Particle Diffusion from Resonance Islands in Aladdin at SRC*

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Abstract

The dynamics of the beam in the resonance islands was studied on the electron storage ring Aladdin at the Synchrotron Radiation Center(SRC). We especially studied the horizontal third- and fourth-integral resonances driven by sextupole fields in the first and second order. A fast kicker was fired to kick the beam into one of the outboard stable islands. The beam took on a quasi-Gaussian distribution and slowly diffused out of the island. The diffusion rate and its dependence on the strengths of the driving sextupoles and the chromaticity sextupoles were measured by tracing the resonance peak of the betatron oscillation on the spectrum analyzer. Beam positions were also recorded through the data acquisition device which was clocked by a pulse-delay circuitry. Interesting results are shown and compared with numerical calculations.

I. INTRODUCTION

Aladdin is a 1-GeV electron storage ring composed of four arc and four straight sections. In normal operation it stores 15 beam bunches with horizontal beam sizes $\sigma_x \simeq 0.82$ mm. In our nonlinear beam dynamics experiment, only one beam bunch is used at an average current of 10 mA. Four pairs of unused horizontal and vertical chromaticity correcting sextupoles are optionally employed to excite desired resonant harmonics, and a fast kicker is fired to drive the coherent betatron oscillation. The details of measurements of the horizontal third- and fourth-integral resonance can be found in Ref [1] and [2]. During the measurement of the horizontal fourth-integral resonance $4\nu_x = 29$, we found that the particles initially captured into the resonance islands did not remain in islands but diffused back to the origin in a few seconds.

The beam lifetime in the resonance island is determined by such transverse diffusion process as quantum fluctuations due to synchrotron radiation, multiple scattering from residual gases, and intrabeam scattering. The purpose of the experiment was to measure the diffusion rates between the islands and the central region as a function of resonance island size.

II. EXPERIMENTAL MEASUREMENT

The third-integral resonance $3\nu_x = 22$ was chosen for

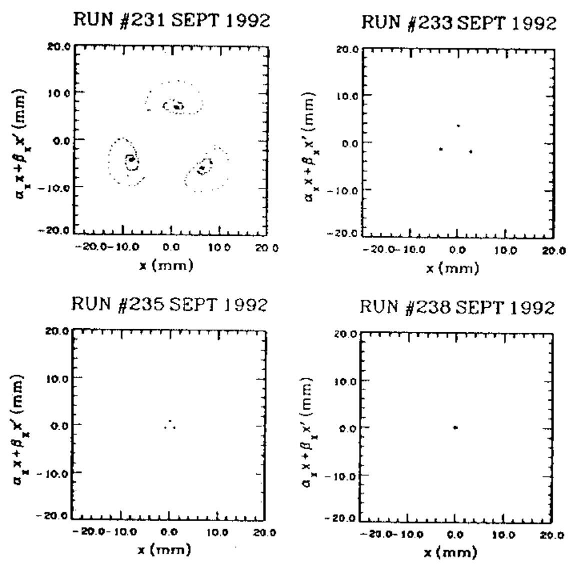

Figure 1
Third-integral resonance measurement at $\nu_x = 7.328$

measurement since the island size is relatively easily controlled with driving sextupoles. Figure 1 is a series of typical results from the BPM measurement of the beam centroid, showing the diffusion process. The first plot clearly shows the resonance island pattern with decoherent damping due to the nonlinear detuning. The beam centroids are damped to the island centers in about 300 turns. As more particles diffuse back to the central region, the amplitude of the beam centroid gradually decreases and finally becomes zero.

An efficient way to measure the diffusion rate is to trace the coherent signal on the spectrum analyzer. Right after the beam is kicked into one of the islands, a coherent betatron signal immediately appears at the resonant tune. Sometime later particles slowly leak out of the island while the signal exponentially decreases at a certain rate.

Figure 2 (a) shows an early measurement of the diffusion time versus the driving sextupole, taken at $\nu_x = 7.325$ with chromaticity $\xi_x = 0.808$. The driving sextupole SF1 is powered to control the island size, while chromaticity-correcting sextupole SFs are adjusted to keep chromaticity constant. Figure 2 (b) plots the diffusion time τ versus I_{SF}. The SF1 was set at $I_{SF1} = 45$ A, and the SFs are adjusted from $I_{SF} = 33.3$ to 35.5 A, which results in the chromaticity changing from 0.500 to 1.664. The diffusion

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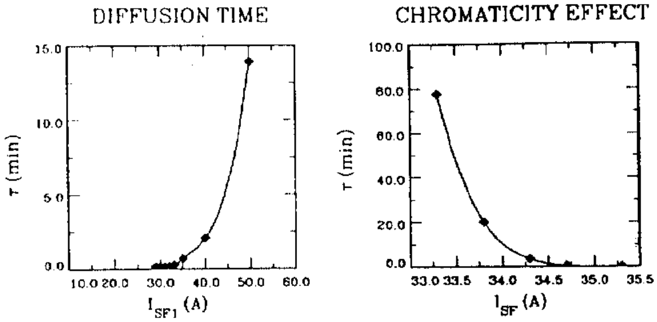
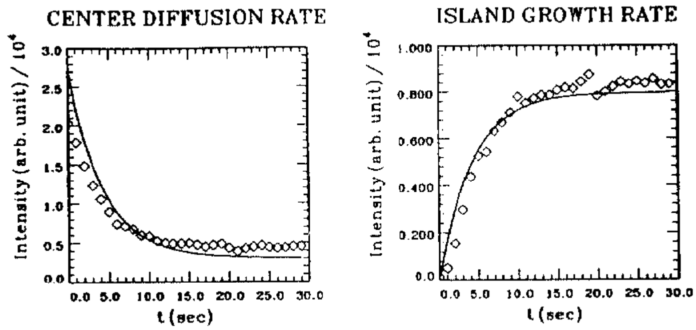


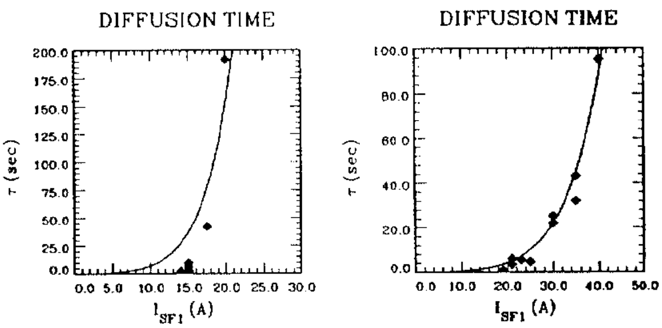
(a) τ vs. I_{SF1} (b) τ vs. I_{SF}

Figure 2
Diffusion time measurement at $\nu_x = 7.325$



(a) Central intensity (b) Island intensity

Figure 4
Measurements of the intensities in the center and islands



(a) $\xi_x = 0.160$ (b) $\xi_x = 1.083$

Figure 3
Diffusion time measurement at $\nu_x = 7.327$

time is exponentially decreased. This indicates that when ξ_x is large, the beam emittance is increased.

A recent measurement illustrates a shorter diffusion time. In Figure 3, we plot the τ against I_{SF1} at $\nu_x = 7.327$ with chromaticities $\xi = 0.16$ and $\xi_x = 1.083$. The solid curves are results fitted with the function,

$$\tau = \frac{\tau_x}{\gamma I_{SF1}^{1/2}} e^{\gamma I_{SF1}^{1/2}}, \qquad (1)$$

where γ is a fitting parameter depending upon the chromaticity and the details of the diffusion process and τ_x is the damping time given by $\tau_x = 0.022$ seconds. We obtain $\gamma = 2.617$ and $\gamma = 2.128$ for $\xi_x = 0.160$ and $\xi_x = 1.083$ respectively, which predicts a beam size about double that expected from quantum emission. Eq. (1) follows from the Sands formula [3] for τ, assuming island size is proportional to $I_{SF1}^{1/2}$. As we see, the Sands formula does not agree with the measurement for the small island size.

The outward diffusion was also measured with the laser beam analyzer. At specified time intervals, the analyzer takes frames of a synchrotron light spot through a camera mounted at a beamline port. Our camera was focused on the central region and one of the islands. By appropriately choosing an aperture to include one of the synchrotron light images, we obtained beam intensity in arbitrary units.

We first set the driving sextupole $SF1$ to $I_{SF1} = 50$A, and then decreased the central stability region by jumping the betatron tune close to the resonance $3\nu_x = 22$. The TV monitor clearly showed the diffusion process as particles gradually diffused into islands, and 25 seconds later the beams in the center and islands reached an equilibrium. As one can see from Figure 4, the beam intensity in each region approaches its equilibrium value exponentially.

Assume C and I are beam intensities in the central region and islands, respectively. By solving equations

$$\frac{dC}{dt} = -\frac{C}{\tau_c} + \frac{I}{\tau_i}, \qquad \frac{dI}{dt} = -\frac{I}{\tau_i} + \frac{C}{\tau_c}, \qquad (2)$$

one can obtain

$$C = \frac{C_0}{1+\alpha}[1 + \alpha e^{-\frac{t}{\tau}}], \qquad I = \frac{C_0 \alpha}{1+\alpha}[1 - e^{-\frac{t}{\tau}}], \qquad (3)$$

where C_0 is the initial intensity in the central region; $\alpha = \frac{\tau_i}{\tau_c}$ and τ is the effective diffusion time defined by $\frac{1}{\tau} = \frac{1}{\tau_c} + \frac{1}{\tau_i}$, and τ_c and τ_i are diffusion times for the central beam and for the island beam, respectively

Using Eq. (3) to fit the intensity measurements simultaneously with the least squares method, we obtain $C_0 = 26972$, $\alpha = 7.83$ and $\tau = 4.24$ seconds which give $\tau_c = 4.78$ seconds and $\tau_i = 37.41$ seconds. The fit is shown as the solid curves in Figure 4.

III. NUMERICAL SIMULATION

When an electron passes through a bending magnet, it losses its energy by releasing photons in a random Poisson process. The photon energy is randomly chosen from the appropriate distribution [4]. For reasonable statistics, at least 100 particles should be included and tracked up to 10^8 turns (30 seconds). To speed the simulation, the total effect of the quantum emission and RF acceleration damping is replaced with a Gaussian distribution,

$$P(\Delta x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\Delta x^2}{2\sigma^2}}, \qquad P(\Delta p_x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\Delta p_x^2}{2\sigma^2}}, \qquad (4)$$

where $\sigma = \frac{1}{2}\overline{\Delta x^2}$, based on the central limit theorem [5]. One can then choose Δx and Δp_x once a turn from the random generator and add them to the betatron amplitude. At $\nu_x = 7.327$, typically $\sigma = 3.9860 \times 10^{-6}$ m for

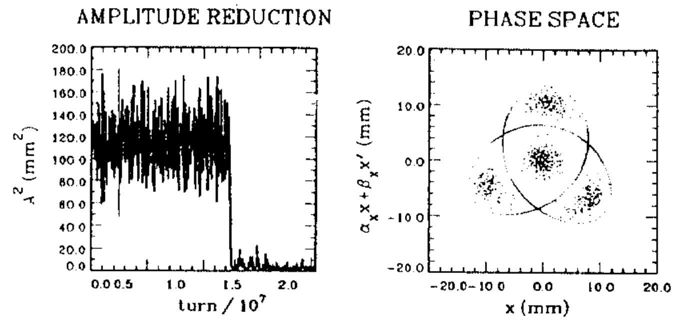


Figure 5
A single particle diffuses into the central region from resonance island at $\nu_x = 7.327$

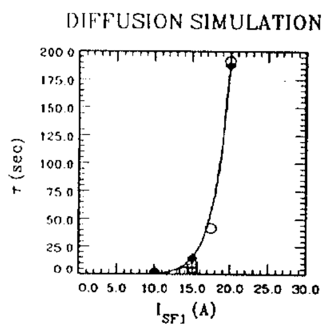
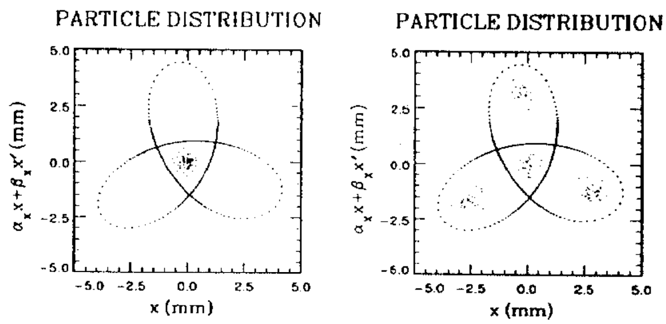


Figure 6
A simulation of the particle diffusion from the resonance island; $\nu_x = 7.327$ and $\xi_x = 0.16$



(a) At the 4 msecond (b) At the 7.5 second

Figure 7
Simulation of the outward diffusion process

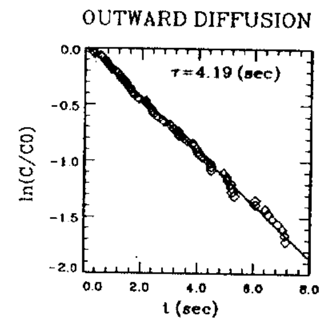


Figure 8
The outward diffusion of the central particles

Aladdin. Figure 5 illustrates that a single particle, initially located at the stable fixed point in the resonance island, diffuses into the central region from the resonance island.

The particle diffusion from resonance islands is first simulated. Figure 6 shows the simulation results corresponding to the case $\xi_x = 0.16$ as shown in Figure 3 (a). A total of 100 particles are initially located at the fixed point in one of the islands. Some time later particles take on a quasi-Gaussian distribution and slowly diffuse out of the island. If particles pass across the separatrix, they are lost from the island. In this way the diffusion rates are obtained by fitting the number of lost particles. The simulation results (rhombus) are consistent with the experimental results (circles).

In the simulation of particle diffusion from the central region to the island, the linear lattice parameters at the beamline port are used, where $\beta_x = 0.9354$ m and $\epsilon_{x0} = 8.5377 \times 10^{-7}$ (π rad-m). A total of 100 particles are tracked, starting at zero amplitude. Two frames of particle distributions at different times are selected to demonstrate the outward diffusion process as shown in Figure 7. The diffusion rate from simulation is also shown in Figure 8 plotted as $ln(C/C_0)$ against time. The fitting gives the diffusion time $\tau_c = 4.193$ seconds which is rather consistent with the measured τ_c.

IV. CONCLUSION

The particle diffusion from the resonance island and the inverse process are studied experimentally and numerically. Experimental measurements have partially verified the stochastic diffusion theory applying to the resonance phase map, and reveal the dependence of diffusion time on the machine chromaticity. To completely understand the experiment, we are continuing studies of the beam dynamics in the resonance island.

V. REFERENCES

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