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SYNCHROTRON RADIATION SOURCES*

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Abstract

As a result of the exponential growth of the utilization of synchrotron radiation for research in the domain of the material sciences, atomic and molecular physics, biology and technology, a major construction activity has been generated towards new dedicated electron storage rings, designed optimally for synchrotron radiation applications, also, expansion programs are underway at the existing facilities, such as DORIS, SPEAR, and VEPP. In this report the basic properties of synchrotron radiation will be discussed, a short overview will be given of the existing and new facilities, some aspects of the optimization of a structure for a synchrotron radiation source will be discussed and the addition of wigglers and undulators for spectrum enhancement will be described. Finally, some parameters of an optimized synchrotron radiation source will be given.

Introduction

Synchrotron Radiation is emitted when an electron undergoes radial acceleration. The earliest observation occurred probably in the year 1054 when Chinese astronomers recorded a superbright star, which is now known to be the crab nebula supernova remnant. Recent astronomical observations established the existence of synchrotron radiation originating from this source. Approximately 900 years later a more informed effort was made towards observation of this phenomenon. Following the prediction (1944) by Iwanenko and Pomeranchuk[1] that synchrotron radiation would limit the maximum achievable electron energy in a betatron, Blewett[2] provided for the first indirect observation of synchrotron radiation by correlating the observed shrinking of the electron orbit in General Electric's betatron with the predicted radiation loss. Actual first observation of the visible part of the synchrotron radiation spectrum was made a few years later, also at the GE laboratories. Following this, more extensive investigations of the properties of synchrotron radiation were carried out at the 300 MeV Cornell synchrotron[3] and at the 660 MeV synchrotron of the Lebedev Institute in Moscow.[4] The first sustained research program in atomic spectroscopy using synchrotron radiation was carried out at the National Bureau of Standards[5] where an ultraviolet beam exiting from the 180 MeV synchrotron was used.

Synchrotron Radiation research so far has been carried out mainly with photon sources obtained parasitically from electron synchrotrons or e^-e^+ colliding beam facilities originally built for "high energy" physics research. As will be discussed further below, these structures are not optimum for the production of the highest brightness photon sources. Following the early impressive research results obtained at facilities such as NBS (Washington), INS-ES (Tokyo), ADONE (Frascati), DESY (Hamburg), and TANTALUS (Wisconsin), the need for "dedicated to synchrotron radiation" research facilities was recognized.[6,7] As a result, worldwide, up till the present, three facilities have been converted to dedicated usage and one new facility, specifically designed for synchrotron radiation utilization, i.e. the SOR electron storage ring in Tokyo, has been constructed. In addition to this, an upsurge of construction of new facilities is underway both here and abroad.

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Properties of Synchrotron Radiation

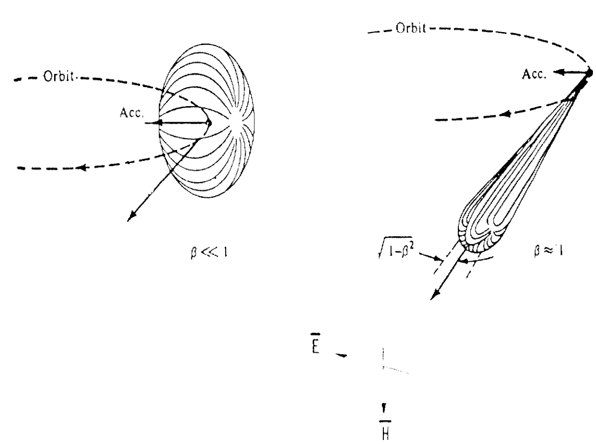
The theory of electromagnetic radiation from relativistic electrons undergoing centripetal acceleration in a magnetic field predates, of course, the time that this was first named synchrotron radiation. At the turn of the century, Schott[8] calculated the radiation for classical atomic models and derived expressions for the angular distribution. Following the early construction of electron synchrotrons, Schwinger[9] developed the classical radiation theory for an arbitrary electron trajectory. Further basic equations were developed by Sokolov and Ternov.[10] Here, some of the essential expressions and spectral distributions will be given, following mainly the textbook treatment by Jackson[11] (since the momentum of the emitted photon is negligible in comparison to the momentum of the radiating electron 'classical electrodynamics' is applicable) and extensive review by Green.[12]

For an electron orbiting in a magnetic field and thus undergoing radial acceleration, in the rest frame of the electron, the radiation pattern has the well known dipole radiation distribution, i.e. $P_\nu \propto \sin^2\theta$, as given in Figure 1, for the case $\beta \ll 1$. For a relativistic electron this radiation pattern has to be transformed by Lorentz transformation into the laboratory system. The transformation is given by

$$\tan\theta_\ell = \sin\theta_r / \gamma(\beta + \cos\theta_r)$$

which, for $\theta_r = 90^\circ$ yields $\theta_\ell \simeq 1/\gamma$, i.e. the angular distribution in the laboratory system has a typical opening angle of $1/\gamma$. This is illustrated also in Figure 1 for the case $\beta \gtrsim 1$. The frequency spectrum, as detected by a stationary observer, may be understood by considering that this narrow cone of radiation, tangent to the e^- orbit, sweeps past an "observer" in the e^- rest frame in a time $\Delta t_r \simeq \rho/\gamma c$. Transformation to the laboratory frame introduces a $1/\gamma^2$ factor giving $\Delta t_\ell \simeq \rho/\gamma^3 c$. For typical values of $\gamma \sim 2\ 10^3$ and $\rho \sim 2$ this single e^- "flash" lasts $\sim 10^{-18}$ seconds. This short duration pulse will contain, in accordance with Fourier synthesis, a spectrum of harmonics of the revolution frequency up to $\omega \simeq (\Delta t_\ell)^{-1} = \gamma^3 c/\rho$, or in the present example, 10^{18} Hz. Expressed in photon energy (or wavelength) this would correspond to

FIGURE 1. SYNCHROTRON RADIATION ANGULAR DISTRIBUTION FOR SLOW AND RELATIVISTIC ELECTRONS ON A CIRCULAR ORBIT



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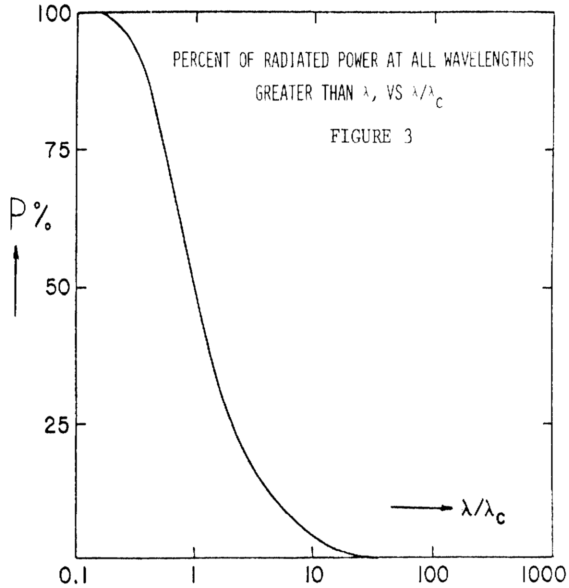
$\varepsilon = \hbar\omega \sim 1$ keV ($\sim$12 Å). Because of the δ function character of this orbiting source there is a strong accentuation of the higher harmonics of the frequency of rotation. For the case of more than one electron, associated with the betatron and synchrotron motion in the e^- ring, a continuum spectrum is observed. The properties of this spectrum may be deduced from the general equation[3,9] for the spectral and angular distribution, which expresses the instantaneous power radiated per unit wavelength and per radian of orbit for a monoenergetic electron. By integrating over all azimuthal angles Ψ, where Ψ is the angle between the direction of photon emission and the orbital plane, an expression for radiated power versus wavelength is obtained which may be cast in universally applicable form by characterizing the spectrum by a critical wavelength $\lambda_c = 4\pi\rho/3\gamma^3$ (or ε_c given by $\varepsilon_c = 3\hbar c\gamma^3/2\rho$), as follows:

$P_K(\lambda) = 5.95\ 10^{-16}\gamma^4 k\ G_2/\rho$ Watts/kλ,mA,mrad θ,all Ψ

for which $G_2(\lambda_c/\lambda)$ is given in Figure 2, θ is the orbit arc angle and $k = \Delta\lambda/\lambda$. As may be noted, this power spectrum peaks at about $\lambda \simeq 0.7\ \lambda_c$. Of great practical interest is the total magnitude of radiated power (at all wavelengths greater than λ) versus wavelength. This is given in Figure 3, indicating that essentially all ($\sim$95%) of the radiated power is contained in a wavelength domain of $0.2\ \lambda_c - 10\ \lambda_c$.



PERCENT OF RADIATED POWER AT ALL WAVELENGTHS GREATER THAN λ, VS λ/λ_c

FIGURE 3

For purposes of spectroscopy, the photon flux, rather than power flux, is of relevance. This is given in Figure 4 in the form of a universal spectral distribution function for synchrotron radiation, expressing

$N_k(\lambda) = k_\gamma\ F_1$ Photons/kγ,sec,mA,mrad θ,all Ψ

Typically, for a 1 GeV, 0.5 Ampere storage ring, using a 1% $\Delta\lambda/\lambda$ wavelength bite, the photon flux per mrad of orbit arc is approximately 10^{14} photons/second.

As indicated above, the photon radiation (per single electron) angular extent is proportional to $1/\gamma$. A good approximation of this, versus wavelength, is given by[12] $\sigma_r = 0.6(1/\gamma)(\lambda/\lambda_c)^{0.4}$ which indicates that for a typical 1 GeV electron synchrotron source the vertical divergence is of a magnitude of approximately 0.5 mrad for $\lambda \simeq \lambda_c$. This extreme natural "collimation" is beautifully demonstrated in the picture (Figure 5) of synchrotron radiation emerging from the VEPP3 Storage Ring, Novosibirsk. (Visibility here is the result of fluorescent recombination of the air molecules which were ionized by the X-ray beam.)

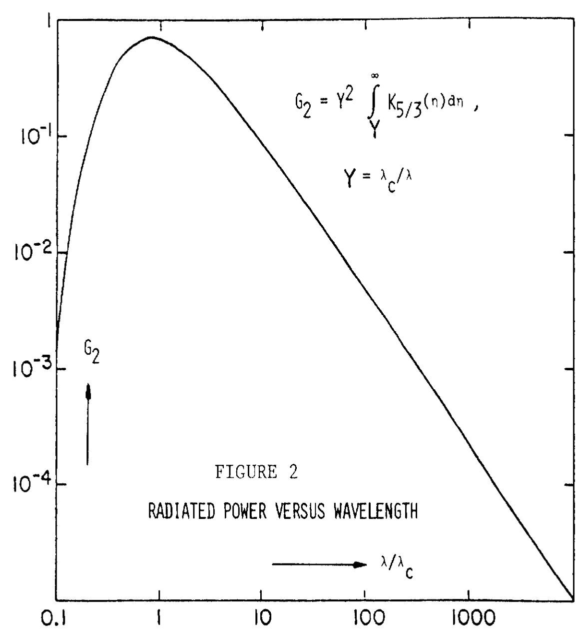


FIGURE 2

RADIATED POWER VERSUS WAVELENGTH

$P_k(\lambda) = 5.95\ 10^{-16}\gamma^4 k\, G_2/\rho$ WATTS/kλ,MA,MRAD θ; ALL Ψ

$P_{TOT} = 9.58\ 10^{-16}\ \gamma^4/\rho$ WATTS/MA,MRAD, ALL Ψ, ALL λ

$[\lambda_c = 4\pi\rho/3\gamma^3,\ k = \Delta\lambda/\lambda]$

Synchrotron radiation is polarized. In the plane of the electron orbit (Ψ=0) it is 100% polarized with the $\bar{E}$ vector parallel to the electron acceleration vector. Out of the orbit plane ($\Psi \neq 0$) its degree of polarization is dependent on wavelength and magnitude of the angle, Ψ, and the radiation is elliptically polarized ($\bar{E}_\perp$ and $\bar{E}_\parallel$, 90° out of phase). Detailed knowledge of the degree of polarization is of relevance, not only because it is anticipated that increased use will be made of this particular property of synchrotron radiation in future applications, but also because the magnitude of reflectance from optical surfaces, as typically used in the experimental photon lines, is E vector orientation dependent. Details of the relative magnitude of the $\bar{E}_\parallel$ and $\bar{E}_\perp$ components, as a function of Ψ for various wavelengths are given in Figure 6, indicating that for shorter wavelengths the degree of polarization drops faster with increasing value of the angle, Ψ.

In addition to the high photon flux, natural collimation and high degree of polarization, other properties

UNIVERSAL SPECTRAL DISTRIBUTION FUNCTION FOR SYNCHROTRON RADIATION

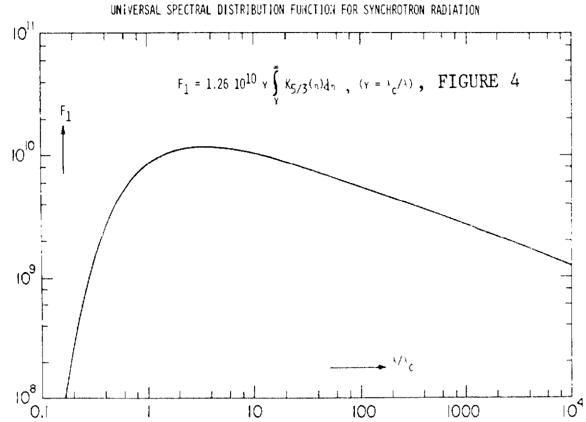


FIGURE 4

$N_k(\lambda) = k_\gamma F_1$ PHOTONS/Kλ, SEC, MA, MRAD θ, ALL Ψ $[\lambda_c = 4\pi\rho/3\gamma^3,\ k = \Delta\lambda/\lambda]$

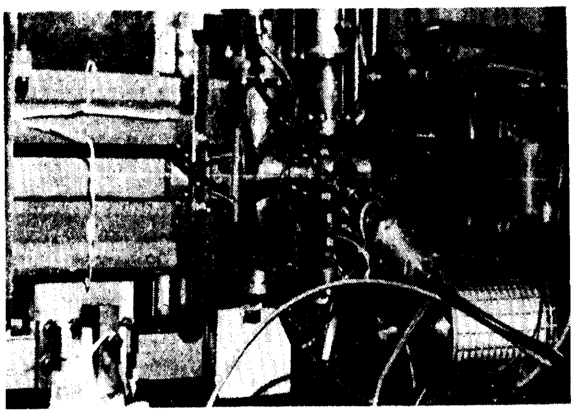
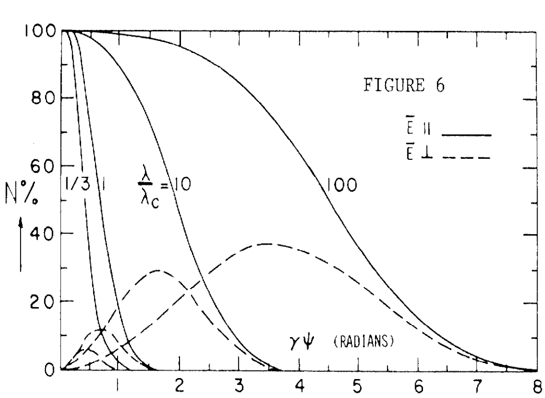


FIGURE 5

SYNCHROTRON RADIATION EMERGING FROM THE VEPP3 STORAGE RING, NOVOSIBIRSK

of interest to experimental utilization are the time structure of the emerging photon beam, specifically for fluorescent lifetime experiments (typical time structure, electron bunch length ~1 nec, with bunch separations of up to the orbital time of ~0.5 μsec); the small source size of the photon source (strong amplitude damping caused by the emission of synchrotron radiation results in beam sizes of fractions of mm^2, limited only by the excitation of betatron and energy oscillations due to the quantum nature of the photon emission); and the high vacuum environment of the source (typically 10^{-9}-10^{-10} Torr) which is principally of importance to users of the soft X-ray and VUV domain of the wavelength spectrum, i.e. no "windows" separate source from target.

For completeness sake, a comparison of the inherent brightness (Ph/sec,eV,(mrad)^2,cm^2) of synchrotron radiation sources with the brightness of more conventional photon sources, heretofore available, is made. This is detailed in Table I indicating not only that, typically, in the X-ray domain of the wavelength region improvements of 10^4 to 10^7 are anticipated, but also that, typically, in the VUV domain strong photon sources are now available in a wavelength region where essentially none existed. As a result of this access to strong "line" sources (say $\Delta\lambda/\lambda \simeq 10^{-4}$) by means of precision tunable monochromators, from a spectrum continuum ranging from 0.1 Å to the visible region (~5000 Å), the full spectrum of electromagnetic radiation potentially available is now indeed accessible for research.



VERTICAL ANGULAR DISTRIBUTION OF PARALLEL AND PERPENDICULAR POLARIZATION COMPONENTS

TABLE I, COMPARISON RADIATION SOURCES

→ Å

	0.1 – 10 (Hard) X-Ray (Soft)	100: VUV	1000: UV	Vis. / 10000: Infrared
Other Sources	Bremsstrahlung Continuum, Characteristic Emission	(Some Line Sources)	Gas Discharge	Lasers
NSLS	Wiggler (5T) Arc Elem. (1.2T) 2.5 GeV Ring	Arc Elements 0.7 GeV Ring		
Brightness* Synch. Rad. / Other	10^7 x Bremss. Cont.; 10^4 x Char. Emiss.		Typ. 10^2x He (800Å)	$\approx 10^{-10}$

Multipole Coherent Wiggler (N = 100)
Narrow Band Emiss. 10^8 x Char. Emiss.

EXAMPLE: X-ray Ring, 2.5 GeV, 0.5 A, Arc Source, at λ = 10 Å
$B \simeq 5\ 10^{15}$ Ph s^{-1} $(eV)^{-1}$ $(mrad)^{-2}$ $(cm)^{-2}$

Synchrotron Radiation Facilities

The existing and new facilities under construction with principal parameters are enumerated in Table II. Because of the inherent source stability and 100% duty factor all new dedicated sources are built as e$^-$ storage rings.

Rather than attempting to highlight some of the established facilities here, only the status of the synchrotron radiation sources under construction or with significant expansion programs will be mentioned. TANTALUS will be replaced at Wisconsin by the 1 GeV, 1 Ampere storage ring ALLADIN with a capability of 24 arc sources, of up to 120 mrad azimuthal extent per port. With a value of λ_c of 11.6 Å its usable spectral range will extend to about 2 Å. This could be reduced further by means of high field wigglers. Commissioning of this storage ring is expected in 1980. At Stanford, major construction is underway to extend the experimental capability of SSRL; five new synchrotron radiation beam lines will be added to Spear, which will result in a total of 25 experimental stations. Fifty percent parasitic usage is planned for 1980. An example of an arrangement of photon lines already in existence at SSRL (Spear) is given in Figure 7. At Cornell University a new synchrotron radiation facility (CHESS) is under construction, which will operate parasitically from the 8 GeV e$^-$e$^+$ storage ring. With this electron

TABLE II, SYNCHROTRON RADIATION SOURCES

IN OPERATION

		GeV	λ_c
USSR:	VEPP4 (NOVOSIBIRSK)	7.0	0.27 $(1.1)_W$
	VEPP3 (")	2.0	2.9 $(1.0)_W$
	VEPP2M (")	0.7	23
	*ARUS (EREVAN)	4.5	1.5
	*SIRIUS (TOMSK)	1.4	9.4
	*PAKHRA (MOSCOW)	1.3	10
	*FIAN, C60 (MOSCOW)	0.7	28
	N-100 (KARKHOV)	0.1	3100
GERMANY:	[PETRA (HAMBURG)	18.0	0.18]
	*DESY (")	7.5	0.4
	DORIS (")	5.0	0.5
	*BONN I (BONN)	2.5	2.7
	*BONN II (")	0.5	77
USA:	SPEAR (STANFORD)	4.0	1.1 $(1.6)_W$
	**SURF II (WASHINGTON, DC)	0.25	344
	**TANTALUS I (WISCONSIN)	0.24	258
FRANCE:	DCI (ORSAY)	1.8	3.4
	**ACO (")	0.54	39
JAPAN:	*INS.ES (TOKYO	1.3	10.1
	***SOR (TOKYO)	0.4	95
ITALY:	ADONE (FRASCATI)	1.5	8.3 $(4.6)_W$
SWEDEN:	*LUSY (LUND)	1.2	11.8

IN CONSTRUCTION

	GeV	λ_c
***JAPAN (TSUKUBA), PH. FACT.	2.5	3.0 $(0.6)_W$
***UK (DARESBURY), SRS	2.0	3.9 $(0.9)_W$
***GERMANY (BERLIN) BESSY	0.8	20
[PEP (STANFORD) (1979)	18.0	0.16]
CESR (CORNELL) (1979)	8.0	0.35
***ALADDIN (WISCONSIN) (1980)	1.0	11.6
***NSLS (BROOKHAVEN) (1981)	2.5	3.0 $(0.6)_W$
***NSLS (NAT'L. LAB) (1981)	0.7	31

* Synchrotrons.
** Dedicated to Synchrotron Radiation research.
*** Designed for Synchrotron Radiation research exclusively.

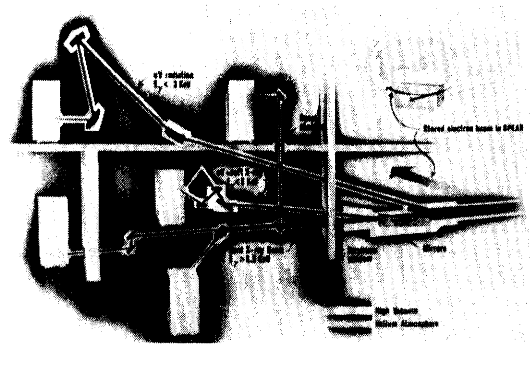


FIGURE 7, EXAMPLE OF ARRANGEMENT OF PHOTON LINES
(COURTESY SSRL, STANFORD)

energy a critical wavelength of 0.35 Å is obtained making synchrotron radiation available of very short wavelengths, i.e. ~0.06 Å (~200 keV). Two beam lines will be constructed at CHESS. At NBS, following the recent conversion to higher source brightness and higher energy performance, some of the research activities are being expanded. At NSLS (National Synchrotron Light Source), Brookhaven National Laboratory, two dedicated high photon flux facilities are under construction, i.e. a 2.5 GeV, 0.5 A storage ring for the X-ray domain and a 700 MeV, 1 A storage ring for the VUV domain. In addition to the basic arc sources of the X-ray ring (λ_c=3.0 Å), superconducting wavelength shifters will be incorporated (λ_c=0.6) for which five special insertions have been reserved in the eight-fold structure, providing for very high brightness photon sources into the wavelength region of about 0.12 Å ($\varepsilon \sim 100$ keV). For the VUV ring (λ_c = 31 Å) the incorporation of undulators (see below) is planned. Commissioning of both storage rings is scheduled for 1981.

Overseas, completion of the Daresbury (SRS) 2 GeV, 1A storage ring (λ_c = 3.9 Å) is expected during 1980, its injector booster synchrotron (600 MeV) has already performed successfully. Active construction is underway of the Japanese "Photon Factory" at Tsukuba. This is a 2.5 GeV, 0.5 A storage ring making use of a 2.5 GeV, 50 mA linear accelerator injector. (This high energy injector is also intended to be used for a future separate e^+e^- colliding beam ring.) In Berlin, the BESSY facility is being constructed, a 800 MeV storage ring, which will mainly be used for metrology, applied research and industrial development (microelectronics, etc.) and research in the material sciences. At DESY, the DORIS storage ring will be converted to dedicated usage following the addition of a small positron accumulator ring (PIA) to the injection chain for PETRA (e^+e^-). With new magnet chambers, 30% of the synchrotron radiation in one quadrant of DORIS will be made available. With this, simultaneous usage by 30 experimental stations will be possible. Finally, at VEPP3, in Novosibirsk, plans exist to expand further the dedicated usage for synchrotron radiation research (now up to 25%) and parasitic facilities are being developed around the VEPP4 e^+e^- storage ring.

Synchrotron Radiation Source Parameters and Brightness

Since these sources are designed for diversified research in spectroscopy the characteristics of the electron beam are important as they relate to the properties of the emitted radiation. As will be indicated below, in general small beam emittances will permit high photon source brightness values. This is desired for experiments requiring small angular spread on a small sample, such as for protein crystallography. Although there may be specific experiments such as for EXAFS and Topography where source brightness could be sacrificed in favor of higher total photon flux, the storage ring structure should be designed for optimum photon source brightness in order to realize the objective of high brightness and high flux density. The basic relationships correlating photon source brightness and electron beam sizes have been worked out by Green[12] and a summary of this is given in Table III in which it is stated that for optimum photon source brightness $\sigma_x, \sigma_y (\sigma_{x'}, \sigma_{y'})$ should be minimum, i.e. the electron beam emittances should have a minimum value. This, so far, may have ignored some of the characteristics of the photon line instrumentation, such as the use of slits to improve the resolving power ($Q=\lambda/\Delta\lambda$) of monochromators, for example. To illustrate this, a schematic arrangement of photon beam line instrumentation is shown in Figure 8.

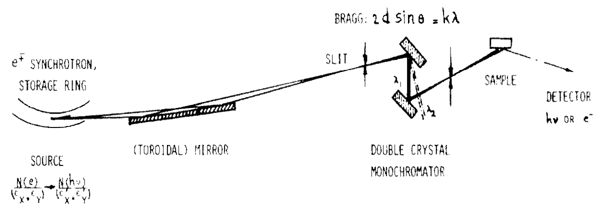


FIGURE 8

SCHEMATIC ARRANGEMENT OF PHOTON BEAM LINE INSTRUMENTATION

Two cases may be considered now, one whereby no optical element is placed between the photon source and the slit of the beam line instrument and one whereby typically a focusing mirror is interposed between source and monochromator. Depending on the case at hand, either a minimum vertical angular spread of the electron beam is desired or a minimum vertical beam size is desired for optimum flux transmission.[12] Whereas both cases may have to be satisfied simultaneously (multiplicity of beam lines) the vertical emittance should be minimized, and with this, because of finite coupling of the x-y motion, that ε_x should be minimized. Consequently the ring design should be guided by the objective of a minimum ε_x value.

Small β function values are desired at the location of the sources in the magnets of the synchrotron radiation lattice. Normally it is not possible to obtain minimum β_x and β_y (Courant-Snyder betatron

TABLE III, PHOTON SOURCE BRIGHTNESS

e^- BEAM: GAUSSIAN, $\sigma_x, \sigma_{x'}, \sigma_y, \sigma_{y'}$, $\quad \sigma_x(s) = [\varepsilon_x \beta_x(s)]^{1/2}$

ASSUME AT OR NEAR "WAIST" $\quad \sigma_{y'}(s) = [\varepsilon_y/\beta_x(s)]^{1/2}$

$h\nu$ BEAM: $\sigma_X, \sigma_{X'}, \sigma_Y, \sigma_{Y'}$ $\quad \sigma_{Y'} = (\sigma_r^2 + \sigma_{y'}^2)^{1/2} \simeq \sigma_r$ FOR $\sigma_{y'} < \sigma_r$

AZIMUTHALLY EXTENDED SOURCE, $\sigma_Y = (\sigma_y^2 + \ell_S^2 \sigma_r^2)^{1/2} \simeq \sigma_y$ FOR $\ell_S \sigma_r < \sigma_y$

SIMILARLY, $\sigma_X, \sigma_{X'}$ WITH $\sigma_{X'} < \sigma_r$ AND $\ell_S \sigma_r < \sigma_X$

CENTRAL PHOTON SOURCE BRIGHTNESS $\dfrac{d^4N}{dX\,dX'\,dY\,dY'} \sim \dfrac{1}{\sigma_r^2 \sigma_x \sigma_y}$ FOR FIXED γ, I, λ

FOR OPTIMUM BRIGHTNESS, $\sigma_x, \sigma_y (\sigma_{x'}, \sigma_{y'})$ MINIMUM, OR $\varepsilon_x, \varepsilon_y$ MINIMUM

RECALL $\varepsilon_y = K^2 \varepsilon_x$, THEREFORE ε_x MIMIMUM FOR OPTIMUM $h\nu$ SOURCE BRIGHTNESS

[MORE DETAILED TREATMENT: $B(X = X' = Y = Y' = 0) = N_k(0,\lambda)/2\pi\sigma_x\sigma_y$.

WITH $N_k(0,\lambda) = 3.46\ 10^{15}\ k\ I\ \gamma^2\ H_2(\lambda_c/\lambda)$ PH./SEC, $\lambda, d\psi, d\theta$

AND $H_2(\lambda_c/\lambda) = (\lambda_c/\lambda)^2 K^2_{2/3}(\lambda_c/2\lambda)$; $k = \Delta\lambda/\lambda$ (H_2 = 1.4 FOR $\lambda = \lambda_c$)

amplitude function) values simultaneously in a "regular" lattice structure. Therefore, specifically for the purpose of incorporating wavelength shifting wigglers with high photon source brightness values, special (low-β) insertions are incorporated in the new dedicated electron storage rings.

Lattice Optimization

Electron energy and ring lattice parameters determine the horizontal emittance ε_x, i.e. the equilibrium emittance ε_x in an electron storage ring is the result of longitudinal and transverse oscillation amplitude damping associated with the mean energy loss due to the emission of synchrotron radiation, limited by the random excitation of the oscillation amplitudes due to the discrete quantum nature of the photon emission. The vertical emittance is determined in any practical magnet structure by an irreducible magnitude of longitudinal-vertical coupling which results in a typical magnitude of $\varepsilon_y \cong 10^{-2}\,\varepsilon_x$ (i.e. it is difficult to reduce the coupling coefficient K below $\sim$0.1). The analysis of the relationship of beam emittance ε_x and lattice structure parameters may be found in Sands.[13] A minimum summary of this, synchrotron radiation integrals and quantities of relevance are given in Table IV. The horizontal emittance, for a separated function, isomagnetic structure is given by $\varepsilon_x = C_q\gamma^2(\oint H ds)2\pi\rho^2$. For various synchrotron radiation structures, designed for different maximum energies (γ), a figure of merit characterizing the lattice structure may therefore be defined, given by $(\oint H ds)/\rho^2$ where ρ is the bending radius of the magnets in the structure and H is the dispersion "invariant"*, as defined in Table IV. This quantity has been evaluated for a number of structures either in existence or under construction at the present time for dedicated synchrotron radiation sources. This is summarized in Table V. For lower energy storage rings, the lattice figure of merit tends to be larger because the objective of small ε_x (and ε_y) must be balanced by the objective of highest ε_{crit} values for a particular structure without going to (costly) higher e^- energies. While taking this aspect into account, it is evident, however, that a superior structure is obtained by using basic achromatic bend elements to construct a synchrotron radiation storage ring. This fact was recognized in the early design[16] of the storage rings for the National Synchrotron Light Source and incorporated in both the 2.5 GeV structure and the 700 MeV structure.

TABLE IV

SUMMARY OF SANDS, SYNCHROTRON RADIATION INTEGRALS, QUANTITIES OF INTEREST

GENERAL: $I_1 = \oint \eta G\,ds$ SEPARATED: $I_1 = (\oint \eta\,ds)/\rho = 2\pi\alpha R$ WITH $\alpha_p = (\Delta C/C)/(\Delta p/p)$

LATTICE: $I_2 = \oint G^2 ds$ FUNCTION: $I_2 = 2\pi/\rho$

$I_3 = \oint G^3 ds$ ISO MAGN.: $I_3 = 2\pi/\rho^2$ $G(s)$, CURVATURE FUNCTION

$(\oint 2\pi R)$ $I_4 = \oint \eta G^3 ds$ $(\oint 2\pi\rho)$ $I_4 = 2\pi\alpha R/\rho^2$

$I_5 = \oint \mathcal{H} G^3 ds$ $I_5 = (\oint \mathcal{H}\,ds)/\rho^3$ $\eta(s)$, DISPERSION FUNCTION

AND $\mathcal{H} = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2 = (\eta^2 + J^2)/\beta$ WITH $J = \alpha\eta + \beta\eta'$ (FOR REGULAR FODO: $J \simeq 0$)

QUANTITIES OF INTEREST: SYNCHROTRON RADIATION PER TURN: $U_0 = (C_\gamma/2\pi)E_0^4 I_2$; $C_\gamma = 8.85\ 10^5$ M GeV^{-3}

DAMPING TIMES: $\tau_i = 2T_0/D_i$ WITH $D_{x,y} = \left(\frac{I_4}{I_2}\right)$; $D_\epsilon = D_x + D_y$

ENERGY SPREAD: $\left(\frac{\sigma_\epsilon}{E}\right)^2 = C_q\gamma^2 \frac{I_3}{2(I_2+I_4)} \simeq C_q \frac{\gamma^2 I_3}{2I_2}$; $C_q = 3.8\ 10^{-13}$ M.

EMITTANCE: $\varepsilon_x = C_q\gamma^2 \frac{I_5}{(I_2 - I_4)} \simeq C_q\gamma^2 \frac{I_5}{I_2}$; $\varepsilon_y = k^2\varepsilon_x$

SYNCHROTRON RADIATION LATTICE, FIGURE OF MERIT: $\left(\frac{2\pi}{C_q}\cdot\frac{\varepsilon_x}{\gamma^2}\right) = (\oint \mathcal{H}\,ds)/\rho^2$

[BALANCED BY $\varepsilon_c \propto E^2B$; $P_{RF} \propto E^3B$, I.E. MORE ECONOMICAL TO $\uparrow \varepsilon_c$ WITH $\downarrow \rho$

*Analagous to the Courant-Snyder invariant ε, however, H is invariant only through drift spaces and focusing elements of the lattice, it changes through dipoles.[14,15]

Wiggler and Undulators

A "wiggler" is a structure of several short sections of alternating polarity magnetic field which, in its totality, does not result in a net orbit deflection of the electrons and is intended to provide sources of lower critical wavelength magnitude. Recalling that $\varepsilon_c \propto E^2B$ and $P_{rf} \propto E^3B$ and that the cost of rf power is high it is by now generally recognized that wigglers are the poor man's way to higher excitation. Assuming the use of 5T fields for the wiggler, a factor of 4, typically, in the critical energy, can be obtained. The use of a wiggler in a low β insertion can provide for a shorter wavelength source of very high brightness.

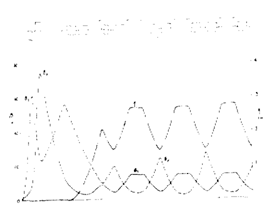
It is relevant for a synchrotron radiation source that the high field wiggler be located in a straight section of zero dispersion in order to avoid antidamping and enlargement of the beam emittances. Here the difference between a specifically optimized synchrotron radiation structure and a e^+e^- storage ring structure, parasitically used for synchrotron radiation research, is relevant. In colliding beam facilities the objective is to enhance e^+e^- luminosity given by[17] (at less than maximum energy): $L_{e^+e^-} \simeq [\pi f(\Delta\nu^2)k_b\gamma^2/r_e\beta_{oy}]\varepsilon_x$. With the incorporation of a wiggler in the structure, in order to increase ε_{crit}, the desire is to locate it in a high momentum dispersion straight section in order to increase the horizontal emittance and thereby the e^+e^- luminosity. The opposite is valid for a dedicated facility where the wiggler would always be located in a zero (or near zero) dispersion straight section in order to reduce the emittance. An example has been calculated whereby the use of 5 high field wigglers in the NSLS 2.5 GeV structure has been assumed. This is given in Table VI indicating a significant difference in achievable emittances depending on the location of the wigglers in the storage ring lattice.

An undulator[19-23] is a structure consisting of many low field wigglers in sequence, either in the form of a flat pole wiggler or helical wiggler. The basic

TABLE V

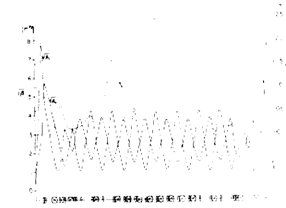
STRUCTURE COMPARISON OF SOME REPRESENTATIVE SOURCES

SPEAR, 3 GeV, e^+e^-, FODO



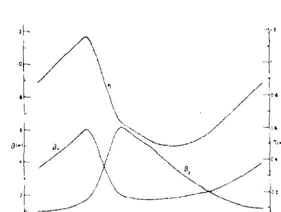
$(\oint H ds)/\rho^2 = 0.2$

PHOTON FACT., 2.5 GeV, SR, FODO



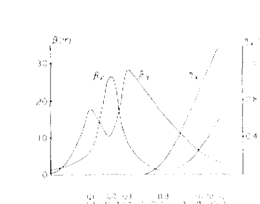
$(\oint H ds)/\rho^2 = 0.28$

ALADDIN, 1 GeV, SR, DOUBLET



$(\oint H ds)/\rho^2 = 2.9$

NSLS, 2.5 GeV, SR, TRI-ACHR.



$(\oint H ds)/\rho^2 = 0.055$

TABLE VI, LOCATION OF WIGGLERS IN THE STRUCTURE

Parasitic Synchrotron Radiation Source e^+-e^- colliding beams	Dedicated Synchrotron Radiation Source
Objective: Enlargement ϵ_x, (increase ϵ_{CRIT})	Increase ϵ_{CRIT}, (decrease ϵ_x)

Recall $L_{e^+e^-} \simeq \frac{\pi f (\Delta\nu)^2 k_b \gamma^2 \epsilon_x}{r_e \beta_{o,y}}$

Wiggler effect on ϵ_x: $(\epsilon_{x,w}/\epsilon_{x,o}) = \frac{1+n_w \ell_w r_\rho^3 \bar{\mathcal{H}}_w / 2\pi\rho_0 \bar{\mathcal{H}}_o}{1+n_w \ell_w r_\rho^2/2\pi\rho_0}$ with $r_\rho = (\rho_0/\rho_w)$

$\mathcal{H} = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2$, "invariant". Behavior in[18]  $\bar{\mathcal{H}} \simeq \mathcal{H}_{in} + \frac{\beta_x \theta}{3}$

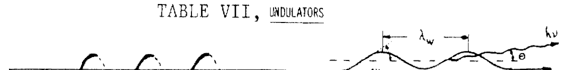
Assume (NSLS 2.5): $n_w = 5$, $\ell_w = 3\lambda_w$, $\lambda_w = 0.14$ m, $\hat{B} = 5$ T., $\rho_w = 1.6$ m, $\theta = 16.7$ mrad, $\rho_0 = 6.8$ m, $\beta_x \simeq 1$ m.

Location: Average Dispersion Straight	Zero Dispersion Insertion
$\bar{\mathcal{H}}_w \simeq \bar{\mathcal{H}}_0 \simeq 0.06$ m.	$\bar{\mathcal{H}}_w = 9\ 10^{-5}$ m, $\bar{\mathcal{H}}_0 = 0.06$ m.
$\rightarrow \epsilon_{x,w} \simeq 2.4\ \epsilon_{x0}$	$\rightarrow \epsilon_{x,w} \simeq 0.6\ \epsilon_{x0}$
[Similarly: $\sigma_{\epsilon,w} \simeq 1.5\ \sigma_{\epsilon,0}$	$\sigma_{\epsilon,w} \simeq 1.5\ \sigma_{\epsilon,0}$]

relationships are given in Table VII. Its great attraction is the on axis brightness proportionality with n_w^2, where n_w is the number of undulator periods, making possible two to three orders of magnitude enhancement with a reasonable structure. Typically, for the NSLS VUV storage ring (700 MeV) an undulator is planned for which a possible photon flux, at λ=200 Å, of approximately 4 10^{16} ph/sec/1% $\Delta\lambda/\lambda$ has been calculated[24] using a n_w=50 structure. Because the brightness enhancement is dependent on the $\sigma_{y'}$, $\sigma_{x'}$ values in the structure straight sections, more modest (rather than low) β values are required in the lattice insertions for optimum brightness enhancement.

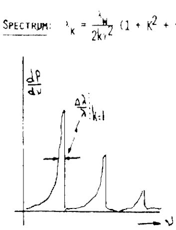
A significant drawback of the helical wiggler is the enhanced horizontal-vertical coupling of the x-y motion, resulting in significantly increased vertical emittances, i.e. the turn on of the photon source for the user of the undulator line would seriously affect other experiments using the arc sources in the structure. An attractive feature of the flat pole wiggler is the possibility of orienting it vertically, providing for a rotated plane of polarization compared with the arc sources. This permits a more favorable arrangement of specific experimental equipment (diffractometer table horizontal).

TABLE VII, UNDULATORS



Approx. sinusoidally transverse B field: $B_n = B_0 \sin(2\pi z/\lambda_w)$; n_w periods.

Spectrum: $\lambda_k = \frac{\lambda_w}{2k\gamma^2}(1 + K^2 + \gamma^2\theta^2)$; $k = 1,3,5,\ldots$; with $K = \gamma\psi_0 = \frac{e B_0 \lambda_w}{2\pi m_0 c^2}$



I.E. quasi monochromatic peaks in the frequency distribution.

For $\theta = 0$, line width $(\frac{\Delta\lambda}{\lambda})_k \simeq \frac{1}{nk}$; for finite θ, $(\frac{\Delta\lambda}{\lambda})_k \simeq \gamma^2\theta^2/(1 + K^2/2)$

Minimum $(\Delta\lambda/\lambda)$ demands $\sigma_{x'} \simeq \sigma_{y'} \simeq 1/\gamma\sqrt{n_w} \rightarrow$ Insertion!

Maximum SR power in $k = 1$ mode, for $K \simeq 1$, i.e. $B_0(T) \cdot \lambda_w(cm) = 1$

Example: ACO Supercon. Wiggler

$n_w = 23.5$; $\lambda_w = 4$ cm; $\bar{B}_0 = 0.25$ T; $\hat{B}_0 = 0.4$ T; $\lambda_{k=1} = 270$ Å

Photon Flux: Total flux $\propto n_w$, however, central brightness $\propto n_w^2$ as

$$\frac{dP_\nu}{d\Omega}\Big|_{\substack{k=1\\ \theta=0}} \; \frac{2 n_w^2 e^2 \gamma^2 K^2}{c(1+K^2)^2}$$

Helical Wiggler	Flat Pole Wiggler
Circularly polarized radiation	Plane polarized radiation
coupling $0.1 \rightarrow 1$, $\epsilon_y \simeq \epsilon_x$	orientation, plane of polarization!
	reduction in ϵ_x (or ϵ_y), at $\eta = 0$

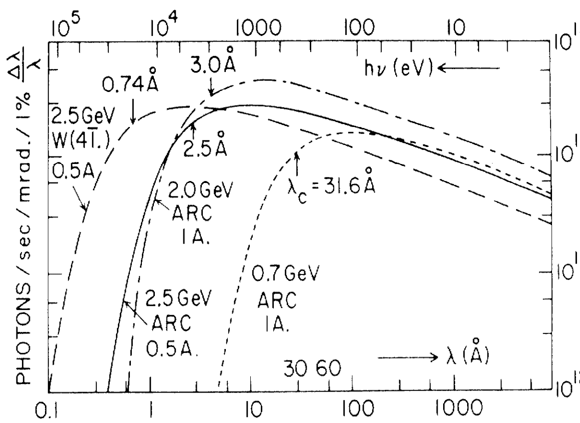
Optimized Synchrotron Radiation Source

At NSLS, two separate sources are under construction, of which the principal parameters are:

	X-ray Source	VUV Source
Current, energy (A, GeV)	0.5; 2.5	1.0; 0.7
Circumference (m)	170 m	51 m
λ_c (Å)	2.5 (0.6)$_w$	31.6
Emittance, ϵ_x (m-rad)	8 10^{-8}	9 10^{-8}
SR Power (kW)	300 (5 wigglers)	12
Source $4\sigma_x$, $4\sigma_y$ (mm^2)	0.5x1.5 (arc) 0.1x0.9 (wiggler)	0.4x1.2 (arc)

The synchrotron radiation spectra, as they apply to the NSLS parameters are given in Figure 9.

FIGURE 9
SYNCHROTRON RADIATION SPECTRA FOR THE NSLS DESIGN PARAMETERS



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