Comparison of Aperture Determinations on RHIC for Single Particles Tracked 10^6 Turns and 100 Particles, Having Randomly Generated Initial Coordinates, Tracked for 1000 Turns*

George F. Dell

Brookhaven National Laboratory
Upton, NY 11973, USA

Abstract

Aperture determinations from 100 particles tracked for 1000 turns using randomly selected initial coordinates are compared with results from 10^6 turn runs when initial coordinates are defined by $\epsilon_x = \epsilon_y$ and $X_i' = Y_i' = 0$. Measurements were made with ten distributions of magnetic field errors. The results from tracking 100 particles for 10^3 turns are equivalent to those from 10^6 turn runs, have a distribution of considerably less width, and require only one tenth the computer time.

I. INTRODUCTION

Aperture determinations are made by probing phase space in a direction defined by the initial coordinates. Tracking on lattices, such as RHIC at BNL and various SSC lattices, indicates there is repetitive transfer of emittance between the horizontal and vertical planes. This transfer can be complete to either plane and depends upon the random field errors as well as the sextupoles used for chromaticity correction. This is illustrated in Figure 1 for the configuration that defined the aperture. The resulting time dependence of ϵ_x and ϵ_y causes the particle to probe in directions other than that defined by its initial coordinates.

Most frequently investigators have used initial coordinates defined by $\epsilon_{x(i)} = \epsilon_{y(i)}$ and $X_i' = Y_i' = 0$. In this paper a method that is patterned after the observed time dependence of ϵ_x and ϵ_y in Fig. 1 is used for selecting the initial coordinates of the test particles. The total emittance $\epsilon_{t(0)}$ is distributed in any way that satisfies the relation $\epsilon_{t(0)} = \epsilon_{x(i)} + \epsilon_{y(i)}$, and X' and Y' can be nonzero. One hundred particles having randomly selected $\epsilon_{x(i)}$, $\epsilon_{y(i)}$, X_i, X_i', Y_i, and Y_i' are tracked for 1000 turns.

The study has been made using the rhic92(0.0) lattice when $\beta^* = 2$m as well as $\beta^* = 6$m. The tunes are $\nu_x = 28.827$ and $\nu_y = 28.823$. The effects of nonlinear fields are represented by thin lens kicks located at the center of all quadrupoles and at both ends and the center of all dipoles, where they are given the weights of 1/6 and 2/3, respectively. Multipole expansions of random field errors are generated from the rms errors $(\sigma a_n, \sigma b_n)$ of Herrera etal[1] according to a Gaussian distribution that is truncated at $\pm 3\sigma$. The expansion is made with $2 \le n \le 16$ for dipoles and $2 \le n \le 10$ for quadrupoles. Aperture determinations were made for ten sets of random field errors generated by using different seeds to initialize the random number generator. No systematic errors were included. Tracking was performed on the NERSC CRAY.C computer at LLNL using a special version of PATRICIA.[2] Measurements were made at $\Delta P/P = 0$. The test particles were always launched at the beginning of an inner arc of RHIC. The amplitude of the test particle was checked at every element to assure it remained within the vacuum chamber.

II. TRACKING

1. Generation of initial coordinates – The steps used in generating the coordinates for multiparticle launching are:
   1. Define the initial total emittance $\epsilon_{t(0)}$.
   2. Select the initial horizontal emittance randomly: $\epsilon_{x(i)} = \epsilon_{t(0)} * \text{RANF}$
   3. Determine the initial vertical emittance $\epsilon_{y(i)}$: $\epsilon_{y(i)} = \epsilon_{t(0)} - \epsilon_{x(i)}$
   4. Select the initial coordinate X_i randomly:
      a). $X_{\max} = \sqrt{(\epsilon_{x(i)} * \beta_x)}$
      b). $X_i = X_{\max}(1. - 2. * \text{RANF})$
   5. Determine X' from the Courant-Snyder relation:
      a). $X_i' = \left(-\alpha_x * X_i \pm \sqrt{(\epsilon_{x(i)} * \beta_x - X_i)}\right) / \beta_x$
      b). Select the sign of $\sqrt{(\epsilon_{x(i)}\beta_x - X_i^2)}$ randomly
   6. Repeat step 4 and 5 with X replaced by Y.
2. Multiparticle Tracking – One hundred particles having randomly generated initial coordinates were launched and tracked in sequence. If any particle failed, the motion was considered unstable, and the run was terminated. The total emittance $\epsilon_{t(0)}$ was decreased in steps from a large value until all particles survived for 1000 turns. The results are expressed in terms of an equivalent X defined as $X_i = \sqrt{\epsilon_{t(0)} * \beta_x / 2}$ and are thus consistent with the convention used for the standard launch when $\epsilon_{x(0)} = \epsilon_{y(0)}$.
3. Single particle tracking for 10^6 turns – With $\epsilon_{x(0)}$ and $\epsilon_{y(0)}$ always equal, X_i was decreased from a large amplitude at which the particle was lost to an amplitude where the test particle first stayed within

*Work performed under the auspices of the U.S. Department of Energy.

0-7803-1203-1/93$03.00 © 1993 IEEE

the vacuum chamber for 10^6 turns. The smallest of the ten measurements was used as the aperture.

III. DISCUSSION

The aperture measurements at $\beta^* = 2$m for multiparticle and 10^6 turn runs are shown as histograms in Figure 2. The number in each bin indicates the set of random errors used for the determination. The test particle survived at the amplitude corresponding to the left side of the bin and failed at the amplitude corresponding to the right side of the bin. Measurements were also made when $\beta^* = 6$m. These are included in Table 1 and are reported elsewhere[3]. It is noted that:

1. The apertures defined by the worst case from 10^6 turn and multiparticle runs are essentially equal.
2. The spread in the distribution of results is smaller for multiparticle than for single particle tracking.
3. The computer time required for the multiparticle studies is one tenth that for the 10^6 turn studies.

Analysis of results from the single particle tracking of Figure 2(a) shows a correlation between the degree of emittance transfer and the worst and best case apertures. The worst case shows large emittance transfer; the best case shows modest emittance transfer The smear plots generated from the first 300 turns and the last 300 turns of these 10^6 turn runs are essentially identical. Hence there is little emittance growth with time. It is also found that the shape of smear plots, generated from 300 turn runs, has only a weak dependence on initial emittance at initial amplitudes for which the particles remain within the vacuum chamber for at least 1000 turns. Hence there is a weak dependence of emittance growth on initial amplitude. Since runs stable for 10^6 turns are not in the region of rapid emittance growth, it seems justifiable to conclude that the best and worst case apertures are determined by emittance coupling, rather than emittance growth. This assertion has been at least qualitatively verified by tracking with initial coordinates defined by $\epsilon_x = \epsilon_t$ and $\epsilon_y = X' = Y' = 0$ – a configuration achieved in multiparticle tracking as well as in single particle tracking with total emittance transfer between the horizontal and vertical planes.

Table 1: Aperture Determinations for RHIC92
$X_i\text{mm} = \sqrt{\epsilon_{t(0)} * \beta_x/2}$ where $\beta_x = 50$m

	SINGLE PARTICLE TURNS		MULTIPARTICLE TURNS
β^*	1000	10^6	1000
2	8.3 ± 0.1	6.7 ± 0.1	6.5 ± 0.1
6	16.3 ± 0.1	14.3 ± 0.1	13.7 ± 0.1

IV. REFERENCES

1. PATRICIA, 1980 version by H. Wiedemann, SLAC, modified for multipoles by S. Kheifets, SLAC; further modifications at BNL by G.F. Dell.
2. J. Herrera, R. Hogue, A. Prodell, P. Thompson, P. Wanderer, and E. Willen, IEEE PAC, Washington D.C., March 16-19, 1987, pp 1477-1479.
3. G.F. Dell, Proceedings on the Stability of Particle Motion in Storage Rings, Brookhaven National Laboratory, Oct. 19-24, 1992.

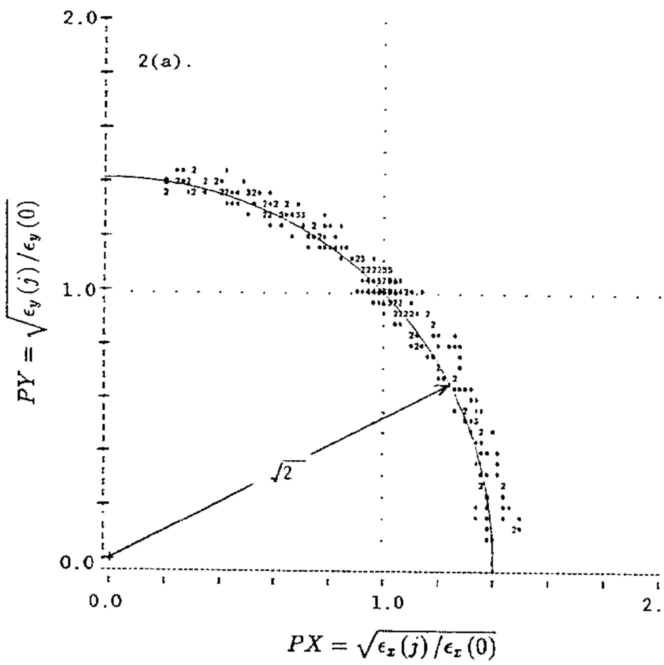

Figure 1: Normalized smear plot showing emittance transfer during the first 300 turns of a 10^6 turn run at $\beta^* = 2$m with seed #9.

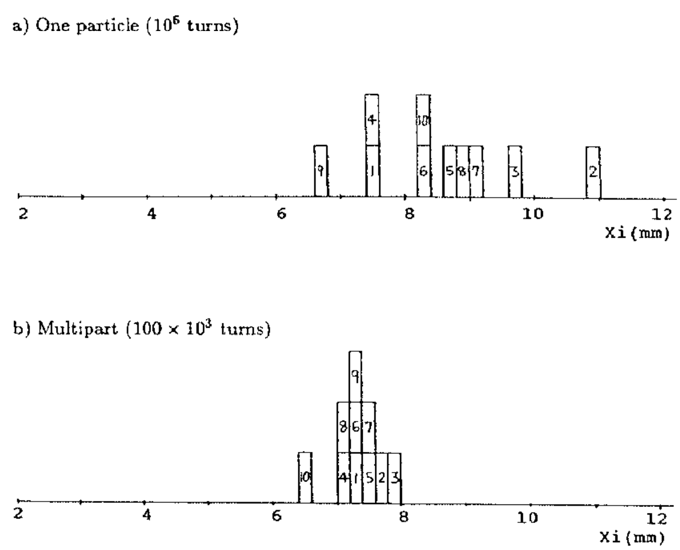

Figure 2: RHIC92, $\beta^* = 2$m. Aperture determinations for: a) one particle/seed tracked 10^6 turns, and b) 100 particles/seed tracked 1000 turns.