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Summary

The limitation of beam currents in PETRA by the beam-beam effect is investigated. The horizontal and vertical betatron frequency shifts ΔQ are measured as a function of bunch currents for different operating conditions. The direct measurement of ΔQ by coherent excitation of the eigenmodes of the coupled betatron oscillations of the bunches is described. This method allows to determine independently the horizontal and vertical beam dimensions at the interaction point.

Behaviour of the beams near the limit

During injection the two beams are separated vertically with an electrostatic field. After switching off the electrostatic field one beam or both are blown up vertically. For currents above the limit the life time is very short, it can be less than 1 sec. For smaller currents near the limit the beams are blown up but the life time is longer, it can be of the order of an hour. Figs.1 and 2 show the two beams on a television monitor. In fig.1 the beam currents are .3 mA, and both beams are blown up. Fig.2 shows the beams at half the current, and they are flat.

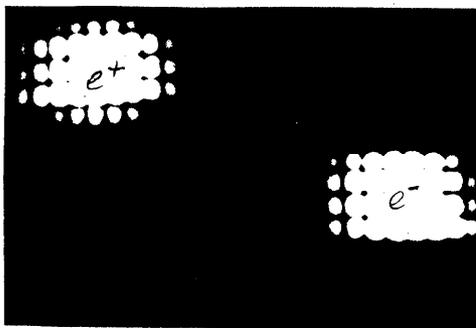


Fig. 1: Beam dimensions at $I^+ = I^- = .3$ mA

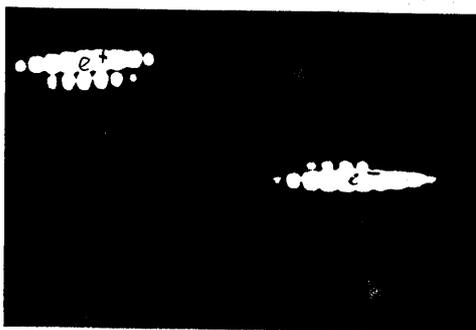


Fig. 2: Beam dimensions at $I^+ = I^- = .15$ mA

Sometimes only one beam is blown up, and the other one is flat. If one then excites the two beams on their vertical betatron frequency it can happen that the second one is being blown up and the first one becomes flat, i.e. the two beams may behave like a flip-flop system.

Measurement of the coherent tune shift

Fig. 3 shows the pickup signal of two colliding bunches (one in each beam) which are excited at their vertical betatron frequency. The currents of the bunches are below the limit. The excitation frequency increases from left to right, and there are two resonance frequencies where both beams are oscillating vertically. These frequencies are the two eigenfrequencies of the resonant system of the two bunches which are coupled by the space charge forces.



Fig. 3: Vertical eigenfrequencies of two colliding bunches

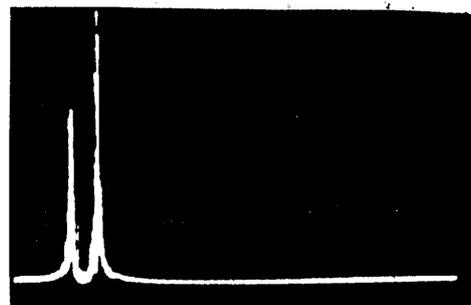


Fig. 4: Vertical eigenfrequencies of two separated bunches

Fig.4 shows the response when the two bunches are separated by the electrostatic field. The small frequency splitting is due to "long range forces" and a small quadrupole component in the separating field. When only one electron or positron bunch is stored the measurement gives only the left peak in fig. 3.

The distance of the two peaks in fig. 3 gives the coherent tune shift per revolution, and that is twice the coherent tune shift per interaction, since we have two interaction points in this case. In linear approximation, the coherent tune shift is twice the incoherent tune shift (1,2,3). One can see this as follows.

The kick which each bunch gets at the interaction point is proportional to the distance of the two bunches

$$\Delta z'_+ = b(z_- - z_+) \quad (1) \quad \Delta z'_- = b(z_+ - z_-) \quad (2)$$

assuming that the two bunches have the same current. The constant b is given by

$$b = \frac{4\pi}{\beta_0} \xi$$

where ξ is the space charge parameter

$$\xi = \frac{N_b r_e \beta_0}{2\pi\gamma\sigma_z(\sigma_x + \sigma_z)} \quad (3)$$

with

N_b = number of particles per bunch,

r_e = classical electron radius,

β_0 = amplitude function at the interaction point,

γ = relative energy

σ_x, σ_z = standard deviations for horizontal and vertical beam dimensions.

The transformation of the coordinates of the two bunches $\{z_+, z'_+, z_-, z'_-\}$ can be written in matrix form where the transfer matrix for the interaction point is

$$M_{ip} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -b & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ b & 0 & -b & 1 \end{pmatrix} \quad (4)$$

The transfer matrix for one half of the ring is then

$$\bar{M}_{1/2} = M_{1/2} \cdot M_{ip} \quad (5)$$

where $M_{1/2}$ is the transfer matrix for one half of the ring without beam-beam interaction. The eigenfrequencies of $\bar{M}_{1/2}$ are given by

$$\det(\bar{M}_{1/2} - e^{i\mu_{res}/2}) = 0 \quad (6)$$

Eq.(6) yields

$$\cos \frac{\mu_{res}}{2} = \cos \frac{\mu}{2} - 2\pi\xi \sin \frac{\mu}{2} \pm 2\pi\xi \sin \frac{\mu}{2} \quad (7)$$

For small ξ one obtains from Eq.(7)

$$Q_{res} = Q + 2\xi \pm 2\xi$$

where Q is the betatron wave number. Thus, in linear approximation, the first eigenfrequency is the undisturbed betatron frequency and the second eigenfrequency is larger by four times the incoherent tune shift per interaction.

The measurement of the coherent tune shift has two practical difficulties. If the excitation is too strong the second resonance frequency is shifted, as shown in fig.5. With increasing excitation voltage the coherent amplitude increases also, the oscillation

becomes nonlinear and the frequency of the coherent oscillation becomes smaller. Only the right corner of the triangle in fig.5 is independent of the strength of the excitation. It gives the largest frequency split, which occurs for the weakest excitation where the linear approximation is valid.

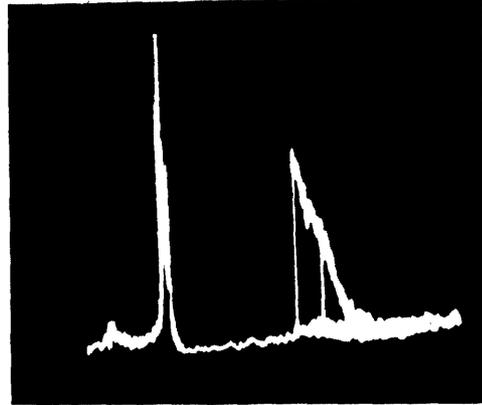


Fig. 5: Eigenfrequencies at strong excitation

Another difficulty arises if the excitation and the observation of the two eigenmodes are not gated. In that case the excitation and the observation depend on their respective position in the ring. This can easily be understood if one assumes for instance that the antenna measures the electric field at the interaction point. Then only the second eigenmode can be observed where the two bunches have opposite displacements at the interaction point. The first eigenmode where the two bunches have the same displacement cannot be observed since the signals of the two bunches compensate. Such positions occur periodically in the ring according to the betatron phase advance. Then there are other positions where only the first mode can be observed, and between these positions there are regions where both modes can be detected.

The same consideration holds for the excitation of the two modes which therefore can be done only in certain regions of the ring. For the measurements in PETRA only the excitation was gated whereas the position for the observation was optimized such that both modes could be observed.

The problem becomes more complex if there are two bunches in each beam. In that case one obtains four eigenfrequencies. If the bunch currents are equal and if the optics is symmetric, two eigenfrequencies coincide and the difference between the largest and the smallest frequency is eight times the incoherent tune shift per interaction.

By measuring both the horizontal and the vertical coherent tune shift one can, in linear approximation, independently calculate the horizontal and vertical beam dimensions at the interaction point (4). A comparison of the measured luminosity with the luminosity calculated from these beam dimensions shows good agreement.

The coherent tune shift has been measured as a function of several parameters. Figs. 6 and 7 show the frequency splitting as a function of the bunch current for different energies. It is remarkable that the vertical splitting goes through a maximum. For larger currents at least one of the two beams is blown up. The maximum vertical incoherent tune shift is about .016 in this case whereas the horizontal one increases up to .027.

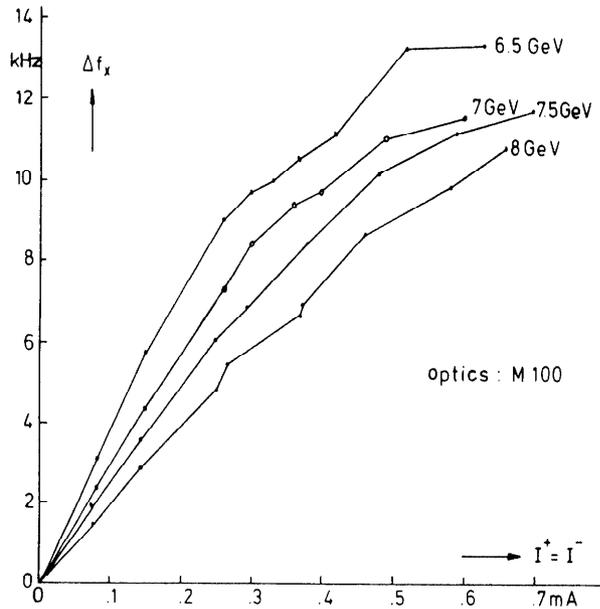


Fig. 6: Horizontal frequency splitting at different energies

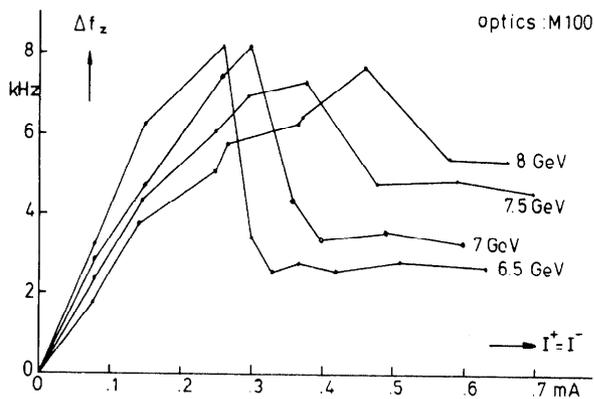


Fig. 7: Vertical frequency splitting at different energies

Measurement of the luminosity

The vertical tune shift can be calculated from the measured luminosity since the beam height is small as compared to the beam width. Fig. 8 shows an example of such a calculation. The luminosity was measured as a function of the current for two different frequencies of the accelerating voltage. The luminosity as well as the vertical ΔQ have a maximum which is shifted to larger currents if the rf frequency decreases.

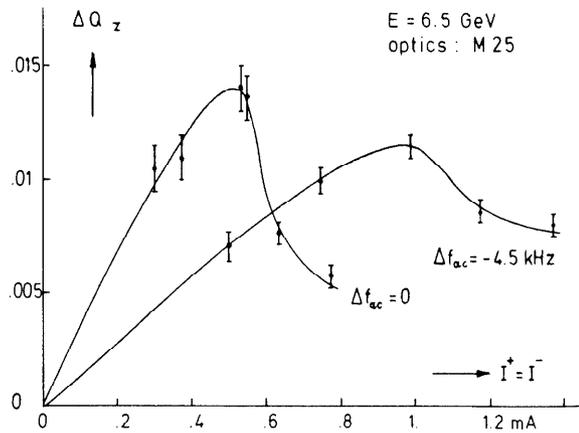


Fig. 8: Vertical tune shift at different rf frequencies

For smaller rf frequencies the damping of the horizontal betatron oscillation becomes smaller and the beam width increases. Then the same ΔQ is obtained at higher currents and the maximum of the luminosity becomes larger.

The largest luminosity is obtained for the largest beam width permitted by the aperture. In this limiting case neither the luminosity nor the tune shift show a relative maximum as a function of current. At 8.5 GeV such a maximum is also not observed.

Maximum tune shifts

The maximum tune shifts, which are measured or calculated, are about .03 for the horizontal plane and about .02 for the vertical plane. In the small energy range between 6.5 GeV and 8.5 GeV these values hardly vary with energy. They also do not show a notable difference in measurements done with one bunch or with two bunches per beam. The betatron wave numbers were for all measurements about $Q_x = 25.18$ and $Q_z = 23.28$.

A comparison of the maximum tune shifts at different energies is difficult since the rf frequency was optimized for maximum luminosities. Thus the beam width and the horizontal damping have an energy dependence which is different from the usual scaling law.

References

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- 3) A. Piwinski, Interner Bericht DESY H2-75/3 (1975)
- 4) A. Piwinski, PETRA-Kurzmitteilung Nr. 149 (1978)