Relativistic Electron Beam Ion Hose Instability In Beam-Induced Channels*

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Summary

Ion hose (or ion resonance) instability of a relativistic electron beam propagating in beam-induced channels, inside a shielded drift tube, is investigated. Due to the nature of the shielded drift tube, only perturbations whose axial wavelength is an integral fraction of the tube length is permitted. These perturbations cause the beam to oscillate transversely. The coupling of this transverse oscillation with the stabilizing effects of a time changing ion channel forces the beam to go unstable (overstabilization). Using the known characteristics of the beam propagation in such an environment allows us to estimate the critical axial wavelength, onset time, and oscillation frequency of this stability, which compared very favorably with experimental results.

Introduction

The ion hose (or ion resonance) instability has been understood to be the result of the electrostatic coupling between the transverse oscillations of the beam and the ion channel, for the case where the channel to beam line density ratio (f) is fixed.[1,2] Physically, due to the large difference in masses between the beam electrons and channel ions, the characteristic time scale for their response to any electrostatic perturbations is widely divergent. The electron, being light, responds more readily than the heavy ion, and their motion is largely uncoupled, and stable for small value of f, or neutralization fraction. For larger value of the neutralization fraction, the transverse motion of the beam is reduced in the deeper electrostatic potential well of the ion channel. At sufficiently large f (depending on the perturbation's axial wave number κ_z), the time scales for transverse motions of the beam and the channel become similar (resonance). Their motions are strongly coupled and grow, i.e. the beam and channel are overshooting in chasing one another (overstabilization).

In a shielded drift tube, the surviving and large amplitude noises tend to have discrete axial wavelength due to the boundary conditions, then the resonance condition is rarely met for an electron beam propagating in a pre-formed channel, unless the neutralization fraction is tuned to a specific value. In this context, relativistic electron beams, propagating in beam-induced channel, are more susceptible to ion hose instability, since here f is varying function in time and propagation distance. If the rate of change in time for f is slow compared to the instability growth rate, then large scale transverse displacement of the beam can be observed. Recent experiments have confirmed this scenario.[3] We shall demonstrate in this paper, the fact that f is time varying, can be manipulated, under certain conditions, to give us an extra physical observable to verify the instability experimentally.

*Work supported by IR Funds at NSWC.

Analysis

We consider an intense relativistic electron beam propagating through an initially low pressure neutral gas, inside a shielded cylindrical drift tube, at a speed near the speed of light. The beam induces its own ion channel, with the channel electrons assumed to be expelled instantaneously when $f < 1$. We neglect secondary ionization in the low pressure regime considered here. In this circumstance, the neutralization fraction can be approximately given by

$$f(t) = \frac{N_i(t)}{N_b} \simeq \sigma c n_g t = \frac{t}{t_0}, \qquad (1)$$

where N_i and N_b are line densities of the channel ions and beam electrons, respectively, σ is the ionization cross section, and n_g is the neutral gas density ($\sim$ pressure, P). Equation (1) is only an estimation, since the beam radius (R) is a rapidly changing function of time until the equilibrium condition[4] for radial focussing $f > \frac{1}{\gamma^2} + \frac{2T_\perp}{\nu mc^2}$ is achieved. Here, γ, $T_\perp$, and ν are the beam relativistic factor, transverse temperature, and Budker's parameter, respectively. However, this approximation makes the problem analytically tractable, and is reasonable within the framework of the rigid treatment, where the beam and channel are treated as rigid but flexible rods.

The transverse displacement of the beam and channel centroids, D_b and D_i, can then be obtained from

$$N_b \gamma m_e \frac{d^2 D_b}{dt^2} = F_{bi}(D_i - D_b),$$
$$f N_b m_i \frac{d^2 D_i}{dt^2} = F_{ib}(D_b - D_i), \qquad (2)$$

for $f < 1$, where m_e and m_i are the electron and ion rest masses, and $F_{ij}(x)$ is the electrostatic force exerted on column i by column j. For two columns with Gaussian profiles, we get[5]

$$F_{ij}(x) = \frac{2e_i e_j N_i N_j}{x}\left(1 - e^{-x^2/(R_i^2 + R_j^2)}\right),$$
$$\simeq \frac{e_i e_j N_i N_j}{R^2} x, \text{ for } x << R, \qquad (3)$$

and $R = R_i = R_j$. Equations (2) - (3) can be rewritten as

$$(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z})^2 D_b = \alpha \frac{t}{t_0} (D_i - D_b), \quad (4a)$$

$$\frac{\partial^2}{\partial t^2} D_i = \eta\alpha(D_b - D_i), \quad (4b)$$

where $\alpha = e^2N_b/\gamma m_e R^2 \simeq c^2 I_b(kA)/17\gamma R^2$, and $\eta = \gamma m_e/m_i$. Noting that $\eta << 1$, and that z enters only through $\partial/\partial z$, we seek solutions of the form $D = \hat{D}(t)\, e^{-ik_z z}$, with $\partial/\partial t << k_z c$. In order to assure the validity of the rigid treatment, we also require that $k_z^2 R^2 \lesssim 1$. Then Eq. (4a) can be integrated to give

$$\hat{D}_b = \hat{D}_i + i\frac{k_z c}{2} e^{-i\delta(t)} \int_{-\infty}^{t} e^{i\delta(t')}\, \hat{D}_i(t')dt', \quad (5)$$

where the integrating factor $\delta(t)$ is given by $\delta(t) = (\alpha t^2/2t_0 - k_z^2c^2t)/2k_zc$. Incorporating Eq. (5) into Eq. (4b) gives us

$$\frac{\partial^3 \hat{D}_i}{\partial t^3} + i\delta' \frac{\partial^2 \hat{D}_i}{\partial t^2} = i\frac{\eta\alpha k_z c}{2} \hat{D}_i, \quad (6)$$

where $\delta'(t) = (\alpha\, t/t_0 - k_z^2c^2)/2k_zc$.

Since $\partial/\partial t << k_z c$, if $\delta' \neq 0$, then Eq. (6) can be approximated by

$$\frac{\partial^2 \hat{D}_i}{\partial t^2} = \frac{\eta\alpha k_z c}{2\delta'} \hat{D}_i, \quad (7)$$

which indicates stable oscillations for $\delta' < 0$, and pure slow growth when $\delta' > 0$. It is worthwhile to note at this juncture that, from Eqs. (4a) and (7), when $\delta' \neq 0$, the beam is moving too fast for the channel to respond. However, when $\delta' < 0$, the electrostatic coupling is rather weak, and the system is stable. Whereas, when $\delta' > 0$, the coupling is strong, the beam is forced to pull the heavy channel along, while oscillating inside the potential well of the channel. The result is a small net drift to one side for each oscillation period, i.e. pure slow growth.

In the case where $\delta' \simeq 0$ (resonance), Eq. (6) can be written as

$$\frac{\partial^3 \hat{D}_i}{\partial t^3} = i\frac{\eta\alpha k_z c}{2} \hat{D}_i, \quad (8)$$

which can be solved to give $\hat{D}_i \sim e^{i\omega t}$, where

$$\omega = \frac{1}{2}(1 - i\sqrt{3}) \left(\frac{\eta\alpha k_z c}{2}\right)^{1/3}, \quad (9)$$

which indicates fast and growing oscillations.

From Eqs. (7) - (9), we conclude that, for a relativistic electron beam propagating through an initially neutral gas, the ion hose instability for a given axial wave number k_z will onset at time

$$t_d = \frac{k_z^2c^2t_0}{\alpha}. \quad (10)$$

We also note that, inside a shielded drift tube, (which is assumed to be much shorter compared to beam length) the perturbation axial wavelength is given by (standing waves)

$$k_z = \frac{2\pi}{\lambda_z} = \frac{\pi n}{L}, \; n = 1,2,3... \quad (11)$$

where L is the drift tube length. As a result, if for any physical reasons, the instability first onsets with a high mode number, n, then the resonance condition, $\delta'(t) \simeq 0$, is nearly always met, resulting in a smooth and fast growing oscillation indicated by Eqs. (8) and (9), since the transition time between different mode numbers is short. If the instability onsets with a low mode number, then due to the long transition time, the instability will first grow as indicated by Eq. (8), saturates as indicated by Eq. (7), and eventually the cycle starts again as the system goes into resonance with a new mode number. We end this section by remarking, from Eqs. (7) - (9) and the definition of α, that growth is slow for large ion mass, long perturbation axial wavelength and large radius.

Discussions and Results

As we have previously remarked, the radius at the beam head is expanding under the influence of the beam net self-field force and the beam inherent transverse temperature (emittance). This expansion is reduced as the electrostatic focussing force due to the ion channel is increased. The beam will settle to an equilibrium radius, once the following condition is satisfied[4]

$$f = \frac{t}{t_0} \gtrsim \frac{1}{\gamma^2} + \frac{2T_\perp}{\gamma mc^2}. \quad (12)$$

Because of the absence of the radial focussing at its head, the beam assumes a Trumpet profile, which can be rather sharp for short characteristic neutralization time, t_0. Consequently, for heavier gases (e.g. Argon, Nitrogen), we expect the instability to onset around the time Eq. (12) is satisfied. The axial wavelength of the instability can then be estimated from Eqs. (10) - (12). The reason for this is due to the fact that, for heavier gases, the ionization cross-section is larger (i.e. shorter t_0), and growth is slow due to large ion mass, long axial wavelength, and large radius. As a result, those low mode number perturbations, that come into resonance before the beam pinches, are quickly detuned. For lighter gases (e.g. Hydrogen), growth rate is faster, and beam radius changes relatively slow at low pressure. In this case, low mode number perturbations can grow and be observed even before the beam pinches.

The experiments were performed with 10, 20, 30, 40, 80 and 160 mTorr of Argon, Nitrogen, Neon and Hydrogen as filling gas. Further details about the experiment can be found in Reference (3), we just note here that the beam current I_b = 4kA, γ = 2.4, pulse length 100 ns, L = 100 cm, and the transverse temperature ($T_\perp$) has been previously characterized to be 35 keV, with an equilibrium beam radius

$R \approx 1$ cm in air.[6] Using the relevant experimental parameters, we found $f \geq 0.75$ in order to satisfy the equilibrium condition in Eq. (12).

For Argon, Neon and Nitrogen, this translates from Eq. (10) into an axial wavelength of roughly 25 cm, or mode number n = 8. This axial wavelength is evidenced in Figure (1). The delay time for the instability onset for this wavelength is found from Eq. (10) to be:

$$t_d = 0.63\, t_0, \qquad (13)$$

where t_0 (ns) is 1200/P(mTorr) for Argon and Nitrogen, and 2500/P(mTorr) for Neon.[7] As we can see from Figure (2), the agreement between theory and experiment is excellent for Argon. Similar agreements are also found for Neon and Nitrogen.[3] The oscillation frequencies can be found from Eq. (9) for λ_z = 25 cm to be 17.6, 22 and 25 MHz for Argon, Neon and atomic Nitrogen, respectively. These values agree well with experimental determined frequencies, which are 17 ± 3, 21, and 27 ± 1 MHz for Argon (10 mTorr), Neon (20 mTorr), and Nitrogen (10 mTorr), respectively, since only at these indicated pressures can the beam oscillate at least one full period before either the beam terminates or $f > 1$. Figure (3) shows one of the current and beam displacement traces, from which the onset time and oscillation frequencies are determined.

For Hydrogen, t_0(ns) = 5000/P(mTorr), thus even at the end of the pulse (100 ns), f is only 0.2 for P = 10 mTorr. In this case, the beam radius, R, hardly changes in time, but it is a strong function of z. We can reasonably estimate R to be about 8 cm at the observation point, 30 cm downstream from the injection point. The experimental onset time is about 52 ns, which indicates λ_z = 66 cm, or mode number n = 3. The oscillation frequency for this mode is expected to be 8.6 MHz. This frequency is reasonable in comparison with experimental evidences in Figure (4), where several mode transitions are apparent and the oscillation frequency increases later in the pulse. At higher pressure, no estimates had been made given the uncertainty in estimating the beam radius. We will note, however, that at 80 and 160 mTorr, the onset times are similar to those of Argon and Nitrogen at 20 and 40 mTorr, respectively. This indicates the beam profiles at these t_0 values are too sharp for long wavelength modes, even with the smaller mass of molecular Hydrogen ion.

References

1. R. C. Davidson and H. S. Uhm, Phys. Fluids 21, 60 (1978).
2. H. S. Uhm and R. C. Davidson, Phys. Fluids 23, 813 (1980).
3. R. F. Schneider and J. R. Smith, Phys. Fluids 29, 3917 (1986).
4. R. C. Davidson and H. S. Uhm, Phys. Fluids 22, 1375 (1979).
5. K. T. Nguyen and H. S. Uhm, IEEE Trans. Nucl. Science, NS-32, 2623 (1985).
6. J. R. Smith, et.al., J. Appl. Phys. 60, 4119 (1986).
7. F. F. Rieke and W. Prepejchal, Phys. Rev. A6, 1507 (1972).

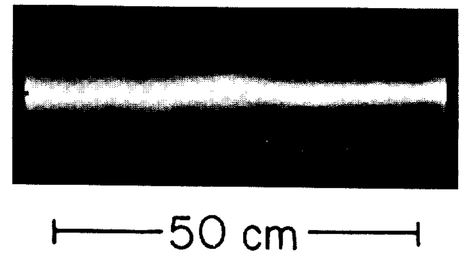

Figure 1. Open shutter photograph (Nitrogen at 40 mTorr)

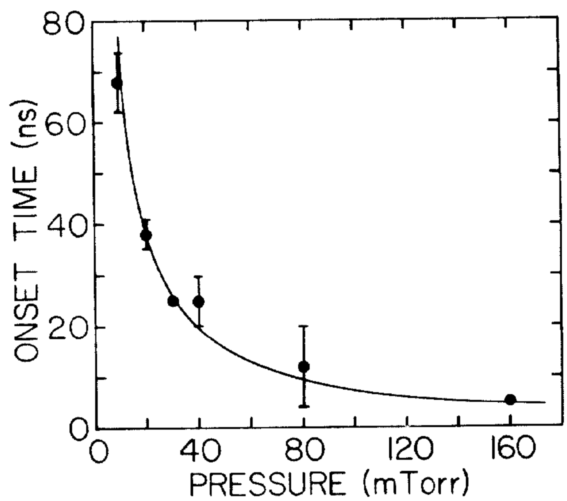

Figure 2. Instability onset time vs. gas pressure (Argon). The solid line is the theoretical curve. The error bars about the experimental points are sample standard deviations.

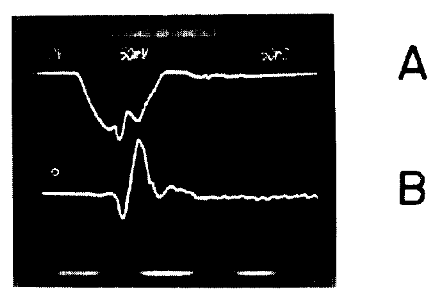

Figure 3. Oscillographs of Argon at 10 mTorr. (A) Current trace, (B) displacement trace.

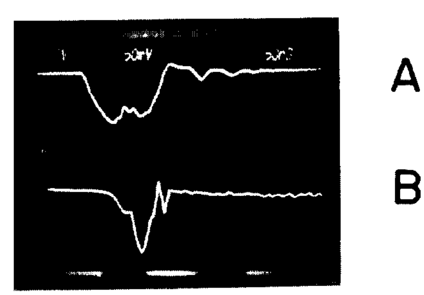

Figure 4. Oscillographs of Hydrogen at 10 mTorr.