High Frequency Betatrons

David R. Winn
Department of Physics
Fairfield University
Fairfield CT 06430-7524

Abstract

We discuss the scaling of betatron technique with the use of high frequency, low-loss magnetic materials (ferrites, metglas, etc.). Because of synchrotron radiation, the maximum betatron energy E_{max}(GeV) scales as ~ 0.013 f(Hz)/B_s^2(T), where B_s is the maximum magnetic field on the orbit and f is the full-wave acceleration frequency. Eddy current losses in laminated iron limit f to ~120 Hz, thereby limiting E_{max} to 300-400 MeV for a classical betatron, with a low acceleration gradient, and low current. With low-loss, high frequency materials, one may consider f~1-100 kHz, and energies in the GeV regime, or betatrons with substantially higher current. We discuss practical considerations, potential advantages, and possible applications.

I. INTRODUCTION

The standard betatron[1] has many advantages for generating electron beams compared with other technologies. At energies below ~50 MeV betatrons have a much simpler power modulator and construction than either linear induction or RF linacs, and can be more compact for a given energy. On the other hand, the standard betatron is flux-limited compared with the linacs of similar energy. Compared with the synchrotron, both the energies obtainable and the large amounts of high μ material necessary lessen the the betatron from being the technology of choice in many applications. In this brief note, we point out that the inherent virtues in the betatron technique may be enhanced by the availability of modern high frequency magnetic materials[2]. We refer throughout to the excellent reviews in references [3], [4], [5], [6], [7] .

II. ENERGY LIMITS

The energy limit of a betatron is given to first order by synchrotron radiation. As was pointed out by Iwanenko and Pomeranchuk in 1944 [8], the maximum possible energy of a full core betatron scales as

$$E_{max}(\text{GeV}) \sim 0.013 f(\text{Hz})/B_s^2(\text{T}), \qquad (1)$$

where B_s is the maximum magnetic guide field = 1/2 B_w, the average induction swing during acceleration, and f is the frequency of the full-wave acceleration cycle. Typical early large laminated iron core betatrons were limited by eddy current losses to low frequencies, typically ~120 Hz at 2 T maximum, thereby limiting E_{max} to about 400 MeV for a classical betatron. With modern high frequency low loss magnetic materials [7],[9] at frequencies f > 1-10 kHz one can therefore in principle consider betatrons in the multi-GeV regime.

The induction energy on the equilibrium orbit for relativistic electrons [7] is given by:

$$E = eV \le eB_sRcs^2. \qquad (2)$$

The constant s^2 is the fraction of the area of the orbit of radius R filled with core, introduced as a connection to large recirculating induction machines[7]. The limit of small cores on a circle is that of a recirculating induction linac. As usual in a full-core betatron, R~3.3 m at $B_s = B_w/2 = 1$ T, at E~pc=1 GeV. The energy under the betatron condition is constrained by half or less of the maximum magnetic field swing available in magnetic materials with low loss.

R is also constrained by

$$R = c/8\pi fn, \qquad (3)$$

where n is the number of turns per 1/4 cycle of the acceleration frequency f, for producing beams with small energy spread at maximum value. (The limit of n=1, single turn acceleration, is essentially the linear induction accelerator.)

Substituting, we find that the fn product

$$fn \le B_s c^2 s^2/8\pi E(\text{eV}), \qquad (4)$$

which for a field B=1 T and $s^2=1$ gives fn ~3.6 x 10^6 Hz-turns at 1 GeV. The fn product, scaling as 1/E, highlights that high frequency operation at a fixed energy requires a small number of turns in the acceleration cycle, implying a high acceleration gradient. The small number of turns lessen the orbit stability requirements, while the high gradient allows higher captured current. For other parameters fixed, the acceleration gradient F scales directly with f, and the turns per 1/4 cycle n with 1/f. A 1 T betatron at 300 MeV has a gradient of ~9 keV/Turn at 100 Hz, travelling 120,000 turns; a 1(5) kHz, 600 MeV betatron beam would travel only 6,000 (1,200) turns, with a gradient of 100(500) keV/turn; the electron energy gain exceeds the injection energy from a typical internal electron gun in less than 1 turn.

III. CURRENT LIMITS

If the injected charge in one cycle were independent of the acceleration mechanism, the time averaged current

0-7803-0135-8/91$01.00 ©IEEE

would simply increase with f. However, we note that the peak injection current limit scales roughly as $I_o \sim F^{3/2} f(E_o)$ where F is the the acceleration gradient and $f(E_o)$ is a function that scales between linear-quadratic in the injection energy E_o[10]. Since F scales with f, the initial injected current scales as $I_o \sim f^{3/2}$. However, to maintain a well-defined energy, the current is injected over a small fraction of the betatron cycle time T=1/f, typically about 0.1% to a few 2% of T, and therefore the average beam current I scales as $I \sim I_o f^{-1} \sim f^{1/2}$ (averaged over a cycle).

The captured current limit in space charge equilibrium with a weak focussing force in a standard betatron also scales between B to B^2 at injection [6], [7], depending on the focussing gradients, implying that the highest practical injection energies and gradients are needed. In a large betatron, for example, a typical peak current limit at injection would be only ~1 A at 100 keV, but this limit rises to ~30 kA if it could reach ~1 GeV.

As an example of the potential of rapid acceleration, at 100 keV injection for acceleration to 1 GeV at 1 kHz , the energy after 1 turn is ~350 keV, increasing the equilibrium current limit averaged over the single orbit by a factor of ~2-6, depending on the weak focussing. The radius of the instantaneous orbit after 1 turn shrinks proportionally to the acceleration frequency (gradient), and beam avoidance problems of the injector could be made minimal at large gradients/high frequencies. However, normal betatron orbit solutions assume that the accelerating field does not change appreciably over 1 revolution, which for a 10-20 turn machine would not be as good an approximation.

Therefore, for similar focussing force betatrons, it is reasonable to expect at least a factor of x3-x10 in average current for every decade increase in the acceleration frequency. Typical weak-focussing betatrons injecting at ~100 keV over 2% of the cycle, accelerating to ~0.34 GeV, achieved 20 mA average current, averaged over over 1 cycle, at 60 Hz (and therefore an average circulating current of 2-10 mA if operated continuously - losses typically forced them to operate at a few Hz in pulsed mode). We would therefore expect classical betatrons could be designed for ~0.5-1 GeV at a few kHz to provide ~100-200 mA of average circulating current, a range useful to synchrotron radiation x-ray photolithography, [11] provided low loss material can be afforded .

IV. FREQUENCY LIMITATIONS

The frequency that a betatron is able to be driven is limited by practical considerations of: (1) the large inductance of the betatron core & guide fields, and (2) core losses. A resonant drive with the inductor and a series energy store capacitor avoids switching large amounts of power because the current and voltage are always 90° out of phase. However, besides the Wattless current which stores energy in the magnetic field, there is a working current proportional to the ampere-turns of an effective loss of magnetic field. This is given by: (1) eddy currents, and (2) the ampere-turns equivalent to the hysterisis phase shift. (Reviews of inductive drives at high frequencies are given in [12],[13], and in [9], [7], [3]). For simple estimates, we assume $\mu >> \mu_o$, and a gap height g much less than the length of the flux lines, about 5% of the beam radius, and make estimates based on the 340 MeV betatron made by Kerst [1]., [3] as in Figure 1[5]. Then the inductance L_g for the guide field magnet and L_C, the core inductance, are given approximately by $L_g \sim \mu_o A N^2/g$, $L_C \sim \mu A N^2/l$, where N, A are the respective coil turns and area, l is average length of a flux line in the core circuit, and μ is the core permeability. The guide magnetic field B is given by $B \sim \mu_o I N/g$. The energy is given by $1/2\, CV^2 \sim 1/2\, LI^2$, where C is the energy store, and V and I are the driving voltage and current. The resonant frequency $f_c = 2\pi/\sqrt{LC}$.

An interesting but extreme specific example of the above is to take f_c =1.2 kHz, E=1 GeV, g=8 cm, N=10, and B_s=1.5 T and metglas, $\mu \sim 10^3$. We assume a DC biased (B_w~3.3 T) betatron, like figure 1. We find that $I_{peak} \sim 12$ kA, L ~ 20 mH, 10 mH for the core and guide field respectively, C ~ 1.2 mF, and V_{peak}=50 kV for a 2.2 m radius electron beam. The energy store is prodigious, ~3 MJ. The volume of high μ~1,000, 1.5 T magnetic materials scales like $V \sim 12r^3$ where r is the beam radius. The metglas for this example would have a mass of $\sim 10^6$ kg. At 1.2 kHz (0.2 ms 1/4 cycle saturation time), the losses for typical 0.6 mil metglas with a 3.3 T saturation are measured to be $\sim 10^{-4}$ J/kg, giving ~120 kW of loss which must be resupplied by a 1.2 kHz power source. Parasitic losses could be as small as ~1-2% of the RMS circulating power, ~50-100 kW, depending on the dielectric hysteresis of the energy store, and coil losses and resistance. If the magnet is driven by a 50Ω line, a shunt capacitance is ~4-8 μF. Using this beam as a light source would yield ~1 kW of x-ray power with a peak wavelength of ~2 nm at an average beam current of ~0.5 A.

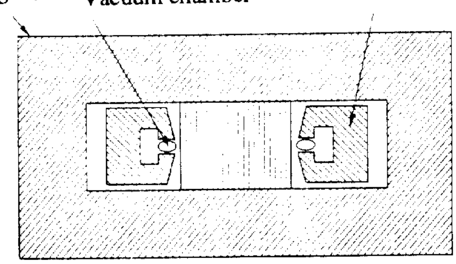

Figure 1. Betatron schematic with 2 independent magnetic circuits for the accelerating flux and guide field, from [5].

More practical applications (less extreme cases) might be in radiotherapy (10-30 MeV) or x-ray imaging of thick objects (1-10 MeV), where the increased performance of a betatron made with low loss ferrites or metglas might make betatrons again competitive with RF linacs, and the compact size and a modulator coupled by power lines may offer advantages. In the case of smaller machines (<10 MeV) high frequency operation up to 100 kHz might be contemplated. At 20 MeV, we estimate core losses could be <1 kW at 10 kHz using the best available magnetic materials. Much higher frequencies may be contemplated with the best available ferrites, especially at lower energies where losses are not as important.

V. COMMENTS AND DISCUSSION

A fast-cycling betatron has many similarities with a recirculating induction linac[7].The demands on a modulator of a high-gradient betatron approaches that of an induction linac as the frequency increases, making the betatron technique more difficult, but offering higher energy and flux. Breaking the core into many sectors[7] offers the possibility of reducing the demands on an individual resonant power modulator, and decreases the individual inductance, allowing faster operation. Low-loss magnetic materials may make the trade-off between the flux, energy and modulator of a single pass induction linac relative to a betatron less distinct. A 10kHz, 1,000 beam-turn ferrite betatron operating at 0.2 T, R=75 cm, would give 45 MeV electrons at high average current; the modulator would have a risetime of 25 μsec over the acceleration cycle, less taxing than an induction linac of similar energy.

In practical terms, the high energy limit is the limit of the cost of large volumes of magnetic material, with $V \sim E^3$. A 1-GeV accelerator would cost ~$25 million US for cores of metglas alone, at $25/kg in large quantity. This probably exceeds the practical cost limit of the synchrotron/storage ring technique for light sources at ~1 GeV, for example. Furthermore, the capacitive store is expensive. However, strong focussing techniques, which would be feasible for such a large accelerator, may result in a large average circulating current. The region between 500 MeV-1 GeV is accessible, and sensitive to the cost of materials.

Although ferrite is a cheaper and a faster material than metglas, the saturation magnetic field is too low to be even remotely cost-effective for high energy. However, for low energy (~1-25 MeV), high frequency (> ~5 kHz) betatrons, ferrites would be the material of choice. We note that the steep scaling of core volume down with lower energy may make low-energy, fast-cycling betatrons attractive in many low energy applications when compared with commercial linacs.

Parasitic resistance, capacitance and inductance, and the overall energy budget, will require careful analysis. Driving the accelerator with high impedence (ferrite loaded) lines may be a possible way to reduce some of these effects. These problems may make superconducting techniques desirable for this application, and are especially appropriate for DC biasing the induction field. The potential to eliminate costly magnetic material with superconducting/superferric magnets can be considered if the conductors (filaments) are small enough to avoid quenching during the induction cycle. Operation of high-current superconductors at high (~0.5-1 kHz) frequencies is problematic but may be possible; for a discussion see [14].

VI. CONCLUSION

The extension of the betatron technique to high frequency magnetic materials has potential for improved betatron technology, allowing it to be extended to higher energies and to higher currents. We expect this possibility may be especially practical for machines with energies below 700 MeV.

VII. REFERENCES

[1] D. W. Kerst, Phys.Rev. 58, 841 (1940)
[2] Private Communication, Prof. W. Bertozzi, MIT (1984).
[3] D.W. Kerst, "The Betatron", Handbuch de Physik, Dd. XLIV, Springer (1959), and loc. cite.
[4] M.S. Livingston & J.P. Blewett, Particle Accelerators, McGraw-Hill, NY (1962)
[5] E. Persico, E. Ferrari, S. Segre, Principles of Particle Accelerators, Benjamin, NY (1968)
[6] W. Scharf, Particle Accelerators and Their Uses, Harwood Academic, NY 1985
[7] S. Humphries, Jr, Principles of Charged Particle Acceleration, Wiley, NY, (1986)
[8] D. Iwanenko and I. Pomeranchuk, Phys.Rev. 65, 343 (1944)
[9] D. Birx, "Induction Linacs", Lecture Notes, US Particle Accelerator School, BNL, M. Month, ed. (1989),
[10] S. Humphries, Charged Particle Beams, chapter 5, Wiley, NY (1990)
[11] D. Andrews and M. Wilson, J. Vac.Sci Tech.,B7 (6), 1696 (1989)
[12] J. Weaver et al., Design, Analysis and Measurement of Fast Kicker Magnets at SLAC, SLAC-Pub-4897, (1989)
[13] B. Brown, "Simply Physics Limitations on Pulsed Bending Magnets", Fermilab pub. TM-768, 0353.000 (1978)
[14] M. Wilson, Superconducting Magnets, Oxford, (1983).