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Four Cell Third Order Achromats and Their Application to Multi-Pass Time-of-Flight Spectrometers

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Abstract

A repetitive third order achromatic system was designed. As an example, we chose the ESR storage ring at GSI. Instead of repetition of cells, which is widely used in achromat design based on normal form theory, we utilize cells which are obtained from the the original ones through mirror imaging about the x-y plane, which corresponds to a reversion. In our design, the second half of the ring is the reversion of the first one, and two turns make a third order achromat. The dynamical aperture was determined by tracking with 11th order maps using COSY INFINITY [4]. The 9th order time-of-flight resolution was calculated statistically with beams of different emittances.

I. Introduction

In the past few years, various third order achromatic systems containing at least five repetitive identical cells have been found using normal form theory [1] [2] [3]. The number of bending magnets needed ranges from 5 to 150. By introducing mirror symmetry into the consideration, we developed a new theory which requires only four cells and one bend per cell to get a third order achromat [4] [5] [6]. According to the theory, the linear map of every cell should be a rotation with the phase advance $\mu_{x,y} = n * 90deg$ $(n = 0,1,2,3)$ plus no more than one nonzero chromatic term. Based on this, a family of achromatic systems which demand the fewest conditions on the first cell (10 for second order 15 for third order) were found and are listed in table 1. Here F (Forward) stands for the base cell of a system, R is the reversed cell which reverses the order of the elements in the forward cell, S is the switched cell which flips the bending direction of the forward cell, and C is the combined cell which switches the bending direction and reverses the order of elements simultaneously.

Inspired by the fact that due to symplecticity, the only aberrations left in an achromat are $(t|\delta^n) = 0$ $(n = 0,1,2,\cdots)$, we adopted the idea of making ESR a high resolution time-of-flight energy spectrograph by redesigning it as a third order achromat. The designing process is presented in section II., where the strengths of the multipoles are found quite feasible. Section III. shows our analysis of the system which includes the long term stability of the system and the resolution it can achieve.

*This work was supported by the U.S. National Science Foundation No. PHY 89-13815 and the Alfred P. Sloan Foundation.

θ_x	$\theta_y = 0$	$\theta_y = 90$	$\theta_y = 180$	$\theta_y = 270$
0	F R S C	F R S C	F R S C	F R S C
90	F R F R	F R F R	F R F R	F R F R
180	F R S C	F R S C	F R S C	F R S C
270	F R F R	F R F R	F R F R	F R F R

Table 1: Achromatic systems with $(a|\delta) = 0$ after the first cell. In case $(x|\delta) = 0$, exchange R and C.

II. Design of the Achromat

A. *First Order Design*

In order to design a circular machine as a third order achromat, no switched (S) or scwitched-reverse (C) sections can be used, and so we use the patten FRFR. This means that the first order map of the forward cell has to have $\mu_x = 90deg$ or $270deg$. The ESR ring contains six dipoles, twenty quadrupoles, eight sextupoles as well as RF cavities, beam cooling devices, and the injection-extraction system. Two long straight sections divide it into two identical parts, each of which is symmetric about its center. (Figure 1) [7]. It is much easier to take half rather than a quarter of the ring as the forward cell. Consequently, the other half should be the reversed cell, and an achromat corresponds to two turns of the ring. Since there are five conditions for a symmetric cell to meet [6], it is sufficient to fit only the quad strengths to find a desired solution. Thus the first order layout of the existing ESR ring is preserved. Also because of the symmetry, the forward cell is a first order achromat. The field gradients of the quads are displayed in Table 2.

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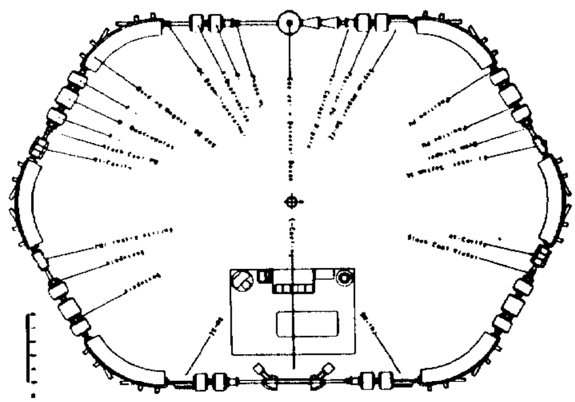

Figure 1: The original ESR

Element	Field Gradient(T/m)
Quadrupole1	5.2141708710097464
Quadrupole2	-3.7780943756357911
Quadrupole3	-0.2757583566872618
Quadrupole4	-1.0861967098567489
Quadrupole5	2.1018617072758571

Table 2: The field gradient of the quads

B. *Second and Third Order Achromat*

Ten sextupoles were placed symmetrically in the forward cell. The values of the field satisfying the conditions for the second order map were found using the nonlinear optimizer in COSY INFINITY (Table 3). The same was done with the third order correction except that there are more octupoles after the third bend than before the first one. The positions of some of the multipoles were carefully chosen to limit the required field strengths. (Figure 2), and the results are quite realistic (Table 4).

Element	Field Gradient(T/m^2)
Sextupole1	-12.655855454352217
Sextupole2	8.0550942902064335
Sextupole3	-14.549046698823922
Sextupole4	15.697993340520378
Sextupole5	0.8844613366417970
Sextupole6	-53.905125612548678
Sextupole7	66.227372392466883
Sextupole8	-16.648568962020669
Sextupole9	-0.1067654043405714
Sextupole10	3.4540652334940187

Table 3: The field gradient of the sextupoles

Element	Field Gradient (T/m^3)
Octupole1	-285.07749404229136
Octupole2	368.94910085314347
Octupole3	614.99793829516426
Octupole4	680.30311596846740
Octupole5	-5442.7323531935168
Octupole6	4250.2409995782432
Octupole7	-837.91940849587759
Octupole8	-1920.2296769772195
Octupole9	3595.6586358546853
Octupole10	-2187.7620170696497
Octupole11	2622.2366537113714
Octupole12	-1494.9597751428264
Octupole13	-757.00955057500670
Octupole14	305.17394857470414
Octupole15	-146.44242626707558

Table 4: The field gradient of the octupoles

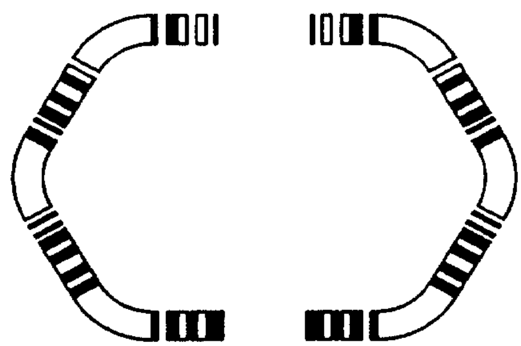

Figure 2: The upgraded ESR, the long multipoles are quads and the short ones are sextupoles and octupoles

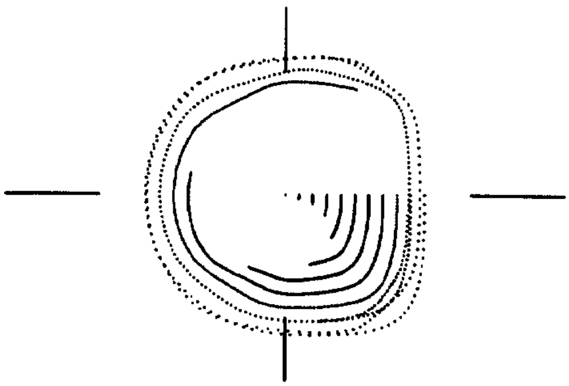

Figure 3: 200 turn tracking of the x-a motion of on-energy particles

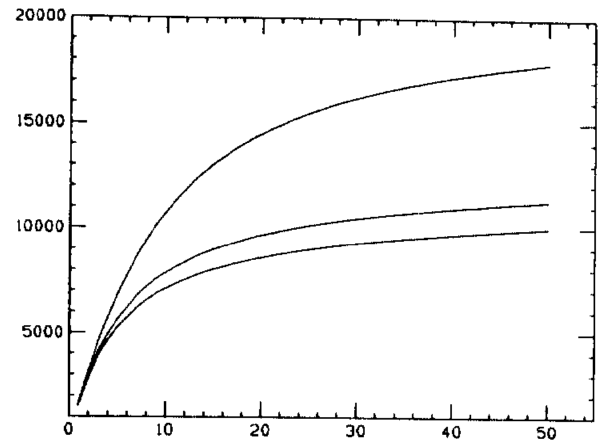

Figure 4: Resolution vs number of turns at different emittances

III. Analysis of the Ring

A. *Dynamical Aperture*

Since our goal is to make ESR a multi-pass time-of-flight spectrograph, the long term behavior becomes of vital importance. It was studied with an 11th order one turn map which was generated by COSY INFINITY and used for non-symplectic tracking. The 200 turn dynamical apertures for both horizontal and vertical motion were determined by analyzing phase space plots. For particles of momentum spread $\pm 0.5\%$ to survive 200 turns, they are roughly 100 π *mm mrad* horizontally and 15 π *mm mrad* vertically. As an example, Figure 3 shows the horizontal motion of on-energy particles up to 200 turns.

B. *Resolution*

The resolution of this machine was determined in a statistical way. First, the 9th order one turn map was computed. Secondly, a large number of particles (1000) inside a certain phase space area were produced randomly by COSY. Then these particles were sent through the one turn map n times, therefore the n-turn time-of-flight of each particles were computed. Considering the random errors of the detector, which was assumed to be about 100 *ps*), the predicted energy deviations of every particle were calculated. Finally the difference between the predicted and initial energy deviations was obtained and the resolution of the ring was determined by calculating the inverse of the average differences. The dependence of the resolution on the number of turns and the emittance is presented in Figure 4 .

IV. Conclusion

A multi-turn third order achromat has been designed based on the layout of the ESR. The strengths of the magnetic multipoles are feasible and the dynamical aperture seems realistic for operation. For a rather large phase space, the resolution can be impressively high.

Acknowledgment

We would like to thank G. Hoffstätter and R. Degenhardt for several useful discussions.

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