Transport Properties of the CEBAF Cavity*

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Abstract

The transport properties of the CEBAF 5-cell cavity are studied. The 3-D cavity fields are calculated by use of the 3-D program MAFIA and are incorporated in a modified PARMELA. Numerical simulation results show that the cavity has finite dipole, quadrupole and skew quadrupole field components, which are due to the asymmetric field in the fundamental and the higher-order-mode couplers. The azimuthal focusing of the cavity disappears for high energy particles as $\frac{1}{\gamma^2}$. The dependence on the initial energy and cavity phase is given. The cavity-steering effects were measured on the CEBAF 45 MeV injector and are in good agreement with the numerical simulation.

I. Introduction

The CEBAF superconducting cavity has five cylindrical symmetric cells and two end-couplers as shown in Figure 1. One is the fundamental-power(FP) coupler which couples RF power to the cavity. The other is the higher-order-mode (HOM) coupler which is designed to couple the higher-order-mode field, generated by the beam, to an RF load. The FP and HOM couplers do not have cylindrical symmetry, and these asymmetric structures generate asymmetric fields at their adjacent regions. The deflecting fields on the axis are no longer zero. Cavity-steering effects are important issues in nominal linac operation, and they are also a concern in a proposed CEBAF FEL, where two beams share the linac. In the proposal [1], both FEL and physics beams are injected into the cavity with different energies and are accelerated in the cavity at different phases. Since the steering effect of the cavity depends on both the phase of the RF field and the energy of the particle, the steering effect must be understood to successfully transport the two beams. To study these issues, a full 3-D modeling of the cavity is required, which is the motivation for this paper.

*This work was supported by the U.S. Department of Energy, under contract No. DE-AC05-84ER40150.

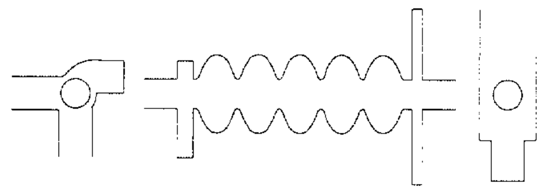

Figure 1. CEBAF 5-cell Cavity.

The 3-D fields of the CEBAF 5-cell cavity are calculated by use of MAFIA [2]. A particle tracking program PARMELA [3] is modified to take the full MAFIA fields into the calculation. The properties of the 5-cell cavity are studied by use of PARMELA and are compared with the experimental results. A more detailed account can be found in a CEBAF report [4]. PARMELA simulations of CEBAF cryomodules can be found in reference [5].

II. Field Distribution of the CEBAF Cavity

The CEBAF cavity is a 3-D structure (Figure 1). The maximum radius of the cell is 9.4 cm and the beam pipe radius is 3.5 cm. In this paper, we are interested only in the transverse effects of the fundamental mode field. The frequency of this mode is 1497 MHz, which is well bellow the cutoff frequency of the 3.5 cm beam pipe. To have a good boundary condition, the 3.5 cm pipe is extended up to 10 cm beyond the couplers on each side of the cavity where the field of the fundamental mode has vanished. The FP coupler has two ends. One end is terminated by the superconducting material while the other end is a waveguide leading to the RF power system. The length of the waveguide is taken as 20.7 cm in the calculation, which is a short position [6] for the fundamental mode. The HOM coupler is used to transfer the HOM field to a load, and the actual length of the two arms is more than 30 cm. Since it only perturbs the field distribution in the beam pipe region and does not propagate the fundamental mode, the length of the two arms is shortened to 15.5 cm in the calculation. Figure 2 shows the E and the B fields on the axis. The E_z

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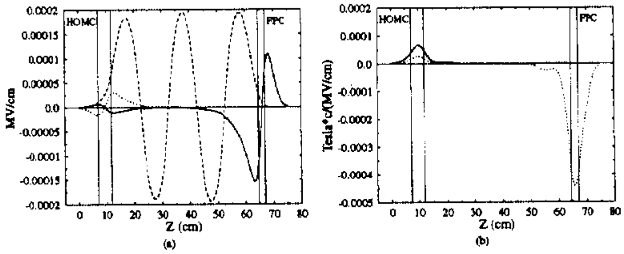

Figure 2. Field distribution on the axis (cav503f). Downstream FPC. a) Electric field, solid line: E_x; dotted line: E_y ; dashed line: $E_z/100$. b) Magnetic field, solid line: B_x ; dotted line: B_y.

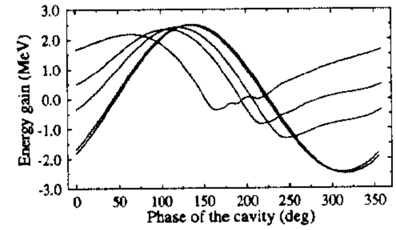

Figure 3. Energy gain vs. initial cavity phase for the particles with different initial energies. Gradient = 5 MV/m. From the left to the right: 0.5, 1.0, 1.5, 5.0, 1000.0 MeV (cav503f, downstream FPC).

field shown in the figure is scaled by 0.01. The flatness of the E_z field is about 2.5% in the 5 cells.

III. Numerical Simulation of the Beam Transport in the Cavity

A new subroutine CBFCAV3D is incorporated into PARMELA for the CEBAF 3-D cavity simulation. This subroutine takes the MAFIA result of the previous section as the field distribution in the cavity. In PARMELA, the particle advances step by step in the cavity under the Lorenz force given by

$$\mathbf{F} = e(\mathbf{E}\sin(\omega t(z)+\phi_0) + \mathbf{v}\times\mathbf{B}\cos(\omega t(z)+\phi_0)) \quad (1)$$

where ϕ_0 is the initial phase of the cavity. The momentum change of the particle after passing through the cavity is given by

$$\Delta\mathbf{P} = \int_{z=0}^{z=L} \mathbf{F}\frac{dz}{v_z} \quad (2)$$

A. *Acceleration of the cavity*

First we want to know the energy gain and the maximum acceleration phase for the particles with different initial energies. For a given momentum change ΔP, the energy change of the particle is

$$\Delta E = \gamma m_0 c^2\left(\sqrt{1+\frac{2P_0\Delta P+\Delta P^2}{(\gamma m_0 c)^2}}-1\right) \quad (3)$$

where P_0 is the initial momentum. Figure 3 shows the energy gain of the particles with different initial energies as functions of the initial phase of the cavity. The gradient of the cavity is 5 MV/m. As one can see, 1) the maximum energy gain is different for different initial energy particles, 2) the phase for the maximum energy gain (on-crest phase) is different for different initial energy particles and 3) the acceleration is not symmetric about the crest for low-energy particles. At 5 MeV, the electrons are quite relativistic, and the acceleration curve is almost the same as that of the 1 GeV electrons.

B. *Multipole components of the cavity deflection*

The transverse momentum changes, ΔP_x and ΔP_y, determine the deflection of a particle by a cavity. The deflection angles corresponding to the momentum changes are $\Delta\alpha_x = \frac{\Delta P_x}{P}$ and $\Delta\alpha_y = \frac{\Delta P_y}{P}$. Generally, ΔP_x and ΔP_y are functions of (x, y), and can be expanded as Taylor expansions of x and y. The coefficients of the expansion are related to the multipoles and can be obtained by means of the Fourier transform.

The z Fourier component of the electric field E_z can generally be expressed by [7]

$$E_z(r,\theta,z,\beta_z) = \sum_{n=0}^{\infty} A_n J_n(\gamma_r r)\cos(n\theta)e^{-i\beta_z z} + \sum_{n=0}^{\infty} B_n J_n(\gamma_r r)\sin(n\theta)e^{-i\beta_z z} \quad (4)$$

where $\gamma_r^2+\beta_z^2 = k^2 = \frac{\omega^2}{c^2}$, $J_n(\gamma_r r)$ is the Bessel function. Assume $\beta \approx 1$ and that the trajectory is a straight line. From Panofsky-Wenzel theorem we have, to order of r, a transverse momentum change of the form

$$\begin{aligned}\Delta\mathbf{P}_t &= \frac{ie}{\omega}\left(\frac{\gamma_r A_1}{2}\mathbf{x_0} + \frac{\gamma_r B_1}{2}\mathbf{y_0} - \frac{\gamma_r^2 A_0}{2}(x\mathbf{x_0}+y\mathbf{y_0})\right) \\ &+\frac{ie}{\omega}\left(\frac{\gamma_r^2 A_2}{4}(x\mathbf{x_0}-y\mathbf{y_0}) + \frac{\gamma_r^2 B_2}{4}(y\mathbf{x_0}+x\mathbf{y_0})\right) \\ &= D_x\mathbf{x_0} + D_y\mathbf{y_0} + F(x\mathbf{x_0}+y\mathbf{y_0}) \\ &+Q(x\mathbf{x_0}-y\mathbf{y_0}) + S(y\mathbf{x_0}+x\mathbf{y_0})\end{aligned} \quad (5)$$

F is the azimuthal-focus strength, D_x and D_y are the dipole strengths in the x and y planes, and Q and S are the quadrupole and the skew quadrupole strengths respectively. The synchronous condition requires that $\gamma_r^2 = k^2-\beta_z^2 = \frac{\omega^2}{c^2}-\frac{\omega^2}{v^2} = -\frac{\omega^2}{v^2\gamma^2}$, which goes to zero as γ goes to infinity. From Eq.(4), if E_z is to have finite acceleration, dipole and quadrupole components, the coefficients of the expansion must satisfy

$$A_0=\text{const.} \qquad A_1, B_1 \propto \frac{1}{\gamma_r} \qquad A_2, B_2 \propto \frac{1}{\gamma_r^2}$$

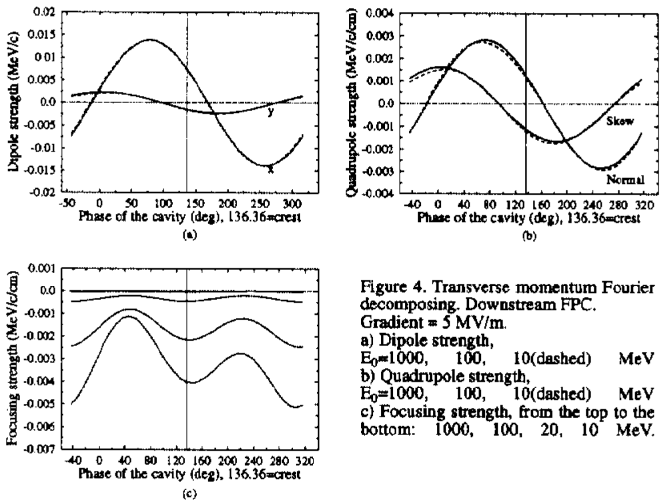

Figure 4. Transverse momentum Fourier decomposing. Downstream FPC. Gradient = 5 MV/m.
a) Dipole strength, E_0=1000, 100, 10(dashed) MeV
b) Quadrupole strength, E_0=1000, 100, 10(dashed) MeV
c) Focusing strength, from the top to the bottom: 1000, 100, 20, 10 MeV.

For large γ, D_x, D_y, Q and S become finite while F vanishes as $\frac{1}{\gamma^2}$. If the trajectory change is taken into account, F varies as $\frac{1}{\gamma}$ [8] instead of $\frac{1}{\gamma^2}$.

To calculate the coefficients of Eq.5, we first calculate the ΔP_x and ΔP_y for a number of particles initially distributed on a circle of radius a at $z = 0.0$ with certain energy and zero transverse momentum, and then do Fourier transforms of ΔP_x and ΔP_y. The coefficients are shown in Figure 4. The cavity gradient in Figure 4 is 5 MV/m.

IV. Experimental Results

Experiments were conducted to measure the steering effect of the CEBAF cavity on the 45 MeV CEBAF injector. In these experiments, we run the injector at about 20 MeV. The cavity measured is the second last cavity in the second cryomodule, and the results are shown in Figure 5, as a function of RF phase. The beam position is relative to the position of the particle on crest and is measured at about 17 m downstream from the cavity. Figure 5a is for a case with 18.42 MeV initial energy and 5.26 MV/m cavity gradient. Figure 5b is for a case with 17.28 MeV initial energy and 5.106 MV/m cavity gradient.

The data shown in Figure 5 contain both cavity steering effects and transverse kicks from the tilt misalignment of the cavity. They are functions of the RF phase. The maximum coupler kicks are about 50^0 off crest in both the x and y directions, which are not symmetric about the crest phase, while the kicks from the misalignment of the cavity have maximums on the crest phase and are symmetric about the crest (cosine-like). They can be removed by symmetrizing the data shown in Figure 5 about the crest phase. The differences of the symmetrizing are only from the coupler steering, which is shown in Figure 6. To compare with the PARMELA simulation, the position displacement is converted to transverse momentum. Agreement of the experiment with the simulation is very good.

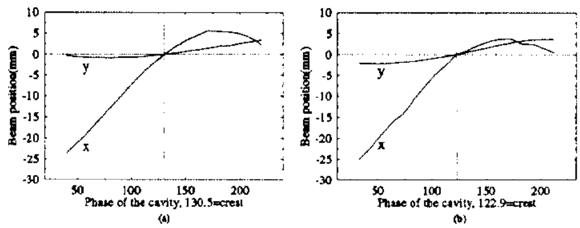

Figure 5. Experiment results of cavity-steering effect. a) measured on 4/30/92, b) measured on 5/7/92.

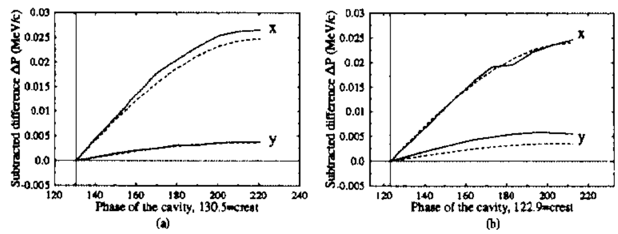

Figure 6. Symmetrized Figure-5 results. a) measured on 4/30/92, b) measured on 5/7/92. Solid=experiment; Dashed=PARMELA.

V. Conclusion

Cavity steering and focusing studied in this paper is from the fundamental mode field only. The modified PARMELA reveals that the cavity fundamental mode has finite multipoles which are due to the asymmetric HOM and FP couplers. Experimental results agree with the steering effects calculated.

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