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A Quasi-Isochronous Operation Mode for the LNLS UVX Storage Ring

Liu Lin^{*} and C.E.T. Gonçalves da Silva, Unicamp and LNLS Laboratório Nacional de Luz Síncrotron - LNLS/CNPq Caixa Postal 6192 - 13081-970 Campinas - SP - Brazil.

Abstract

We present an operation mode with very small momentum compaction factor for the LNLS UVX electron storage ring under construction in Campinas, Brazil. We establish conditions for longitudinal single particle stability in this quasi-isochronous mode including second order longitudinal and transverse effects. The results indicate that it is possible to operate this ring with the momentum compaction reduced by a factor of 100 with respect to the normal operation mode.

INTRODUCTION

The growing interest in very short electron and photon bunches provided by quasi-isochronous storage rings makes both the proposal of such rings for experiments and the analysis of higher-order longitudinal dynamics of considerable importance. The condition of quasi-isochronicity requires a momentum compaction factor α several orders of magnitude smaller than the values normally found in storage rings used for synchrotron light sources. This leads to an orbit length which is nearly independent of the particle energy deviation. This is a necessary condition to produce ultra-short electron bunches as the bunch length scales with $\sqrt{\alpha}$. The expected problems with quasi-isochronous rings are mainly related to higher order longitudinal dynamics and beam instabilities. When the zeroth-order (energy independent) momentum compaction factor approaches zero, its higher order terms in energy deviation can become dominant and introduce new features in the longitudinal dynamics. In addition, several beam instabilities have thresholds which depend on α . These problems, however, can be overcome as indicated theoretically by C.Pellegrini and D.Robin^[1], H.Wiedemann^[2] and L.Lin and C.E.T. Gonçalves da Silva^[3] and demonstrated experimentally by Hama et al. [4] for the UVSOR ring.

In this paper we present a small momentum compaction operation mode for the LNLS UVX electron storage $ring^{[5]}$ under construction in Campinas, Brazil. We study the single particle dynamics of this quasi-isochronous mode using a general form for the momentum compaction factor which includes the effect of second order terms and the transverse betatron oscillations. Tracking studies are also presented.

OPTICAL FUNCTIONS FOR UVX QUASI-ISOCHRONOUS MODE

The LNLS UVX lattice consists of six long straight sections matched to six arcs with two dipoles and two quadrupoles between the dipoles. In the standard operation mode each arc is made achromatic and, as a consequence, the dispersion function η_x is always positive. To lower the momentum compaction we combine two standard superperiods into one with four dipoles and we force the dispersion function to negative values at the two central ones. Thus, we obtain a three-fold symmetric quasi-isochronous ring preserving three first-order dispersion free straight sections.

We set the betatron phase advance so that the horizontal and vertical tunes in this mode are the same as in the standard mode. This allows continuous transference from the standard to the quasi-isochronous mode without crossing resonance lines during the process. This scheme has the advantage of avoiding the need to establish new injection conditions in this mode relaxing, therefore, the requirements for dynamic aperture.

The zeroth-order momentum compaction factor α_0 can be controlled by adjusting the negative part of the dispersion. In this particular quasi-isochronous mode for UVX we have reduced α_0 by a factor of 100 as compared to the standard operation mode. This reduces the bunch length by a factor of 10, in this case from 8 mm (27 ps) to 0.8 mm (2.7 ps).

Figure 1 shows the optical functions and the magnet lattice for one quasi-isochronous superperiod in UVX. We have 4 families of sextupoles in the dispersive section and two families in the non-dispersive section. We use the dispersive sextupoles to simultaneously correct the chromaticities and set the first order momentum compaction to a desired value. The non-dispersive sextupoles are used to optimize the dynamic aperture.



Figure 1: Optical functions and magnet lattice for one UVX quasi-isochronous superperiod.

NON-LINEAR SYNCHROTRON OSCILLATIONS

In order to describe the longitudinal motion of the electrons considering higher order and transverse motion effects we need to include them into the expression for the momentum compaction factor. The path difference for one revolution around the ring for an arbitrary particle with respect to the ideal one can be derived by geometric considerations:

$$\frac{\Delta L}{L_0} = \frac{1}{L_0} \oint ds \left(\frac{1 + x / \rho}{\cos(\theta)} - 1 \right)$$
(1)

where $\theta^2 = x'^2 + y'^2$. We expand (1) keeping terms up to second order and express the particle amplitudes x and y as

$$\begin{aligned} x &= x_{\beta} + \eta_0 \delta + \eta_1 \delta^2 \\ y &= y_{\beta} \end{aligned}$$
 (2)

^{*}Graduate student at IFQSC - Universidade de São Paulo.

where δ is the relative energy deviation. Noticing that the integrals containing terms which are linear with x_{β} and y_{β} vanish due to their oscillatory character, we have

 $\Delta L/L_0 = k_0 + \alpha_0 \delta + \alpha_0 \delta^2$

where

(3)

$$\alpha_0 = \frac{1}{L_o} \int_0^{L_o} \frac{\eta_o}{\rho} ds \tag{4}$$

$$\alpha_{I} = \frac{1}{L_{0}} \int_{0}^{L_{0}} \left(\frac{\eta_{I}}{\rho} + \frac{\eta_{0}'^{2}}{2} \right) ds$$
 (5)

$$k_0 = \frac{I}{2L_0} \int_0^{L_0} ds \left(x_{\beta}^{\prime 2} + y_{\beta}^{\prime 2} \right)$$
(6)

 α_0 and α_1 , respectively the zeroth and first order momentum compaction, are of chromatic nature and k_0 is of geometric nature. k_0 represents the effect of transverse oscillations on the orbit length. It can be estimated by using the smooth approximation for the betatron oscillations:

$$k_0 \approx \frac{\pi}{2L_0} \left(\varepsilon_x v_x + \varepsilon_y v_y \right) \tag{7}$$

The first order dispersion η_0 is determined only by the first order magnetic elements whereas the second order term η_1 is affected not only by those elements but also by the sextupoles. This makes the sextupole the natural 'knob' to vary the first order momentum compaction in a controlled way without changing the zeroth order value.

The longitudinal equations of motion are:

$$\dot{\varphi} = \omega_{rf} \left(k_0 + \alpha_1 \delta + \alpha_2 \delta^2 \right) \tag{8}$$

$$\dot{\delta} = \frac{e\hat{V}_0}{E_0T_0} (\sin\psi - \sin\psi_s) - \frac{U_0J_\varepsilon\delta}{E_0T_0}$$
(9)

where ω_{ff} is the angular frequency of the rf cavity, \hat{V}_0 is the peak voltage in the cavity, T_0 is the revolution period, U_0 is the energy radiated in one turn by the ideal particle and $J_{\mathcal{E}}$ is the radiation damping partition number. In deriving eq.(9) we have expressed the accelerating voltage by a sinusoidal wave form $V(\psi) = \hat{V}_0 \sin \psi$. The ideal particle arrives at the accelerating cavity exactly at the synchronous phase ψ_s .

The last term on the right hand side of equation (9) represents the damping of the longitudinal oscillations. In our analysis we will neglect this damping term. In this case the equations of motion can be derived directly from the Hamiltonian:

$$H = \omega_{rf} \left(k_0 \delta + \frac{\alpha_0}{2} \delta^2 + \frac{\alpha_1}{3} \delta^3 \right) + \frac{e \hat{V}_0}{E_0 T_0} \left(\cos \psi + \psi \sin \psi_s \right)$$

which is the Hamiltonian for the dynamics of longitudinal phase motion including sextupoles and transverse effects.

We consider firstly the case where $k_0=0$. In the usual case $(\alpha_1=0)$ the longitudinal phase space (ψ, δ) presents just one stable and one unstable fixed point at $(\psi_s, 0)$ and $(\pi - \psi_s, 0)$, respectively. The second-order term $(\alpha_1 \neq 0)$ creates additional stable and unstable fixed points at, respectively, $(\pi - \psi_s, -\alpha_0/\alpha_1)$ and $(\psi_s, -\alpha_0/\alpha_1)$. We call attention to the existence of this new stability zone on the other flank of the rf wave, centered at $\delta = -\alpha_0/\alpha_1$, which does not appear in the linear theory. We will call the stable phase region around the usual

stable fixed point a normal bucket and the stable phase region around this new stability point an anomalous bucket. We define a critical α_1 :

$$\alpha_{IC} = \sqrt{\frac{E_0 T_0 \omega_{rf} |\alpha_0|^3}{12e\hat{V}_0 (-\cos\psi_s + (0.5\pi - \psi_s)\sin\psi_s)}}$$

According to the value of α_1 , the phase diagram will assume a different aspect. For $|\alpha_1| < \alpha_{1C}$ the normal buckets are very similar to the buckets in the linear theory, but with a great asymmetry between the two branches of the separatrix. In this case, the energy aperture is very large, as well as the separation between the normal and anomalous buckets. As $|\alpha_1|$ increases the buckets approach each other. For $|\alpha_1| > \alpha_{1C}$ the buckets change their form and the stable region decreases very rapidly reducing the energy and phase aperture. For $|\alpha_1| = \alpha_{1C}$ we have just the transition between the two cases as can be seen in figure 2.



Figure 2: Effect of α_l onto phase space separatrix. When $|\alpha_l|$ is small (top), the buckets are large and well separated. As $|\alpha_l|$ increases, the buckets approach each other and for $|\alpha_l| > \alpha_{IC}$ (bottom) their size decrease rapidly. The middle diagram shows the transition when $|\alpha_l| = \alpha_{IC}$.

We have seen that when the first order momentum compaction factor becomes dominant the anomalous bunches approach the normal bunches. Eventually, the anomalous bunches move within the physical energy acceptance of the storage ring. In this situation, it might be possible to observe the anomalous extra bunches intercalated in phase with the normal ones, doubling the number of bunches. It is interesting to see qualitatively what happens at this new point of stability. From the equation for the longitudinal phase, $\dot{\phi} \propto \alpha \delta$, we see that a necessary condition for a stable solution (oscillatory phase) is that $\alpha \delta$ changes sign periodically. In the usual case, we have a constant α and δ oscillates around zero changing sign periodically. In the anomalous case, the term which periodically changes sign is α while δ has always the same sign. This is possible only because the nonlinearity introduces the energy dependence of the momentum compaction factor. Expressing $\alpha = \alpha_0 + \alpha_1 \delta$, we see that the momentum compaction will oscillate around zero for values of δ in the neighborhood of $\delta = -\alpha_0/\alpha_1$, which is exactly the energy of the anomalous stable point. In this situation, the anomalous bucket is entirely confined to the negative δ half-plane (α_0 and α_1 of the same sign) or to the positive δ half-plane (α_0 and α_1 of opposite sign).

We consider now the effect of the term k_0 , from transverse oscillations, to the longitudinal buckets. We expand δ for small k_0 and analyze the variation of the fixed points with the introduction of the transverse motion:

$$\delta_{fp} \approx \begin{cases} -\frac{1}{2} \frac{k_0}{\alpha_0} \\ -\frac{\alpha_0}{\alpha_1} + \frac{1}{2} \frac{k_0}{\alpha_0} \end{cases} \quad \Psi_{fp} = \begin{cases} \Psi_s \\ \pi - \Psi_s \end{cases}$$

Since k_0 is always positive, we conclude that if α_0 has the opposite sign to α_0/α_1 the fixed points move in such a way that the stable phase space area increases with k_0 . In the other case, α_0 with the same sign as α_0/α_1 , the stable area decreases with k_0 . The transverse motion has thus introduced a distinct behavior depending on the sign of the zeroth and first order momentum compaction values.

For UVX the critical value of α_1 is $\alpha_{1C}=3\times10^{-4}$. We study the case when the anomalous bunches are within the physical acceptance of the ring, $\delta_{phys.acc}=3\%$. The rf bucket will just fill the physical acceptance for $|\alpha_1|=4.4\times10^{-3}$. Regarding the sign of α_0 and α_1 we choose positive α_0 to keep the normal bunches at the same phase as in the standard operation mode, and negative α_1 to have the phase space increased with k_0 . We estimate the contribution from transverse motion, considering 10 % emittance coupling, to be $k_0=2.5\times10^{-8}$. Figure 3 shows the normal and the anomalous buckets in this case. There are no noticeable perturbations to the buckets due to transverse motion.



Figure 3: Normal (left) and anomalous (right) buckets for UVX (full line) and the effect of transverse motion on the buckets (dotted). Straight lines are physical limits of the ring.

DYNAMIC APERTURE CALCULATIONS

Tracking studies have been performed to determine the dynamic aperture of this UVX quasi-isochronous mode using the codes Patpet^[6] and Teapot^[7]. Both codes provided similar results. Two families of sextupoles in the non-dispersive region are used to minimize the tune shift with amplitude. We recall that these sextupoles do not affect either the chromaticities or the momentum compaction of the ring. The dynamic aperture simulations include effects of

systematic multipole errors and synchrotron oscillations for 1% energy deviation. The results are shown in figure 4.



Figure 4: Dynamic aperture for UVX quasi-isochronous mode with dispersive sextupoles adjusted to set chromaticies to zero and α_1 to -4.4x10⁻³; and non-dispersive sextupoles set to minimize the tune shift with amplitude. Systematic multipole errors and synchrotron oscillations for δ =1% are also included. The off-energy orbit has been subtracted.

CONCLUSIONS

We have analyzed the longitudinal phase space in quasiisochronous storage rings including second order terms and the effect of betatron oscillations on the momentum compaction factor. The presence of the second order term introduces a new set of stable buckets displaced in energy and intercalated in phase with respect to the original set of stable buckets. When transverse betatron motion is considered, the stable phase region will depend on the particle amplitude. We have shown that this dependence will tend to enlarge the stable region when the signs of α_0 and α_0/α_1 are opposite.

We have also proposed a quasi-isochronous operation mode for the LNLS 1.15 GeV UVX electron storage ring where α_0 is 100 times smaller than in the standard mode. The chromaticities and the first order momentum compaction can be tuned by means of the sextupoles. The mode can be achieved by a continuous transfer from the standard operation mode, avoiding setting new injection conditions. This scheme could not be used for observing the new anomalous bunches since they do not exist in the standard mode. Dynamic aperture calculations show that we can have long lifetimes in this mode although injection is still difficult.

We note that the natural emittance of the beam in this quasi-isochronous mode has increased by a factor of approximately 4 with respect to the standard mode. We plan to continue exploring the condition of simultaneous small emittance and short bunches as well as questions related to beam instabilities which were not addressed in this report.

ACKNOWLEDGMENTS

We wish to thank J. Le Duff, H. Wiedemann and A. R. D. Rodrigues for many interesting discussions.

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