Beam Loading Efficiency in Plasma Accelerators

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Abstract

A brief summary of the dependences of key beam parameters in plasma- based accelerator schemes is given here. A self-consistent example of a 10 TeV center of mass accelerator is considered using these parameters. Finally, a particle-in-cell computer simulation study is presented and the results are compared to those obtained theoretically, with particular attention given to the efficiency of energy transfer from plasma wave to accelerated beam.

Introduction

Recent work, both theoretical [1],[2],[3] and experimental [4],[5], has shown that plasma waves are capable of supporting extremely high electric field gradients that could possibly be used to accelerate an appropriately shaped beam of relativistic charged particles.The results were encouraging enough for workers in the field to begin to address some of the accelerator -related issues , [6], [7]. In this paper we briefly review the results in one of these works [6] and extend it with a detailed parameter study of a 10 TeV center of mass plasma accelerator, and a 2-D particle-in-cell (PIC) simulation of a scheme that promises high beam loading efficiency even in 3 dimensions. In particular, we will focus our attention on the maximum number of particles capable of being accelerated by a plasma wave, the energy spread introduced into the beam, and the efficiency with which the energy stored in the plasma wave can be extracted by the beam.

Basic Concepts

In order to find the maximum number of particles that a plasma wave is capable of accelerating, consider a plasma wave of arbitrary amplitude E_0 whose width is much larger than a collisionless skin depth. It has been shown [6] that a bunch containing N relativistic electrons traversing a plasma leaves behind it a similar "wake" given by

$$E_z{}^{beam} = -\frac{4\pi eN}{A}\cos k_p(z-ct) \qquad (1)$$

where A =area of the beam and k_p =the wave number of the plasma wave. It follows from Eq. (1) that the maximum number of electrons that this wave is capable of accelerating is

$$N^{1-D}_{max} = \frac{E_0}{4\pi e}A \simeq 5\times 10^5\left(\frac{n_1}{n_0}\right)\sqrt{n_0}A \qquad (2)$$

where n_0 = is in cm^3, n_1 =perturbed plasma density, and A is in cm^2. Fig. 1 shows a 1-D particle-in-cell simulation where N_{max} particles are being accelerated by a plasma wave. Eq. (2) is modified slightly when the transverse dimension is included in the analysis and becomes

$$N_{max} = N^{1-D}_{max}\frac{A_{eff}}{A} \qquad (3)$$

where A_{eff} is the effective area of the electron beam, given by

$$A_{eff} = \frac{\pi a^2}{1-k_p a K_1(k_p a)} \qquad (4)$$

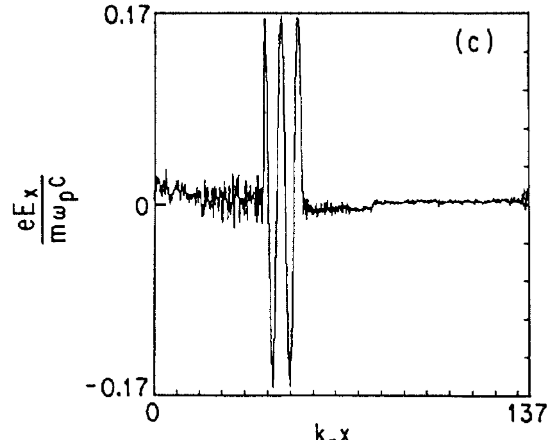

Fig. 1. Total longitudinal electric field in a 1-D simulation loaded with a short bunch of N_{max} (positively charged) electrons on the third peak of an accelerating wave. Simulation consisted of 14,000 particles on 1024 grids;system length 170 c/ω_p, 10 particles per grid, time step=$0.04\omega_p^{-1}$. wave amplitude $n_1 \simeq .16$ (produced by wakefield mechanism), and $\gamma^{beam} = \gamma^{wave} = 60$.

where a =radius of the beam, and $K_1(k_p r)$ is the first order modified Bessel function of the second kind. For beams much larger than c/ω_p in radius, the effective area can be considered to be the true area of the beam (i.e., πa^2). However, if the beam radius is less than a collisionless skin depth (the so-called narrow beam), the beam can still be thought of as having an effective radius of roughly $\sim c/\omega_p$. Physically this means that the beam can extract energy from the plasma wave out to a distance $\sim c/\omega_p$ even though the beam itself is just a fraction of this. Thus, a narrow beam following a plasma wave of half-width c/ω_p can extract virtually all of the energy stored in the wave, making this scheme highly efficient. In fact, extraction efficiencies of the order of 40% to 90% can be obtained for trailing beams of radius $a = 0.001c/\omega_p$ to $.5c/\omega_p$, respectively, in a wave with half-width c/ω_p. Fig. 2 shows the near cancellation of a $.5c/\omega_p$ wave, with a short ($l_b \ll \lambda_p$) bunch of radius $a = .15c/\omega_p$. This scheme (first suggested by Van der Meer [8]) is also appealing due to the fact that the radial variation of the driving beam is negligible, which implies that the energy spread induced will be small. Additionally, since the beam is already small ($a \ll c/\omega_p$) and dense in the acccelerator itself, the requirements on the final focus can be relaxed somewhat.

A simple argument can show that 100% loading of the wave produces 100% energy spread in the beam, even for our very short bunch (i.e., the bunch length, $l_b \ll \lambda_p$, a plasma wavelength. From Fig. 1 it is evident that the first particle in the beam feels the maximum force, as it is sampling only the maximum E-field. Since the field behind the bunch is zero, the last particle in the bunch nessecarily feels no force, and hence is not accelerated.

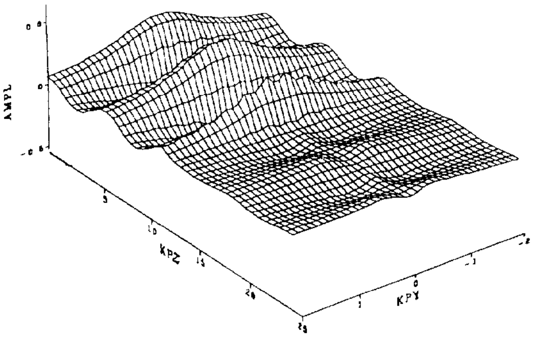

Fig. 2. Longitudinal (sinusoidal) and transverse (exponential decay) dependence of the accelerating field (E_z). A short, narrow ($a = .15c/\omega_p$) beam placed on the third peak of a plasma wave of half- width c/ω_p. The density of the beam has been chosen such that, for this radius, the fields cancel on axis. Note that most of the energy stored in the plasma wave has been absorbed by the narrow beam.

However, if one were to linearly ramp the charge density of the bunch appropriately, and place it at the correct spot in the plasma wave, this energy spread can be avoided, as shown in Fig. 3. This scheme has the added feature that it allows for a natural transition to longer, more realistic, bunch lengths ($l_b \sim \lambda_p$.) This idea of bunch shaping in order to reduce the longitudinal energy spread works equally as well in 2-D, although another source of energy spread is introduced due to the mismatch of the longitudinal fields in the radial direction. The amount of energy spread introduced by this mismatch can be estimated as

$$\frac{\Delta E}{E} \leq \frac{1 - \cos k_p \zeta_0}{\cos k_p \zeta_0}\left(1 - \frac{\Upsilon_a(a)}{\Upsilon_a(0)}\right) \qquad (5)$$

where $\Upsilon(y)$ is the transverse dependence of the longitudinal electric field (see ref. [6]). There is also an energy spread due to phase slippage which is proportional to one over the energy of the driving beam (for the Plasma Wakefield Accelerator). However, this can be made negligible by choosing very relativistic driving beams ($\sim$ 1 GeV).

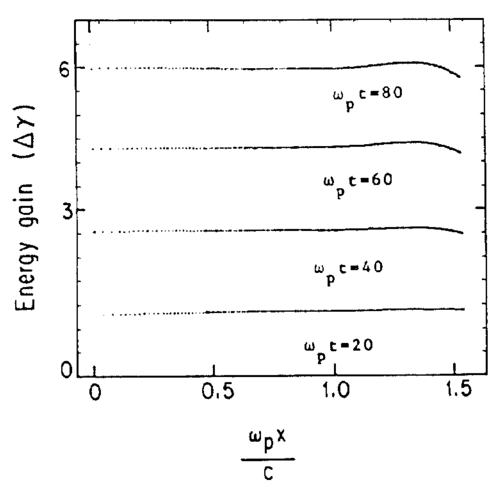

Fig. 3. Time developement of a bunch whose beam current has been shaped so as to minimize the energy spread due to longitudinal field variations inside the beam. Note the effect of phase slippage on the head of the bunch.

Example

We will now use the results above to see whether a plasma accelerator, in theory, would be capable of attaining the necessary parameters for a 10 TeV c.m. collider. Let us take as the required parameters, those presented by B. Richter [9] at the Novosibirsk HEA Conference. Table I contains a comparision of the required parameters (left column) and those obtained using the above equations (right column).

E^*(TeV)	10	10
$\mathcal{L}$ (cm^{-2}s^{-1})	10^{34}	10^{34}
σ_{E^*}/E^*(%)	10	20
β^* (cm)	0.1	0.1
f (Hz)	3000	3000
$N(e^+$ or $e^-)$	4.1×10^8	6.3×10^8
$\epsilon_n(m - rad)$	4×10^{-9}	4×10^{-9}
$\sigma_{r0}(\mu$m)	6.4×10^{-4}	6.4×10^{-4}
σ_z (mm)	3.4×10^{-4}	8.7×10^{-2}

Table I

The following items should be kept in mind concerning this table;

- The maximum bunch length (σ_z) is on the order of the required quarter wavelength of a plasma oscillation, necessary for both acceleration and focusing.($\lambda_p/4 = 78.5 \times 10^{-3}$ mm)
- Such small values for β^* and σ_{r0} would be extremely difficult, if not impossible, for present technology to attain. However, due to the strong radial focusing in the plasma waves, the bunch size just before the final focus would already be $\sigma_{r0} \sim 10^{-2}\mu$m, thus loosening the constraints on the final focus.
- The accelerating gradient consistent with this example is $\sim$ 2 GeV/m. Therefore, the length of such an accelerator would not be unreasonable, even including a conventional section to produce $\sim$ GeV driving and trailing bunches to inject into the plasma portion of the accelerator.
- The beam loading efficiency, or how much energy can be extracted from the wave, for the plasma acclerator case is calculated to be between 15 % and 20 % [6].
- The value given for energy spread (σ_{E^*}/E^*) at the interaction point takes into account both the contribution due to the radial mismatch of the longitudinal fields, and also that due to beamstrahlung.

We would like to conclude this section by saying that any real attempt to design a 10 TeV accelerator would necessarily require extensive research into the technological limitations of bunch shaping and placement, and detailed studies of the effects of beam and plasma inhomogeneities on alignment. In the example we have presented, we have merely shown that nothing found thus far prohibits contemplating plasma waves as a mechanism for acceleration of charged particles.

Simulations

The following simulation was done to illustrate the highly efficient narrow beam scheme discussed above. The simulations were done using the fully relativistic, PIC code WAVE [10]. Fig. 4 shows a diagram of the system at t=0. Note that in this simulation we use the PWFA to excite the plasma wave, although it might equally well have been produced by a beat wave (PBWA). The plasma wave was chosen to have a radius c/ω_p, while the trailing beam has a radius of $a = 0.15c/\omega_p$. The density of the trailing beam was chosen to be 4.4 times the density of the driving beam, in accordance with Eq. 2, which is necessary for perfect cancellation of the field on axis. The longitudinal electric field in the plasma is plotted in Fig. 5, at time $t=64\omega_p$. Notice that, as expected, the field is essentially canceled on axis (center plot), while at the distances $y = \pm c/\omega_p$ some of the driving field still remains. The 2–D analog of Eq. 1 predicts that the field due to the plasma wave alone should be $E_{wave}(y = c/\omega_p) = 0.43$, while the field due to the trailing beam should be given by $E_{beam}(y = c/\omega_p)$=0.24. This implies that roughly half the energy of the plasma wave should remain at this distance, in agreement with what is measured from Fig. 5.

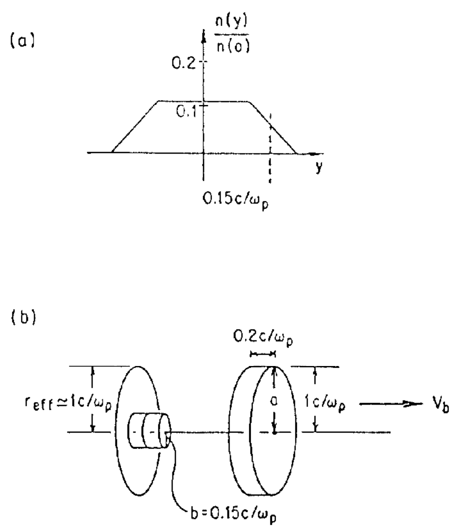

Fig. 4. Illustration of beam profiles used in 2-D simulations. (4a) Density profile of accelerated beam used in narrow beam case.(4b) A scheme for high efficiency and low energy spread. Efficiency depends on the overlap of the wave *fields*, not the actual beam size.

Acknowledgements

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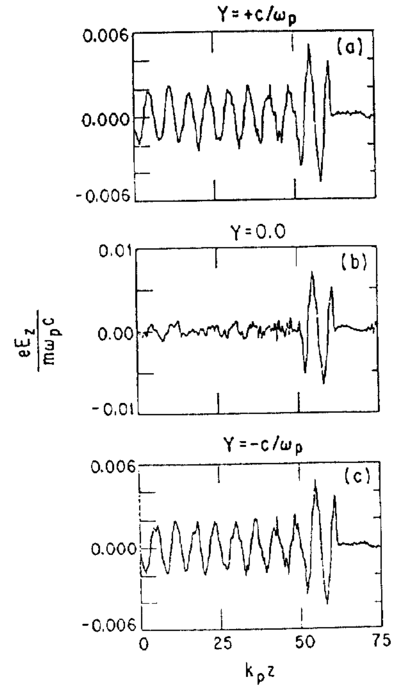

Fig. 5. Accelerating electric field, at $y = -c/\omega_p$, 0, and $-c/\omega_p$ for the beam of Fig. 4. A significant fraction of the energy ($\sim$ 75%) is extracted from the accelerating wave at a distance c/ω_p from the axis (Figs. [a] and [c]), even though the beam radius is only $0.15c/\omega_p$. ($t=68\omega_p^{-1}$)

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