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HIGH CURRENT BEAM STABILITY IN LINEAR ACCELERATORS

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ABSTRACT

Equilibrium and stability constraints for high beam current linear accelerators are examined. Three general classes of linear accelerators are established depending on the geometry of the accelerating structure. For the case of klystron type geometry, of broad current interest for many applications, the equilibrium and stability properties are examined in detail. It is found that equilibrium constraints can readily be satisfied in the presence of a uniform guide magnetic field. Stability criteria for the longitudinal bunching mode and the transverse beam break up mode have been obtained and discussed. Stabilization mechanisms are suggested for the stable acceleration of multikiloamp particle beams.

I. INTRODUCTION

Recent applications, such as, Free Electron Lasers, Heavy Ion Inertial Fusion, Light Ion Inertial Fusion, Collective Ion Accelerators and Radiation Damage Studies Beams have established the need for multikiloamp electron and ion beams with energy of many 10's and 100's of mev. In order to generate these beams it is necessary to extend by nearly an order of magnitude the beam current capabilities of conventional linear accelerators, such as the linear induction accelerator and the radial pulse line accelerator. In addition, novel collective accelerators, such as the coaxial autoaccelerator, utilize driving and accelerated beams with beam currents in the many 10's of kiloamp range. Before such accelerators can be successfully built it is necessary to carefully examine the limitations on high beam current both from the equilibrium and the stability points of view for any given accelerator structure. While each type of accelerator has its own special considerations and problems, our extensive study of the coaxial autoaccelerator reveals that a certain classification of accelerator geometries, illustrated in Fig. 1, will permit a rather general approach to apply to several distinct types of accelerators in order to establish the high beam current limitations due to equilibrium and stability considerations. The critical parameter that separates several distinct regimes of dynamic behavior is the ratio of gap length l_g to the drift region between successive gaps $l_o - l_g$, i.e., $R_c = l_g/(l_o - l_g)$. We distinguish three regimes as follows:
(a) $R_c << 1$, or "Klynstron" regime
(b) $R_c >> 1$, or "Traveling Wave Tube" regime
(c) $R_c \approx 1$, or "Magnetron" regime.

The first regime is illustrated in Figs. 1b-1d and will be discussed at some length in this paper because it includes nearly all the interesting cases for accelerating high beam currents. It is characterized by the fact that the coupling between any two cavities occurs only via modulations on the beam.[1-5] The second regime is illustrated in Fig. 1a. It is the case where there is no drift region, and the accelerator structure behaves as an efficient slow-wave structure. This case is refered to briefly in this paper by way of contrast to case (a), because it appears to hold no promise for the acceleration of high currents on account of the instabilities that are inherent in it[1]. The third and intermediate regime is the most difficult to analyse because there exist gap regions and drift tube regions of comparable size leading to strong coupling of these two regions and to the cavities attached to them. It is for this reason that we refer to it as the "magnetron" regime. Although there exist accelerators, both traditional and collective, in this regime, we shall not discuss them further because of their complexity and the need for individual attention to each particular case.

II. EQUILIBRIUM

The main equilibrium requirement in linear accelerators has been to insure radial confinement of the beam. For low current beams, $I \leq 1ka$, this could be accomplished with alternating gradient focusing magnetic fields. For high current beams and for even higher current hollow beams it seems that a strong uniform guide magnetic field will be required. We shall therefore consider only the case of a uniform

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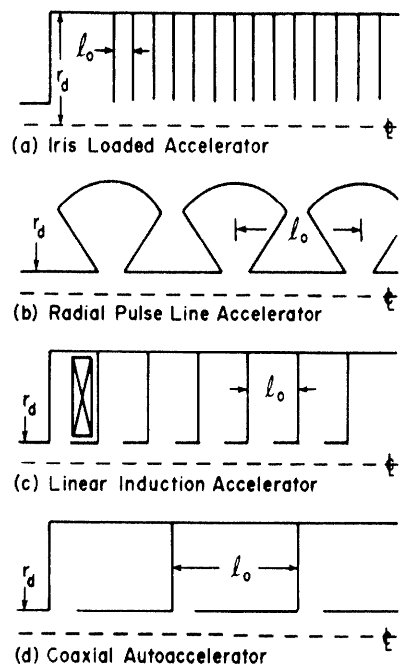

Fig. 1—Schematic Accelerator Diagrams

guide magnetic field B_o, for which the following constraints, resulting from thorough equilibrium studies,[4,5] must be observed.

(a) Radial Force Balance Equilibrium Constraint

For thin solid beams on axis

$$(\omega_{pr}/\omega_{cr})^2 < 1/2 \tag{1}$$

For thin hollow beams near the drift tube wall

$$(\omega_{pr}/\omega_{cr})^2 < r_b/4\delta \tag{2}$$

where

$$\omega_{pr}^2 = nq^2/\gamma^3 m_o \epsilon_o \tag{3}$$

$$\omega_{cr} = qB_o/\gamma m_o \tag{4}$$

$$\gamma^2 = (1 - \beta^2)^{-1} = [1 - (U/c)^2]^{-1} \tag{5}$$

where q, m_o, n, U are the charge, rest, mass, density and streaming velocity of the beam respectively, ϵ_o is the vacuum permitivity and c the velocity of light, all in mks units. The radius of the beam is r_b and δ is the thickness of the hollow beam. Both cases (1) and (2) result in a magnetic field satisfying

$$B_o^2 > \frac{I \times 10^{-8}}{2\beta\gamma\pi r_b^2} \tag{6}$$

(b) Limiting Current Constraint

The beam current must be less than the limiting current I_L.

$$I < I_L = I_o(\gamma_{inj}^{2/3} - 1)^{3/2} \tag{7}$$

$$I_o = \frac{2\pi\epsilon_o m_o c^3}{q\ln(r_d/r_b)} = \frac{8.5 \times 10^3}{\ln(r_d/r_b)} \tag{8}$$

where it is assumed that the beam is coasting without being accelerated and γ_{inj} is the value of γ at injection, before it has been reduced to a lower value because of the establishment of electromagnetic fields in the region surrounding the beam so as to permit the beam to propagate.

(c) Beam Loading of Cavity Constraint

When the beam encounters an uncharged cavity an inductive voltage drop ΔV_g appears across the gap, which decelerates the beam and fills the cavity with energy. For a smoothly rising beam current this voltage is given by

$$\Delta V_g = l_c L_c (dI/dt) \quad (9)$$

For a sharply rising square pulse current it is given by

$$\Delta V_g = cIL_c \quad (10)$$

where L_c is the inductance per unit length of the cavity and l_c is the length of the cavity. The resulting energy drop, per particle, is given by

$$\Delta U_g = q \Delta V_g \quad (11)$$

In order therefore, at high beam currents, not to have the beam stopped and virtual cathodes formed it is necessary to keep small enough the characteristic inductance L_c and impedance $Z_c = \sqrt{L_c/C_c}$, where C_c is the characteristic capacitance of the cavity.

In concluding this section it can be said that the constraints imposed by equlibirum considerations are not severe or critical for the applications listed in the introduction.

III. STABILITY

Traditionally, in accelerator theory and practice, two types of modes are thought to be most important, for stability considerations. These two modes are: (a) The longitudinal bunching mode, which is azimuthally symmetric, $m = 0$, primarily electrostatic in nature comprising slow and fast beam space charge waves[1,2] and (b) The transverse beam breakup mode, which is not azimuthally symmetric, $m = 1$, is electromagnetic in nature and for the case of uniform guide magnetic field it comprises the fast and slow $m = 1$, cyclotron beam electromagnetic waves[3].

The two modes discussed above go unstable in a variety of situations. We distinguish three general cases, as follows: (a) Klystron instabilities (b) Resonant traveling wave instabilities and (c) Universal instabilities. The first two cases relate to the accelerator cavity geometry and the dynamic interaction between the beam and cavities therein. The third case relates to velocity and density gradients in the beam and resistive wall effects, and contributes to enhancement of beam emittance.

We shall discuss at some length the Klystron instabilities because of their possible occurrence in cases (b)-(d) of Figure 1, which are accelerators of current interest. We have found that the stability criteria are best expressed in terms of the gain functions G, which is the ratio of velocity modulation at the second gap to the velocity modulation at the first gap. For stability $G < 1$, so that the velocity modulation does not grow.

For the longitudinal bunching mode we find[2]

$$G = \frac{\omega}{\omega_{pr}} \frac{R_s}{2Z_D} M^2 < 1, \qquad \omega > \omega_{pr} \quad (12)$$

$$G = \frac{(1-\alpha^2\mu^2)}{\alpha\mu(1+\alpha)} \frac{R_s}{2Z_D} M^2 < 1, \qquad \omega < \omega_{pr} \quad (13)$$

where ω is the cavity frequency, M is the transit time gap coupling coefficient with value close to one, R_s is the cavity shunt resistivity for this mode. The other parameters are given by

$$Z_D = (U_o^2 m_o \gamma^3/q)/2I \quad (14)$$

$$\alpha = I/I_o \beta \gamma^3 \quad (15)$$

$$\mu = [1 + (1-\beta)/\alpha]^{1/2} / \beta \quad (16)$$

Figure 2 illustrates the variation of part of the gain function, $G/M^2(R_s/2Z_D)$, with frequency and the transition from the high to the low frequency regime. Assuming, at first, that the factors, $M^2 \lesssim 1$, and $R_s/2Z_D \lesssim 1$, then it follows that curve (B) corresponds to a stable situation for the high current case, $\omega < \omega_{pr}$, whereas curve (A) may correspond to an unstable case. It is possible to transform an unstable situation such as that which occurs for curve (A) when $R_s/2Z_D \lesssim 1$, to

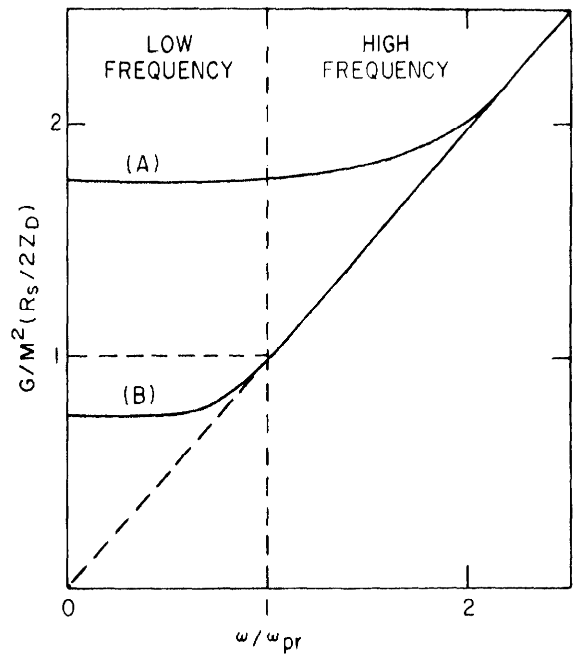

Fig. 2—Variation of the Gain function for the longitudinal bunching mode of the klystron instabiity.

a stable case by adjusting the vaue of R_s with suitable resistive loading of the cavity so that $R_s/2Z_D << 1$, and $G < 1$.

For the transverse beam break up mode for the klystron case we find[3]

$$G = \frac{l_g}{r_b} \frac{\omega_p^2}{\omega\omega_c} \frac{R_s}{\eta} \beta M^2 < 1 \quad (17)$$

where ω_p and ω_c are the rest mass plasma and cyclotron frequencies, $\eta = 377$ is the free space impedance and R_s is the equivalent shunt resistivity of the cavity for the $m = 1$ mode at the beam location.

In order to assess the implications of Eq. (17) for stability, we consider the several factors that appear in it. The factor β is equal to one or less, the factor M^2 is equal to one or less, the factor l_g/r_b is of order one to ten or higher, the factor ω_p/ω_c is always less than one from equilibrium considerations. The factor ω_p/ω can be very large even for moderate beam currents because the cavity frequency ω is typically in the 10's or 100's of MHz range. In order therefore to obtain stability, $G < 1$, the factor $R_s/\eta = R_s/377$, must be made correspondingly small enough. This calls for loading the cavity with dissipation in such a fashion so that the $m = 1$ radial line cavity mode does not grow and its equivalent shunt resistivity R_s at the beam locaton is suitably small. This appears to be the case, ab initio, for the ferite loaded linear induction accelerator, where the ferite core provides the dissipation for the cavity excitations. The transverse beam break up instability is clearly a high current instability with an onset determined by the ratio ω_p/ω exceeding a certain threshold with increasing current. This contrasts sharply with the longitudinal bunching mode klystron instability, which occurs at low currents and shuts off when $\omega_p/\omega > 1$, which occurs with increasing beam current.

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