Beam-Beam Modulational Diffusion in 2 1/2 Dimensions

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Abstract

Qualitative theoretical predictions for single particle modulational diffusion in 2 1/2 dimensions are compared with simulation results. Typical Fermilab Tevatron parameters are used for two beam-beam interactions under the influence of tune modulation created by synchrotron oscillations. When sideband overlap occurs on a pure one-dimensional resonance, diffusive growth is predicted and observed in the other transverse dimension. However exponential amplitude growth is observed in simulation instead of classically predicted root-time diffusive growth. Exponential growth rates are measured in particle tracking and possible impact on Tevatron luminosity upgrades is mentioned.

I. Introduction

Tune modulation is well known to have important effects on the dynamical response of particles to nonlinearities within an accelerator. In particular, theory, simulation and experiment investigating high order beam-beam resonances only agree qualitatively once tune modulation is incorporated[1,2]. Some recent investigation has concentrated on the phenomenon of modulational diffusion [3,4,5]. Salient features of this amplitude growth mechanism relevant to this study are:

- No external noise is present.
- Primary driving resonance overlap from tune modulation creates a thick layer of stochasticity in one dimension of motion.
- Weak coupling drives random-walk amplitude growth in the nonresonant plane. The diffusion rate is highly dependent on the proximity of this coupling resonance.
- Amplitude growth is diffusive, i.e. proportional to the square root of time.

Timescales of modulational diffusion are tens to tens of thousands of modulation periods; in a collider such as the Tevatron these timescales can range up to minutes. This mechanism is therefore a source of emittance growth and luminosity degradation that would significantly impact operations yet be difficult to diagnose.

Since modulational diffusion requires sideband overlap, it is present only in a region of the tune modulation parameter space where such overlap creates a thick layer of stochasticity. We assume tune modulation of the form

$$Q \rightarrow Q + q\sin(2\pi Q_M t)\ , \qquad (1)$$

where the time t is measured in turn number. For a one-dimensional primary resonance (with a strength characterized by the small-oscillation tune, or "island tune", Q_I), the region of parameter space that gives thick layer stochasticity is labeled by "Chaos" in Figure 1. This diagram has previously been investigated in part of experiment E778 at Fermilab [6,7]. The parameter diagram approach is powerful: given knowledge of the tune modulation strengths and frequencies in a particular machine (say, from chromaticity and typical momentum spread) one can set limits on Q_I and resonance strengths such that the chaotic region is never sampled and no modulational diffusion can exist.

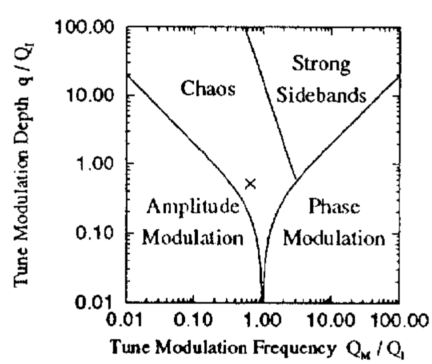

Figure 1. The (q, Q_M) parameter plane. The cross marks the point where simulations were performed.

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Parameter	Symbol	Value
Horizontal base tune	Q_{x0}	20.597
Vertical base tune	Q_{y0}	20.580
Synchrotron tune	$Q_s = Q_M$	$7.8 \cdot 10^{-4}$
Chromaticity	$\xi_{(x,y)}$	3
Momentum offset	σ_p/p	$3 \cdot 10^{-4}$
Linear beam-beam tune shift	ξ	$5 \cdot 10^{-3}$
Tune modulation depth	q	10^{-3}
Beta function at IP	$\beta^\star$	0.5 m

Table 1: Simulation parameters for 1992 Fermilab Tevatron collider run.

II. Simulation Parameters

We use a model of the Tevatron with two offset beam-beam kicks in an otherwise linear machine. All parameters used in the simulation are realistic except the base tunes, which are adjusted as described below. In this model all amplitudes are reported in units of the (round) transverse beam size σ; the beam-beam kick is then given in each plane by

$$\frac{\Delta x'}{\sigma} = \frac{-4\pi\xi}{\beta^\star R^2}\left[1 - e^{-R^2/2}\right]\frac{x}{\sigma}\,, \tag{2}$$

where $R = \sqrt{x^2 + y^2}/\sigma$, $\beta^\star$ is the linear beta function at the interaction point, and ξ is the linear beam-beam tune shift parameter.

Typical collider operations during the Fermilab 1992–3 collider startup used tunes of $Q_x = 20.586$ and $Q_y = 20.575$, with other relevant parameters listed in Table 1. This operating point lies between the $12Q_x$ and $5Q_x$ resonances, allowing a maximum beam-beam tune shift of $\xi = 9 \cdot 10^{-3}$ at each of the two low-beta crossing points before significant portions of the beam are affected by these low order resonances. A typical beam-beam tune shift per crossing is presently $5 \cdot 10^{-3}$, but luminosity upgrades such as the Main Injector will undoubtedly make this value even larger.

This study concentrates on the effects of the primary $5Q_x$ resonance, using this resonance as the source of stochasticity to drive modulational diffusion. Coupling is provided by the $4Q_x + Q_y$ resonance which is also driven by the beam-beam interaction. Moving the horizontal base tune from its nominal operations value to $Q_{x0} = 20.597$ produces a resonance island chain at $a_x = 2.2\sigma$ with $Q_I = 1.51 \cdot 10^{-3}$. This motion turns into a thick stochastic layer suitable for driving modulational diffusion in the vertical plane when realistic tune modulation from synchrotron oscillations and chromaticity is introduced, as shown in Figures 1 and 2. The tune modulation strength and tune used here are $q = 10^{-3}$ and $Q_M = 7.8 \cdot 10^{-4}$, for the Tevatron lattice in collider mode.

Because the beam-beam interaction couples strongly at large amplitudes, particles were launched at an initial vertical amplitude of 0.1σ. This amplitude was allowed to grow up to 1σ, where vertical motion starts to couple back to the horizontal stochastic motion, before tracking was stopped. Although not reported here, other simulations showed a similar growth mechanism operating at larger, more realistic, vertical amplitudes.

III. Simulation Results

Modulational diffusion theory predicts a strong dependence of diffusion rate on the proximity of the coupling resonance to the main driving resonance in the tune domain. This proximity is described by the quantity α,

$$\alpha \equiv \frac{\Delta Q_x}{q}\,. \tag{3}$$

For the resonances of interest here, this quantity is given by the vertical base tune and the tune modulation strength q:

$$\alpha \equiv \frac{20.6 - Q_{y0}}{4q}\,. \tag{4}$$

For values of α ranging from zero to about two, the diffusion is expected to be very strong [5]; the presence of the linear coupling resonance $Q_x - Q_y$ also complicates matters. A region with weaker diffusion, from $\alpha \approx 2$ to $\alpha \approx 5$, was investigated with vertical base tunes ranging from $Q_{y0} = 20.5820 - -20.5920$.

Figure 3 shows typical vertical amplitude growth over a long timescale, approximately 3500 synchrotron periods or $4 \cdot 10^6$ machine turns. This represents over one minute of actual machine operation in the Tevatron. Both log-log and log-linear scales are plotted, and it is apparent that the amplitude growth is exponential (linear on the log-log plot), not root-time as is classically predicted by standard modulational diffusion models. This type of growth was seen over several orders of magnitude of growth rate.

We quantify the exponential amplitude growth rate by the parameter γ:

$$a_y(t) = a_{y0}e^{\gamma t}\,. \tag{5}$$

This rate was measured from a linear fit of $\log a_y$ versus time for approximately fifty particles launched over the base tunes mentioned above. This rate is plotted on a log-linear scale versus the coupling resonance proximity α in Figure 4, and shows structure similar to that of the diffusion rate in the diffusive growth case of [5]. No amplitude growth is evident over ten thousand synchrotron periods at these base tunes when tune modulation is removed, indicating that modulational diffusion is indeed responsible for this amplitude growth.

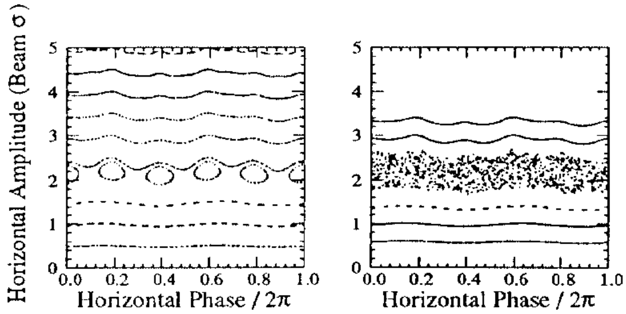

Figure 2: One-dimensional phase space motion at $Q_x = 20.597$, influenced by a strong $5Q_x$ resonance. The left figure is unmodulated; the right figure is influenced by tune modulation as listed in Table 1 and Figure 1.

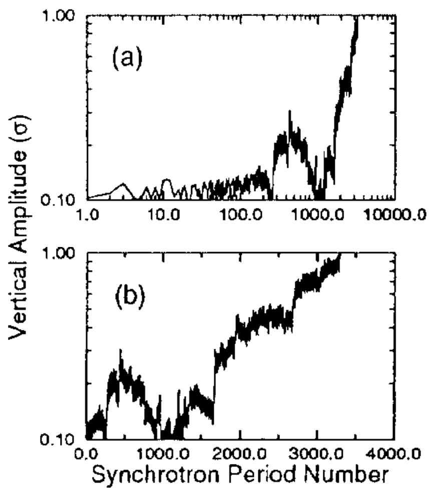

Figure 3: Vertical amplitude growth versus synchrotron period number on log-log and log-linear scales. The vertical tune is $Q_y = 20.5871$, or $\alpha = 3.225$

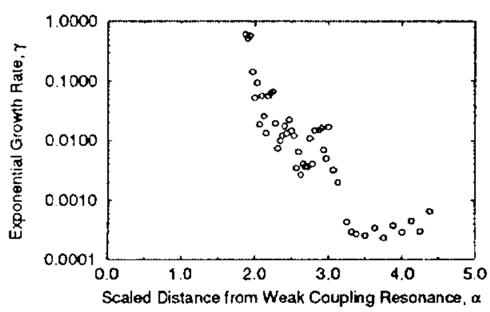

Figure 4: Exponential growth rate measured from simulation as a function of α, the proximity to the $4Q_x + Q_y$ coupling resonance.

IV. Conclusions

Under current operating conditions in the Tevatron, the beam is not expected to be affected by the $5Q_x$ resonance this strongly. However with future luminosity upgrades which will increase the beam-beam tune shift per crossing and the presence of higher-order resonances (in particular $17Q_x$ and $22Q_x$) that may serve as sources of emittance growth [2], modulational diffusion may provide a mechanism for slow luminosity degradation in hadron colliders and storge rings. Because modulational diffusion only occurs when thick-layer stochasticity is present, the tune modulation parameter diagram of Figure 1 may be used to place constraints on acceptable resonance strengths for operations in these machines.

Present work includes a concrete theoretical prediction of exponential growth rates in the case of an amplitude-dependent coupling strength. It has been suggested that the exponential growth observed in this simulation is due to the amplitude dependence of the coupling resonance strength, increasing the coupling strength as the vertical amplitude grows [8]. The collective nature of amplitude growth in the beam-beam modulational diffusion system is also under investigation to see what observable changes in beam emittance this mechanism produces.

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