christofilos, Focussing System for Ions and Electrons, U.S. Patent 2,736,799, [filed March 10, 1950, issued Feb 28, 1956]

[4] E.D. Courant and H.S. Snyder, Theory of the Alternate Gradient Synchrotron, Annals of Physics: 3, 1-48 (1958)

[5] K.R. Symon and A.M. Sessler, Methods of Radio Frequency Acceleration in Fixed Accelerators Proc. of the CERN Symposium on High Energy Accelerators and Pion Physics,1, 46, (1956)

[6] A.A. Kolomenski and A.N. Lebedev, Theory of Cyclic Accelerators, North Holland, Amsterdam, pp190ff

[7] A. Sørenssen, The Effect of Strong Longitudinal Space-Charge Forces at Transition Energy, Proc. 6th Internatl. Conf. on High Energy Accelerators, Cambridge (USA),474, (1957)

[8] L.L. Goldin and D.G. Koshkarev, Nuovo Cimento 2, 1251 (1955)

[9] A. Sørenssen, Crossing the Phase Transition in Strong Focusing Proton Synchrotrons,Particle Accelerators, 6, 141-165,(1975)

[10] J. Wei, Longitudinal Dynamics of the Non-Adiabatic Regime on Alternate Gradient Synchrotrons, PhD Dissertation, State Univ. N.Y. Stony Brook, (1989)

[11] J. Wei and S.Y. Lee, Microwave Instability Near Transition Energy, Workshop on RHIC Performance, BNL-41604,335,(1988)

[12] K. Johnsen, Effects of Non-linearities on the Phase Transition, Proc. of the CERN Symposium on High Energy Accelerators and Pions,1, 106,(1956)

[13] J.E. Griffin, Synchrotron Phase Transition Crossing Using an Rf Harmonic, Fermilab TM-1734 (1991)

[14] N.C. Christofilos, First proposed 'Duck Under' transition crossing. Private Comm. E.D. Courant

[15] D. Möhl, Compensation of Space Charge Effects at Transition by an Asymmetric Q Jump, CERN-ISR/300/GS/69-62

[16] M.K. Craddock, IEEE Trans. Nucl. Sci. NS-24, No.3, 1615 (June,1977)

[17] S. Adam et al, First Operation of a Flattop Accelerating System in an Isochronous Cyclotron, IEEE Trans. Nucl Sci. NS-28, 2721, (1981)

[18] J.A. MacLachlan and J.E. Griffin, Application of a New Scheme for Passing Through Transition Energy to the Fermilab Main Ring and Main Injector, IEEE Part. Accel. Conf. San Francisco, 2826, (1991)

[19] J. MacLachlan et al,Reducing Bunch Disruption in Transition Crossing by Modification of the RF Waveform, Proc. XVth Internat. Conf. on High Energy Accelerators, 1040,(1992)

[20] J. Dey et al, High Level RF System for Transition Crossing Without RF Focusing in the Main Ring at Fermilab, Sb42, This Conf.

[21] C.M. Bhat et al, Operational Experience with Third Harmonic RF for Improved Beam Acceleration Through Transition in the Fermilab Main Ring.