OVERLAP KNOCK-OUT RESONANCES WITH COLLIDING BUNCHED BEAMS IN THE CERN ISR

Stephen Myers*

Summary

A bunched beam can subject a 'perturbed beam' to
overlap knock-out (OKO) resonances when the frequencies
contained within the longitudinal spectrum of the bun-
ches are equal to any of the transverse betatron fre-
quencies of the perturbed beam.   Usually with proton
machines this condition can only be attained when the
revolution frequencies of the two beams are different.
With electron machines and their associated high synch-
rotron frequency, the betatron frequencies of the per-
turbed beam can overlap with longitudinal sidebands re-
lated to the synchrotron motion of the bunched beam.
The worst case occurs when the first harmonic of the
bunch frequency overlaps with the betatron frequencies
of the perturbed beam.   This situation was set up in
the ISR for dipolar and quadrupolar (order 1 and 2)
OKO resonances by operating the rings at largely asym-
metric energies (26 and 11 GeV) and at tune values of
the perturbed beam close to the integer (dipolar) and
the half integer (quadrupolar).   In this mode the
half life of the perturbed bunched beam was reduced
from several months to a few seconds by maximizing the
strength of the OKO resonances.

1.   Introduction

The initial operation of the ISR at tune values
close to an integer resulted in transverse blow-up and
proton losses due to dipolar overlap knock-out reso-
nances acting between the coasting beam and the injec-
ted bunched beam[1].   These effects are eliminated by
reduction of the harmful longitudinal harmonics con-
tained in the bunched beam[1].

Overlap knock-out may also occur when both beams
are bunched.   In this case the synchrotron motion of
the perturbed beam causes particles to traverse the re-
sonance many times and increases the rate of increase
of the transverse emittance.

2.   Elements of Theory

The longitudinal frequencies in a bunched beam are

$$f_\ell = s f_b \pm m_s f_s \qquad (1)$$

where s is the harmonic number of the bunch revolution
frequency (f_b), m_s is the mode of the bunch oscillation
$(m_s = 1$ for dipole etc.), and f_s is the coherent syn-
chrotron frequency of the bunches.

The transverse frequencies contained in a beam are

$$f_t = (p \pm m_t Q_p) f_p \qquad (2)$$

where p is an integer, m_t is the mode of the transverse
oscillation and f_p is the revolution frequency of the
perturbed particle.

In the ISR (where $f_s \ll f_b$) the resonance condi-
tion is satisfied when

$$m_t Q_p = p \pm s \frac{f_b}{f_p} \qquad (3)$$

Fig. 1 shows certain resonance lines in a plot of
q (the non-integral part of Q) against f_b/f_p and as a
function of s/m_t.   Each resonance line is drawn for
a given harmonic of the bunch frequency for the ISR
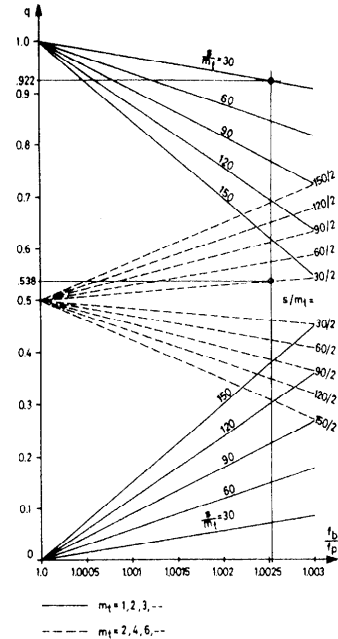(harmonic number = 30) up to the fifth harmonic

Fig. 1.   Resonance
condition for over-
lap knock-out

$m_t = 1,2,3,--$

$m_t = 2,4,6,--$

(s = 150).   For 11.8 GeV/c in one ring and 26.6 in the
other the frequency ratio is (f_b/f_p) 1.00255.   Under
this condition the resonance $s/m_t = 30$ occurs at a q
of 0.922; at this q value the dipolar 'resonance'
$(m_t = 1)$ is excited by the first harmonic of the bunch
frequency (s = 30) and the quadrupole resonance $(m_t = 2)$
is excited by the second harmonic (s = 60) etc.   In
fact all harmonics of the bunch frequency excite a re-
sonance at the corresponding mode number.   Similarly
at q = 0.538 a resonance occurs at $s/m_t = 30/2$ indica-
ting that the first harmonic of the bunch frequency can
excite the quadrupolar resonance $(m_t = 2)$;   the second
harmonic can excite the octupolar $(m_t = 4)$ etc.   The
strength of OKO resonances may be calculated in a simi-
lar way to that for 'normal' beam-beam resonances[2,3]
but by replacing the total current by the component of
the bunched current which drives the resonance.   For
the case of OKO resonances with bunched beams, however,
the situation is greatly complicated by the synchro-
tron motion of the perturbed particles and the time-
space dependence of the beam-beam effect.   In addition,
no theoretical solution exists to evaluate the effects
of a cluster of resonances acting on a single particle.
It has already been seen that different harmonics of
the bunched beam excite resonances of different order
at the same tune value.   For these reasons it is im-
possible to calculate analytically the absolute values
of transverse blow-up rates due to OKO resonances.
However it is still useful to present some of the sim-
plified equations which are required to perform such a
calculation.   In this way the importance of certain
parameters may be appreciated and scaling laws may be
derived.   The distance from an OKO resonance is

$$e = m_t Q_p \pm p - s \frac{f_b}{f_p} \qquad (4)$$

The bandwidth of the dipolar resonance[2] $(m_t = 1)$ is,
for large crossing angle (χ),

$$\Delta e(m_t = 1) = \frac{c_s \mu_o (1+\beta^2) I \beta_z}{4\pi \beta^2 |B\rho| \sigma_z} \sin\chi \cdot erf\left(\frac{z}{\sqrt{2}\sigma_z}\right) \exp(i\mu_z) \qquad (5)$$

where c_s is the relative amplitude of the harmonic

* CERN, Geneva, Switzerland

causing the resonance; β_z, μ_z, σ_z, and z are respectively the amplitude function, the phase advance, the rms beam size and the beam separation in the crossing point.

For higher order resonances[3]

$$\Delta e(m_t \geq 2) = \frac{c_s \mu_0 (1+\beta^2) m_t^2 I \beta_z}{\pi^{3/2} \sqrt{2} \, m_t^{+1} \, \beta^2 m_t! \, |B\rho| \sigma_z \sin\chi} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \times$$

$$\times H_{m_t - 2}\left(\frac{z}{\sqrt{2}\sigma_z}\right) \exp\left(im_t \mu_z\right) \quad (6)$$
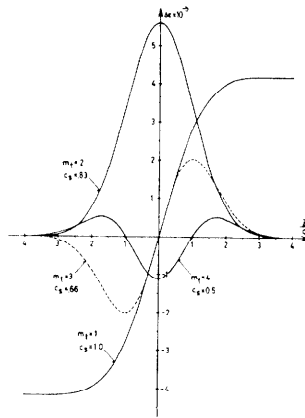
where H_n is given by the recursion equation

$$H_{n+1}(x) = 2x \, H_n(x) - 2n \, H_{n-1}(x)$$

with $H_o(x) = 1$ and $H_1(x) = 2x$.

Fig. 2 shows the calculated bandwidth per intersection as a function of beam separation for the experimental conditions described later.

Fig. 2. Resonance bandwidth as a function of beam separation for $Q = 8.922$ and measured values of c_s.

The transverse increase in the amplitude of a particle which crosses a single isolated mono-dimensional resonance of order m_t is given by[2]

$$\frac{\Delta\sigma_y}{\sigma_y} = \sqrt{\frac{\pi\beta c}{2R}} \frac{1}{m_t} \frac{\Delta e}{\sqrt{\frac{de}{dt}}} \quad (7)$$

where σ_y = the initial rms value and de/dt may be calculated from (4).

For the bunched beam, only the rate of change of momentum of the centre of gravity of the beam (or the synchronous particle) need be calculated. However in the case of the perturbed beam the instantaneous dP/dt of each particle inside the bucket must be evaluated. In this way the transverse blow-up of a particle which crosses a single isolated mono-dimensional resonance of bandwidth Δe (from (5) and (6)) may then be evaluated from (7). In the case of many crossing points the resonance bandwidth must be evaluated from the vector addition of Δe over one turn. This can be evaluated when the relative phase advance $\Delta\mu_z$ in each crossing point is known.

3. Measurements and Results

Both rings of the ISR were initially set for injection at 11.8 GeV/c. The tune values were adjusted so that the expected resonant tune value Q_p was around the

centre of the aperture. An injected bunched beam of around 150 mA (20 bunches) was accelerated from 11.8 to 26.6 GeV/c in ring 2 (R2). This beam acted as the source of excitation of OKO resonances for the other ring (R1). The beam in R1 was displaced from the injection orbit ($\Delta p/p = -0.02$) across the OKO resonance to the outer aperture limitation ($\Delta p/p = +0.025$). A linear dependence of tune (Q_p) on $\Delta p/p$ had been previously set up. During each aperture traversal an XY plotter recorded signals proportional to the beam current and the tune value. In order that vertical excitation would be observed as current loss the vertical aperture was limited by a beam scraper. The rate of acceleration of the pulse was $2.5 \cdot 10^{-4}$ ($\Delta p/p$ per second).

4. Discussion of Results

(i) Q values close to the integer

In the ISR with one beam at 11.8 GeV/c and the other at 26.6 GeV/c ($f_b/f_p = 1.00255$) an OKO resonance condition is (see Fig. 1)

$$Q_p = 8.922 \quad \text{for} \quad \frac{s}{m_t} = 30$$

When the beams are sufficiently separated the strength per intersection of all beam-beam resonances except the dipolar converge to zero (see Fig. 2). The measurements (Fig. 3) show that large separations can cause a very large excitation (a) or no excitation (c)

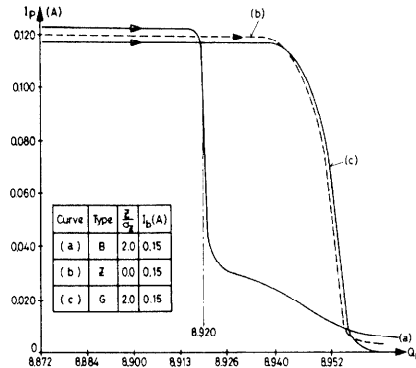| Curve	Type	$\frac{z}{\sigma_t}$	$I_b(A)$
(a)	B	2.0	0.15
(b)	Z	0.0	0.15
(c)	G	2.0	0.15

Fig. 3.   Losses due to dipolar OKO resonances

depending on the type of separation in each intersection. The large excitation occurs when the vector addition of the bandwidths (with μ_z) over one turn is large (Fig. 4(a)). Conversely when the kicks are subtractive (Fig. 4(b)) the excitation can be greatly reduced.
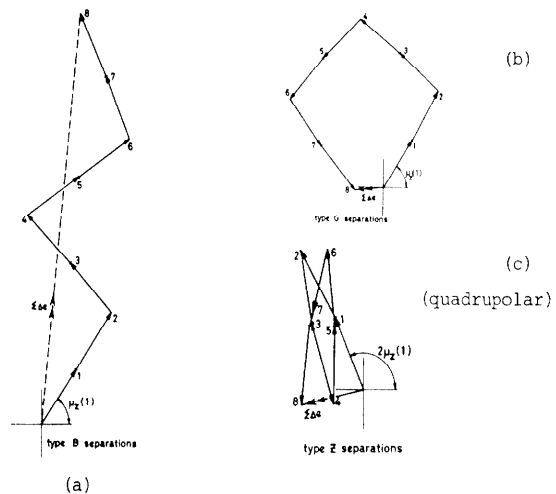

Fig. 4.   Vector addition for various types of beam separations and $Q = 8.922$

3575

Fig. 3(b) also shows that the sum of the excitation is very small when the beams are colliding head-on. In this configuration the calculated beam-beam excitation per intersection is maximum for all even order resonances. Again the absence (or reduction) of excitation may be explained by the vector summation of Δe in each crossing point (Fig. 4(c)). Fig. 5 shows that, as expected, when the bunched beam is dumped the excitation disappears.
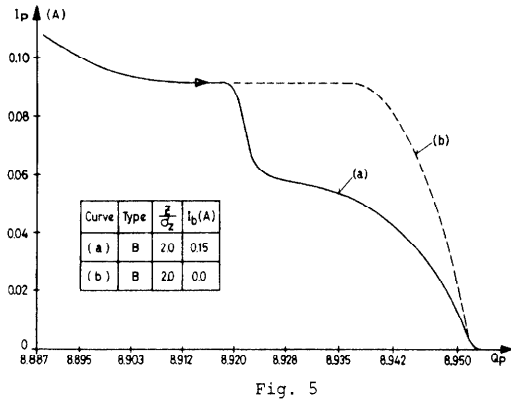

Fig. 5

(ii) Q values close to the half integer

In this case the resonance condition of interest is

$$Q_p = 8.538 ; \quad \frac{s}{m_t} = \frac{30}{2}$$

From Fig. 2 a very small quadrupolar excitation per intersection is expected when the beams have large separations. Fig. 6(b) confirms this expectation. However, once again the expected large excitation per intersection at zero separation produces a very small total excitation (Fig. 6(a)).
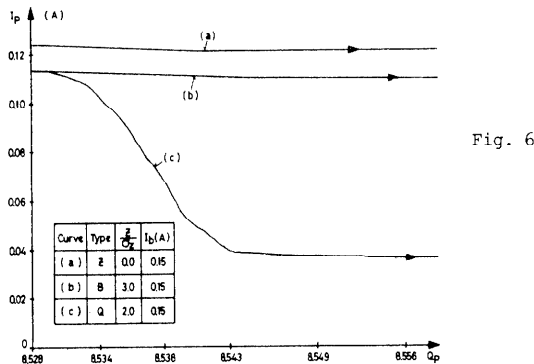

Fig. 6

This is again explained by the vector summation of the resonance bandwidth over the total number of crossing points (Fig. 7). In the case of quadrupolar beam-beam resonances the sign of excitation is independent of beam separation which makes it impossible to devise beam separations which maximize the excitation as was done for the dipolar case. However by decreasing the amplitude of the vectors in some crossing regions the total excitation can be greatly increased (Fig. 7). Application of this type of 'bump' caused an immense increase in the beam losses during traversal of the resonance (Fig. 6(c)). The large Q range over which the beam loss occurs is due to quadrupolar OKO resonances at other Q values being excited by other harmonics of the revolution frequency (e.g. s = 28, 29, 31, 32). These harmonics exist in the ISR since only 20 of the 30 RF buckets contain bunches. Fig. 8 shows that when the exciting beam is debunched the excitation of the perturbed beam disappears as expected.

Fig. 7. Vector addition for 4 and 8 crossing points
Q = 8.538

$$\sum_1^4 = \text{type Q}$$
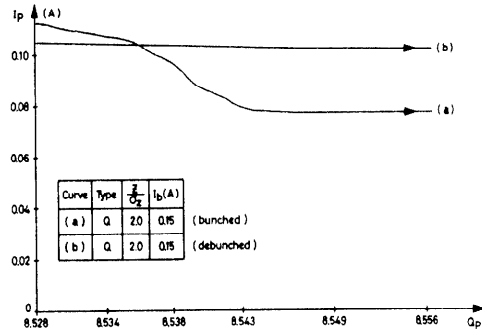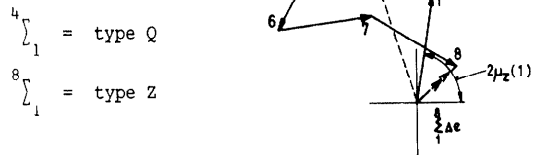
$$\sum_1^8 = \text{type Z}$$

Fig. 8

The variation of the total OKO excitation as a function of the RF voltage (of the perturbed bunches) is shown in Fig. 9. It is clear that increasing the RF voltage and hence the synchrotron frequency causes the beam blow-up rate to increase. This may be explained qualitatively by the fact that each particle crosses the resonance a greater number of times as it is accelerated across the resonance.
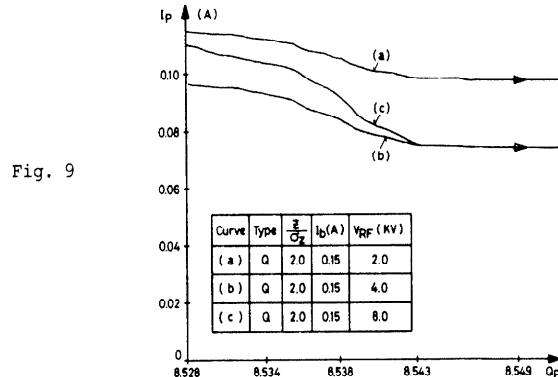

Fig. 9

5.   Conclusions

For machines operating with bunched beams, overlap knock-out resonances of low order may occur at tune values far removed from the tune values of classical resonances. These latent resonances may provide a strong source of transverse beam excitation and blow-up. In a multi-intersection colliding beam machine the beam separation in each intersection, the betatron phase advance between intersections, and the number of bunches in the exciting beam all play an important role in the total excitation of the perturbed particles.

6.   References

1. J.-P. Gourber, H.H. Hereward and S. Myers, IEEE Trans. Nucl. Sci. NS-24, 1405-1407, 1977.
2. G. Guignard, CERN 78-11, 10th November, 1978.
3. G. Guignard, CERN 77-10, 6th June, 1977.