# RF Voltage Modulation at Discrete Frequencies, for Application to Proton Extraction using Crystal Channeling

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## *Abstract*

RF voltage modulation at a finite number of discrete frequencies is described in a Hamiltonian framework. The theory is applied to the problem of parasitic extraction of protons from a circulating beam in a high energy hadron collider, using a bent crystal as a thin "septum" extraction element. Three modes of employing discrete resonances are discussed: a strong, single drive resonance which may be used to excite protons to hit deep within the crystal; a single resonance ramped in such a manner that the island can carry trapped particles from low to high amplitudes; and overlapping resonances to create a chaotic band for separating the moving island and the large amplitude island. Simulations are used to confirm the expected dynamics, and finally a prototypical extraction scheme is described.

## I. Introduction

We explore using RF voltage modulation to affect the flux of protons onto a bent crystal being used to extract the protons from a storage ring. A low flux, high energy proton beam would be useful for both a test beam and for fixed target B-physics experiments at the next generation of hadron colliders [1]. RF noise has also been considered to enhance the flux on the crystal [2].

RF modulation is used to affect the longitudinal phase space dynamics while keeping the beams relatively unaffected at the interaction regions, which are presumed to have zero dispersion [3]. At the crystal, the dispersion would be large relative to the betatron amplitude, so the longitudinal motion affects the beam distribution significantly. This leaves open the possibility of extracting beam (from the halo) while beam collisions occur.

Channeling in a bent crystal could provide an economical way to extract a small flux of protons from a storage ring. Extraction of circulating beam has recently been demonstrated at the CERN SPS [4]. For appropriate beam parameters, a significant fraction of impinging protons channel between the planes of symmetry in the crystal, executing "betatron" oscillations in the effective focusing force. If the crystal is adiabatically bent then channeled protons follow the bend [5, 6] and are extracted from the storage ring. For the Tevatron experiment 853 [7], 7 meters of pulsed kicker magnets are replaced by a 3 centimeter long bent crystal to send 900 GeV protons down an abort beam line.

Using voltage modulation islands can be placed and manipulated in the RF bucket. The position, width and island tunes are well described by analytic theory. These islands will be used to affect the dynamics of single particles. A large island near the RF separatrix gives particles a large step into the crystal, an island with a ramped modulation frequency moves particles from smaller amplitudes to larger, and many overlapping islands form a stochastic layer that buffers between any ramped islands and large outer islands.

## II. Islands in the RF Bucket

The longitudinal dynamics of a proton stored in a ring can be described by an effective Hamiltonian. This form for the Hamiltonian relies on the energy gain from a cavity being small relative to the particle energy, so the discrete system can be approximated by a continuous one. This is equivalent to requiring that the synchrotron tune, $Q_{s0}$, is small, then the longitudinal dynamics for a single proton is described by

$$H(q,p,t) = 2\pi Q_{s0}\left(\frac{p^2}{2} + 1 - \cos\phi\right) \quad , \qquad (1)$$

where $p = 2\delta_p/\delta_{sep}$ gives the relative momentum offset, $\delta_{sep}$ is the offset at the separatrix, $\phi$ is the phase, or timing, of the proton at the RF cavity, and $t$ is time measured in turns around the ring. We do not discuss the above in much greater detail, but refer the reader to a standard treatment of RF phase stability [8]. The synchrotron tune given in terms of other RF parameters is $Q_{s0}^2 = h\eta e V_0/(2\pi\beta^2 E_0)$, and the momentum offet at the separatrix is $\delta_{sep} = 2Q_{s0}/(h\eta)$, where $h$ is the harmonic number and $\eta$ is the phase slip factor.

If the RF voltage is modulated at frequencies near twice the synchronous frequency, resonant islands appear in the RF bucket. Since the tune of a single particle in the RF

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bucket depends on its amplitude in the bucket, choosing the frequency of the modulation sets the position in the bucket where the modulation has the most effect. Defining the amplitude of a particle as $2a^2 = p^2/2 + 1 - \cos\phi$, or in terms of the off-momentum parameter $a = \delta_{max}/\delta_{sep}$, the synchrotron tune is to a good approximation $Q_s(a)/Q_{s0} = 1 - a^2/4$. Since the pendulum system is solvable, it should come as no surprise that there is an exact expression for the tune shift with amplitude in terms of elliptic integrals [9].

The width of the island depends on the strength of the modulation. For a voltage modulation of the form $V(t) = (1 + \epsilon\cos(2\pi Q_m t + \alpha)V_0$, the effective Hamiltonian is

$$H(q,p,t) = 2\pi Q_{s0}\Big(\frac{p^2}{2} + \big(1+\epsilon\cos(2\pi Q_m t+\alpha)\big)\left(1-\cos\phi\right)\Big). \qquad (2)$$

The width of the island is $\Delta a_{1/2} = \sqrt{\epsilon}$, and the island is centered at an amplitude of $a_R$ satisfying $Q_s(a_R) = Q_m/2$. The tune with which particles circulate inside the island, the island tune, is given by $Q_I/Q_{s0} = a_R\sqrt{\epsilon}/2$. A relatively small RF modulation can create a relatively large island and island tune.

## III. Adiabatic Ramped Islands

Islands that are slowly moved from small amplitudes to large ones can be used to re-populate the large amplitude regions of phase space after those particles there have been extracted. It can be moved by ramping the modulation tune from $Q_{m1}$ centered at an amplitude of $a_1$ to $Q_{m2}$ centered on $a_2$. If this ramping is slow enough compared to the time scale of the particle circulation in the island, there remains a stable area in the island and particles may trap and be carried from small to large amplitudes [10]. The precise formulation of the adiabatic condition gives

$$\left|\frac{dQ_m}{dt}\right| < 2\pi Q_i^2 = \frac{\pi}{2}\epsilon a^2 Q_s(a) \quad , \qquad (3)$$

where $Q_s$ is the synchrotron tune at the current position of the island $a$.

In Fig. 1, five trajectories are launched around the initial positions of the island and tracked while the island is ramped; one of the trajectories is launched at the island center. This shows that ramped islands in phase space can move particles from small amplitudes to large albeit with some time structure superimposed. For instance, in the case of the Tevatron example shown in Fig. 1, the time to reach the large amplitude, 500,000 turns, corresponds to about 10 seconds of real time.

## IV. Stochastic Region

The time structure for particles that are brought up from low amplitudes can be alleviated by first passing them through a stochastic layer. This layer can be formed by overlapping several "islands". Then particles move through this layer and into the large, outer island. In Fig. 2(a), 12 trajectories with initial conditions very close to each other and in the stochastic layer, formed by 9 overlapping islands, are tracked. Fig. 2(b) shows that the variance of the amplitude for the trajectories grows exponentially with the time, filling the layer.

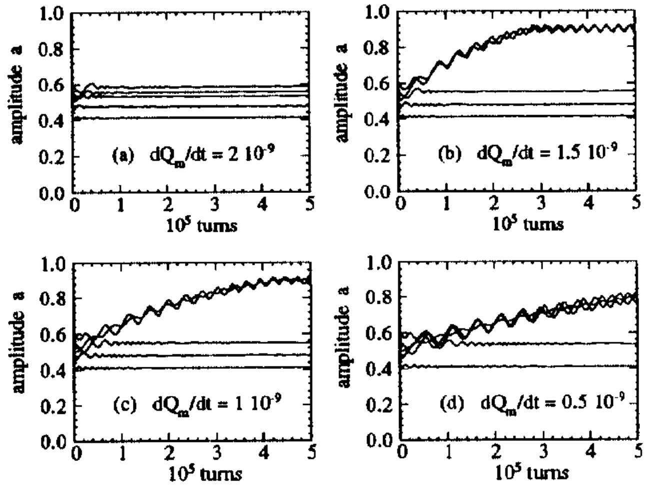


Fig. 1. Effect of ramped island on neighboring trajectories.

## V. Prototype Scheme

In Fig. 3, a prototypical extraction scenario is used and a trajectory that is initially trapped in the ramped island is shown. For the first $2 \times 10^6$ turns the proton is trapped in the moving island, then it enters the stochastic layer, here formed by 5 overlapping islands. It meanders until it is caught in the large island, in which it executes an oscillation out to $a = 0.8$, where the crystal would be placed to extract the proton. Later, the proton detraps from the large island and moves back into the stochastic layer.

## VI. Conclusion

It is possible using discrete RF voltage modulation to control and enhance the flux of circulating protons onto the face of a bent crystal used for extraction. It is thought, but remains to be demonstrated that these techniques give a bigger step size into the crystal resulting in more efficient channeling than RF noise diffusion of the beam. A big, large amplitude island, a ramped island, and a stochastic layer formed from islands make-up a toolbox for controlling the flux of particles and the extraction rate using bent crystals.

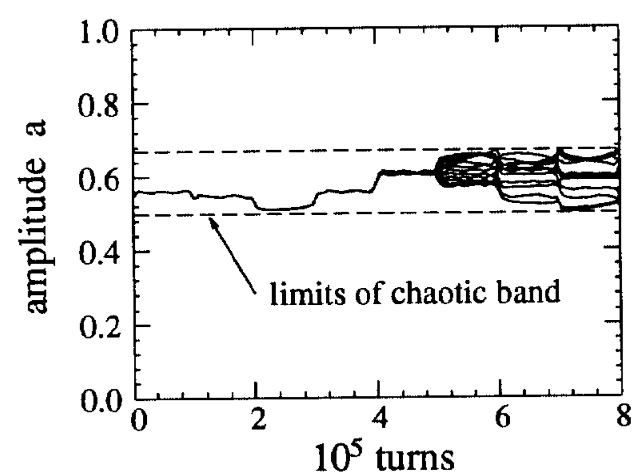


Fig. 2(a). Trajectories tracked in the stochastic layer formed by 9 overlapping islands.

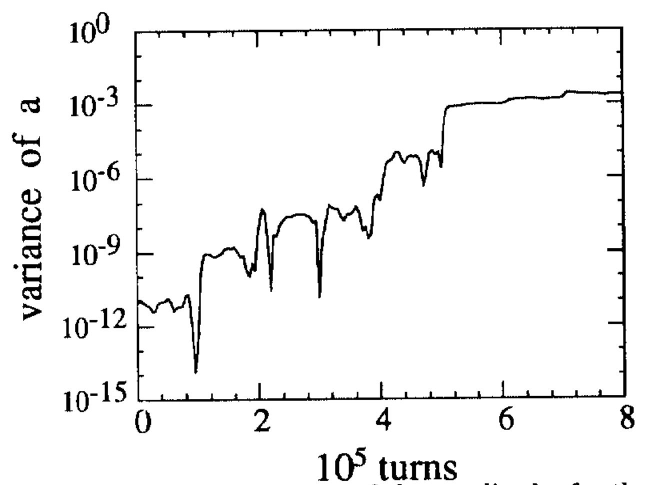


Fig. 2(b). The variance of the amplitudes for the 12 trajectories shown in Fig. 2(a).

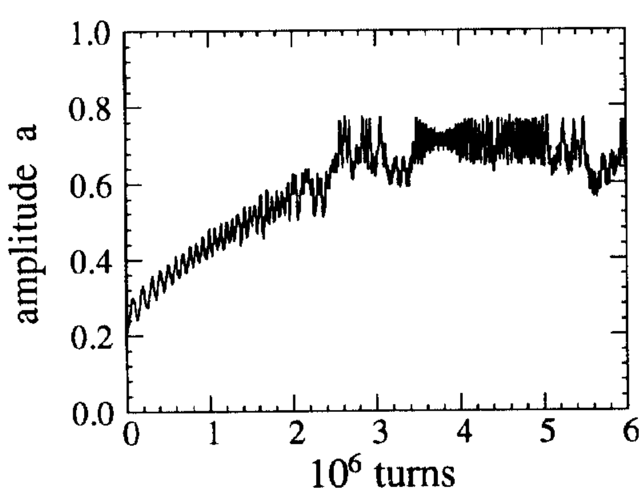


Fig. 3. Amplitude of a proton initially trapped in a moving island, carried to a stochastic layer, and "inserted" into a large amplitude island.

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