A Clean Way to Measure Nonlinear Momentum Compaction Factor α_1

J.P. SHAN, I. KOURBANIS, D. MCGINNIS, K.Y. NG, and S. PEGGS
Fermi National Accelerator Laboratory *, P.O. Box 500, Batavia, IL 60510

Abstract

α_1 is an important lattice parameter for transition crossing. There exist several ways to measure α_1, such as debunching near transition. The extraction of α_1 from debunching rate depends on the momentum spread of beam, which is hard to measure accurately. Here we report another way to bypass this difficulty. Instead of debunching, the beam is stored in a stationary bucket near transition. Since the bucket near transition is very small, the particles inside the bucket will fill the bucket and those outside will be lost if parameters are chosen properly. So the measured bunch length is equal to bucket length, which can be used to extract α_1. The nice thing about this mothed is that the measurement dose not depend on initial distribution of bunch as long as its initial emittance is big enough to fill the stationary bucket near transition. A test has been carried out in the Fermilab Main Ring (MR).

1 Introduction

In a synchrotron or a storage ring, the momentum compaction effect influences the longitudinal motion through the phase slip factor

$$\eta = \frac{1}{T_0}\frac{T-T_0}{\delta} = \eta_0 + \eta_1\delta + O(\delta^2), \qquad (1)$$

where $\eta_0 = \alpha_0 - \frac{1}{\gamma^2} \equiv \frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}$, and

$$\eta_1 \quad = \quad \alpha_0\alpha_1 + \frac{3\beta^2}{2\gamma^2} - \frac{\eta_0}{\gamma^2}. \qquad (2)$$

Here T is the period of revolution for a particle with momentum offset $\delta = \frac{p-p_0}{p}$ and T_0 for a synchronous particle, β and γ follow usual relativistic kinematic notation, and γ_T is the transition gamma for a synchronous particle, α_0 and α_1 are defined in the expansion of orbit length

$$C - C_0 = C_0\alpha_0\delta\left[1 + \alpha_1\delta + O(\delta^2)\right], \qquad (3)$$

where C_0 is the orbit length for reference particle.

Near transition where η_0 vanishes, the nonlinear term

$$\eta_1 \approx \alpha_0(\alpha_1 + \frac{3}{2}) \qquad (4)$$

becomes very important. For a quasi-isochronous electron storage ring, $\eta_1 \approx \alpha_0\alpha_1$ since $\gamma >> \frac{1}{\sqrt{\alpha_0}}$. The first order nonlinear compaction factor α_1 can be calculated analytically [1] for a FODO lattice or numerically from a lattice code such as MAD. But in a real machine, such as the Main Ring, there are a lot of unknown nonlinear components. So it is very important to be able to measure α_1.

There exist several ways to measure α_1, such as nonlinear dependence variation of revolution frequency on the momentum offset [2] or debunching near transition [3]. For a ring with big radius and small aperture, it's very difficult to apply the former method. The extraction of α_1 from debunching rate depends on the momentum spread of beam, which is hard to measure accurately. To bypass this difficulty, a new way to measure α_1 has been proposed [4], which uses a property of a stationary bucket near transition.

2 RF Bucket near Transition

The longitudinal Hamiltonian for stationay bucket ($\phi_s = 0$) including the nonlinear η_1 term can be written as

$$H(\phi,\delta) = \frac{eV}{\beta^2 E}\cos\phi + 2\pi h[\frac{1}{2}\eta_0\delta^2 + \frac{1}{3}\eta_1\delta^3] \qquad (5)$$

where V is RF voltage, h harmonic number, E beam energy.

The separatrix is a Hamiltonian contour through the unstable fixed point in the phase space. There are two sets of fixed points, one at $\delta = 0$ and another at

$$a = -\frac{\eta_0}{\eta_1} \approx -\frac{2\Delta\gamma}{\gamma_t(\alpha_1 + 3/2)} \qquad (6)$$

introduced by the nonlinearity and approximation in Eq. 6 is valid near transition. The nonlinear strength can be parameterized by a quantity

$$x = \sqrt{\frac{\pi h\beta^2 E|\eta_0|^3}{6eV_{rf}\eta_1^2}} \qquad (7)$$

When far away from transition ($x \gg 1$), the bucket around $\delta = a$ is way outside the momentum aperture. Near transition, a becomes smaller and the nonlinear contribution begins to make the bucket unsymmetric in the δ axis and the second set of bucket around $\delta = a$ moving close to the aperture as seen in the Fig. 1. Then comes a piont when the bucket height $\delta_+ = a$ under the critical condition $x = 1$. Even close to transition the bucket height does not depend on RF voltage and the bucket width begin to shrink. The properties of bucket can be summarized in the following

*Operated by the Universities Research Association Inc., under contract with the U.S. Department of Energy.

0-7803-1203-1/93$03.00 © 1993 IEEE

PAC 1993

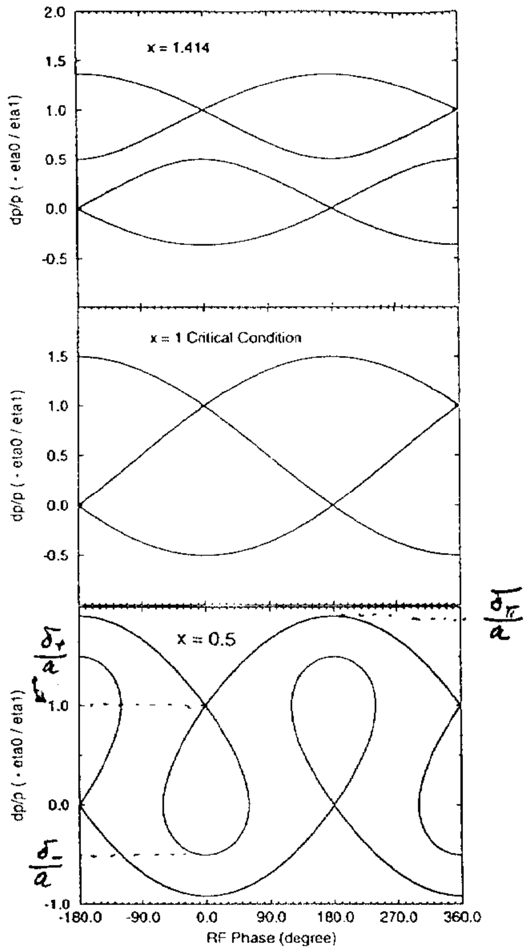

Figure 1: The stationary bucket near transition. The vertical axis is momentum offset normalized to $a = -\frac{\eta_0}{\eta_1}$.

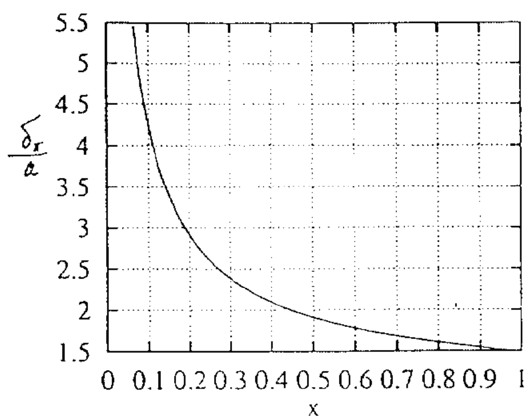

Figure 2: $\frac{\delta_\pi}{a}$ as a function of x

table:

regime	bucket height	width	type
$x \gg 1$	$\delta_+ = \delta_- \propto \sqrt{V}$	2π	A
$x > 1$	$\delta_+ > \delta_- \propto \sqrt{V}$	2π	B
$x = 1$	$\delta_+ = 2\delta_- = a \propto \sqrt{V}$	2π	C
$x < 1$	$\delta_+ = 2\delta_- = a$	$4\arcsin(x)$	D

In the table above, δ_+ and δ_- are respectively the bucket height in the upper and lower phase space as shown in Fig. 1. Another important parameter is the maximum momentum deviation in the separatrix. Since it's always at phase π, let's call this maximum momentum deviation δ_π. The ratio of δ_π over a depends only on x, as shown in Fig. 2. Here we assume $a > 0$.

For type D bucket, the bucket width depends on η_1. So if bucket width can be measured, x can be calculated. Then if know η_0 very accurately, η_1 (thus α_1) can be extracted.

3 Measurement Method and Parameter Choice

One way to measure the bucket length is to let beam fill the bucket. Then the measured bunch length will be the same as the bucket width. So if we can ramp the beam to a energy near transition, probably below trantion to avoid of complication of transition crossing. How far away from transition? The ground rule for choosing parameters is

$$1.5a < \delta_{aperture} < \delta_\pi \tag{8}$$

to make sure that particles inside bucket are captured and those outside are lost to the momentum aperture. To get clean signal, the ratio of δ_π over a should be maximized. From Fig. 2, x should be less than 0.3 to get $\frac{\delta_\pi}{a} > 2.4$. On the other hand, the lower limit of x is set by the resolution of bunch length measurement. For the Fermilab Main Ring, the normal stationary bucket is 20 ns long. Bunch length shorter than 1 ns can not be measured accurately. The lower limit is $x > 0.1$. For a machine with lower RF frequency, such as the Brookhevan AGS, the lower limit could be even smaller.

4 Test in the Main Ring

4.1 Setup

The Main Ring is a synchrotron with following parameters

Radius	1000	m
γ_t	18.86	
f_{rf}	53	MHz
h	1113	

In the experiment, 20 bunches of beam with 1×10^{10} protons per bunch are accelerated from 8 Gev to a energy very close to transition energy at 0.44s. Then the RF voltage is lowered to less than 10 KV by using the technique of paraphasing at 0.47s. The bunch length is measured in the frequency domian by detecting the first (53 MHz) and third harmonic component (159 MHz)[5].

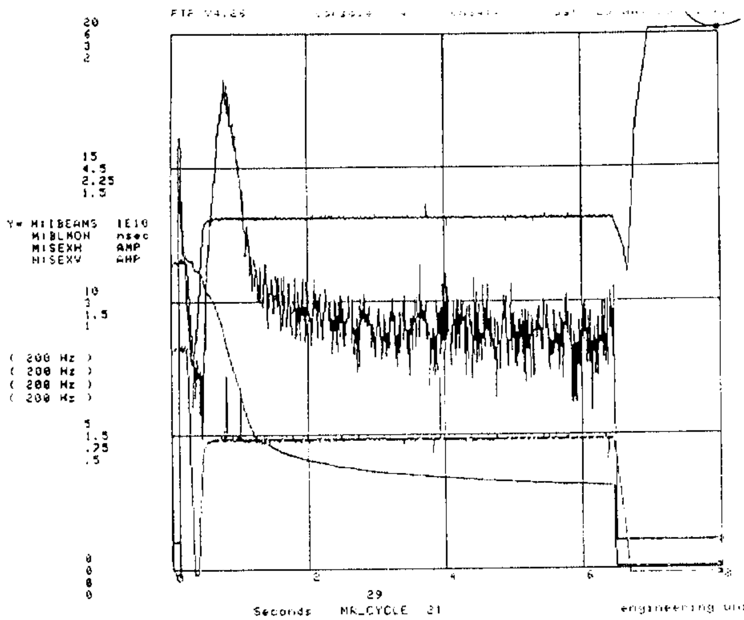

Figure 3: The beam accelerated to 17.63 Gev/c and stay at this energy for 6 s. with $I_h = 1.5$ A and $I_v = 1.6$ A.

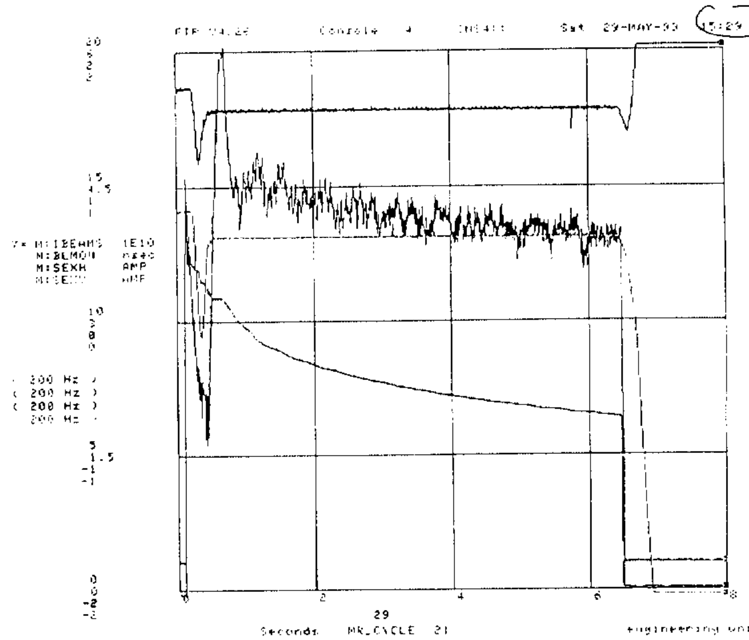

Figure 4: The same as the Fig. 4. with different sextupole currents $I_h = 2.0$ A and $I_v = 0.5$ A.

4.2 Observations

The experiment in the MR has just started. The preliminary measurement presented here was carried out at front porch momentem 17.63 Gev/c.

There are two sets of sextupoles in the Main Ring for correcting chromaticities. Far away from transition, the longitudinal dynamics should not be affected by sextupoles. Near transition we expect that the longitudinal dynamics is very sensitive to the setting of sextupoles. Fig. 3 and 4 are the observations of bunch length and beam intensity at front porch with different sextupole settings. It is obvious that the sextupole has very strong effect on the longitunal motion of beam.

5 Discussion

The preliminary data from the Main Ring are encouraging. The variation of longitudinal bucket width with sextupole strengths has been measured. For cleanest measurement of α_1, fine tuning of experimental parameters and careful isolation of transverse effect are needed. The forthcoming parametric scan are expected to confirm expexted scaling behavior.

Acknowledgements

Thank G. Jackson for the arrangement of beam time in the Main Ring . The stimulating discussions with K. Meisner, X. Lu, W. Gabella and P. Zhou are acknowledged.

References

[1] J.P. Shan, S. Peggs, and S. Bogacz. Fermilab pub92/124, to be published in Particle Accelerators, April 1992.

[2] E. Ciapala, A.Hofmann, S. Myers, and T. Risselada. *IEEE Nsl.*, 26(3), June 1979.

[3] K.Y. Ng, C. Bhat, I. Kourbanis, J. Maclachlan, M. Martens, and J.P. Shan. In *Proc. XVth Int. Conf. on High Energy Accelerators*, 1992.

[4] J.P. Shan. Measuring α_1 by storing beam near transition with low rf volatge. Talk given at MIRF meeting, March 1992.

[5] G. Jackson and T. Ieiri. In *1989 IEEE Particle Accelerator Conference*, pages 863–, 1989.