EFFECTS OF PLANE UNDULATOR (WIGGLER) FIELDS ON BEAM DYNAMICS AT LARGE ORBIT DISTORTION

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Abstract

Effects of plane undulator sine-lake fields on beam dynamics in Storage Rings are investigated. Expressions for tune shifts of betatron oscillations versus their amplitudes are obtained for the case, where the orbit curvature inside the undulator is rather large. It takes place in compact synchrotron light sources with undulator insertions. In the limiting case these expressions for small orbit deflections are coincident with the known relations. Discussion of the results obtained is presented.

I. INTRODUCTION

Previously [1], we have considered the nonlinear effects of a plane undulator, and have derived the expressions for linear and nonlinear vertical tune shifts. Those results were based on using the expressions for the tune shift caused by the frigning fields of the dipole magnet [2], where the vertical field component was described by a sine curve. The problem was solved to the $\sin\alpha = \alpha$ approximation (α is the angle of particle deflection in the undulator field), this being quite sufficient in the majority of case. However, validity of this approximation has not been investigated for the storage rings of relatively low energies (about several hundred MeV) comprising inserts (undulators and wigglers). The aim of this report is to analyze the effect of plane- insert fields on beam dynamics for significant orbit distortions within these inserts.

II. THE FIELD IN THE FIXED COORDINATE SYSTEM

The field in a fixed coordinate system we investigate the plane undulator with parallel poles , infinitely extended in the transverse (horizontal) direction (figure 1).

The magnetic field components of this undulator are written in the known form (e.g.,[3]):

$$B_s = B_0 \sinh(k_u z)\sin(k_u s);$$
$$B_x = 0; \qquad (1)$$
$$B_z = B_0 \cosh(k_u z)\cos(k_u s),$$

where $k_u = \lambda / 2\pi$ is the undulator parameter, λ being is period.

This field can be described by one component of the magnetic vector potential:

$$A_x = -\frac{B_0}{k_u}\cosh(k_u z)\sin(k_u s). \qquad (2)$$

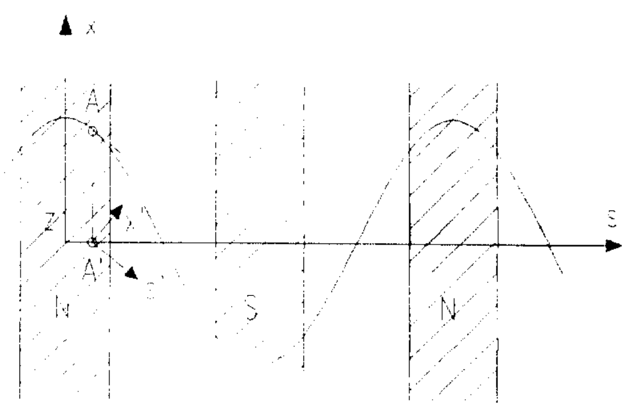

Figure 1. A schematic model for calculations.

III. THE FIELD IN THE NATURAL COORDINATE SYSTEM

To analyze the motion in the vicinity of the equilibrium orbit, we shall go over to the natural coordinate system (σ, χ, z), the origin of which moves along the trajectory

$$x = \frac{1}{k_u^2 \rho}\cos(k_u s), \qquad (3)$$

ρ is the trajectory curvature radius, which is dependent on the effective length of the pole, the field in the gap and the particle energy.

Taking into consideration the infinite extension of poles in the transverse direction, we may go over to the s', x', z' coordinate frame, which moves along the s axis but is turned by an angle

$$\alpha(s) = -\arctan\left(\frac{1}{k_u \rho}\sin(k_u s)\right)$$

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in reference to the fixed system, in terms on which the expressions for fields (1),(2) are written. Proceeding from the expression for the increment $d\sigma$ over the length ds,

$$d\sigma = ds\sqrt{\frac{1}{k_u^2\rho^2}\sin^2(k_u s)+1},$$

the trajectory extension is written as:

$$\sigma(L_u)=\frac{2L_u}{\pi}\sqrt{1+p^2}\,E\left(\frac{p}{\sqrt{1+p^2}}\right); \qquad (4)$$

where L_u is the undulator length;

$p=\lambda/2\pi\rho$;

$E(k)$ is the second-kind complete elliptic integral.

The magnetic vector potential components $\bar{A}$ are written in the s', x', z' system as:

$$A_{s'}=\frac{B_0^2}{Bk_u}\cosh(k_u z)\sin^2(k_u s)\frac{p}{\sqrt{1+p^2\sin^2(k_u s)}}$$

$$A_{x'}=\frac{B_0^2}{Bk_u}\cosh(k_u z)\sin(k_u s)\frac{1}{\sqrt{1+p^2\sin^2(k_u s)}}$$

$$A_{z'}=0. \qquad (5).$$

IV. ANALYSIS OF THE MOTION

In the natural coordinate system employed, the longitudinal momentum of the particle is well in excess of the transverse momentum and this allow us to use the methods of the perturbation theory. It is known[4], that the stabilizing part of the perturbation Hamiltonian leading to the tune shift has the form

$$H_{st}=-\frac{A_{\vartheta'}R^2}{B\rho}, \qquad (6)$$

where R is average radius of the machine;

$\vartheta = s/R$ is the azimutal angle;

$B\rho$ is the particle magnetic rigidity.

Using expression [4] for the tune shift $\nu_z-\nu_{z0}=\frac{1}{2\pi}\int_0^{2\pi}\frac{\partial H_{st}}{\partial|a_z|^2}d\vartheta$,where $|a_z|$ and z are related by $|a_z|^2=z^2\left(|V'|^2-\nu_{z0}^2|V|^2\right)$, and averaging the Hamiltonian over the whole perimeter of the setup, we obtain the following expression for the tune shift

$$\nu_z-\nu_{z0}=\frac{B_0^2 pL_u R}{\pi^2 B^2\rho}\left[\frac{1}{kp}E(k)-\frac{k}{p^3}K(k)\right]\times$$

$$\sum_{j=1}^{\infty}\frac{1}{j!(j-1)!}|a_z|^{2(j-1)}|V|^{2j}k_u^{2j-1}, \qquad (7)$$

where $k=\frac{p}{\sqrt{1+p^2}}$;

$K(k)$ is tte first-kind complete elliptic integral;

$|V|$ is the modules of the Floquet function averaged over the undulator length.

It should be noted that at $p\rightarrow 0$ the term in square brackets $\rightarrow \pi/4$, and after substitution of $|V|^2=\beta_z/2R$, where β_z is the vertical amplitude function, we obtain from (7) the expression for tune shifts given in [1]:

-linear shift

$$\nu_z-\nu_{z0}=\frac{L_u\beta_z}{8\pi\rho^2}; \qquad (8a)$$

-nonlinear shift

$$\nu_z-\nu_{z0}=\frac{\pi L_u|a_z|^2\beta_z^2}{8\rho^2\lambda^2 R}. \qquad (8b)$$

For practical applications, it appears more convenient to use in expression (8b) the betatron oscillation amplitude $a_z^*\approx\sqrt{\beta_z\varepsilon_z}$, (where ε_z is the vertical beam emittance) which is related to $|a_z|$ by $|a_z|^2={a_z^*}^2 R/2\beta_z$. If we restrict ourselves to the first two terms of the series expansion in k $E(k)$ and $K(k)$, then the expression for the tune shift, which are suitable for their practical use, take the form:

-linear shift

$$\nu_z - \nu_{z0} = \frac{L_u p\left(2 + 3p^2\right)\beta_z k_u}{16\pi\rho\sqrt{1 + p^2}\left(1 + p^2\right)}; \qquad (9a)$$

-nonlinear shift

$$\nu_z - \nu_{z0} = \frac{L_u p\left(2 + 3p^2\right)a_z^{*2}\beta_z k_u^3}{128\pi\rho\sqrt{1 + p^2}\left(1 + p^2\right)}. \qquad (9b)$$

The tune shift due to the trajectory extension in the undulator (see expression (4)) is given by

$$\nu_y - \nu_{y0} = \frac{L_u\left(\sqrt{1 + p^2}\left(1 - \frac{p^2}{4\left(1 + p^2\right)}\right) - 1\right)}{2\pi\beta_y} \qquad (10)$$

where y=x,z.

V. CONCLUSIONS

We have derived here the expressions for the tune shift caused by the fields of a plane undulator (wiggler) in the case of a significant orbit distortion. Numerical estimates show that in most cases, in practice, it suffices to use expression (8) because even in the consideration of the effects of caused superconducting inserts in compact storage rings the difference between the results obtained by the use of expressions (8) and (9) is not greated than 10...15%. Yet, the effects by themselves are rather significant and their compensation by means of, for example, magnetic-lens systems is a serious problem.

VI. REFERENCES

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