DISPERSION SUPPRESSION WITH MISSING MAGNETS IN A FODO STRUCTURE
APPLICATION TO THE CERN ANTIPROTON ACCUMULATOR

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Introduction

The lattice of an antiproton accumulator which uses the stochastic cooling method must meet a variety of requirements arising from its double function: storing particles in momentum space, and reducing their phase space volume in both transverse and long-itudinal dimensions. A number of long straight sections, some of them with a zero momentum dispersion function α_p, are required and have to be disposed on a relatively small circumference.

In a classical FODO structure where identical dipoles alternate regularly with quadrupoles, it is impossible to achieve zero values of α_p and of its azimuthal derivative α_p' simultaneously; in other terms, α_p can only be made zero locally. However, if two dipoles or two sets of dipoles have different specific strengths, α_p can be zero all along a dedicated straight section without altering the beta-tron functions of the period of the lattice. For particular values of the betatron phase advance in a period, the strength of one of the dipoles can be made to vanish. In this case the advantage of a zero dispersion in one section is then combined with that of an extra long straight section.

Firstly, a systematic analysis of dipole distri-butions in a dispersion suppressor is presented. Then the requirements to be fulfilled by the lattice of an antiproton accumulator are listed and serve as criteria for the choice of a scheme of dispersion suppressor.

Dispersion suppressors with missing dipoles

Structure of a superperiod

The superperiod of the lattice consists of a succession of periods the focussing properties of which are identical; in all the quadrupoles, the field gradients are the same in magnitude. Their sign differs between an F and a D type quadrupole. The zero dispersion region is enclosed between two dispersion suppressors which will be described in detail in section II.iii. These dispersion suppres-sors are symmetrically disposed around the zero α_p region so that a superperiod of the machine is symmetric (Figure 1).
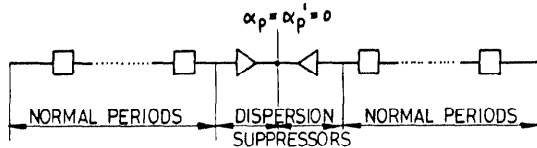

Figure 1. Superperiod

Parameters of a period

In a FODO structure (Figure 2), the focussing strength of a quadrupole, defined as the ratio of the integrated field gradient Gl to the magnetic rigidity $B\rho$, is related to the betatron phase advance μ per period and to the length L of a period by the formula[1]

$$\delta = 4x/L \qquad (1)$$

with

$$x_F = -x_D = -x = -\sin(\mu/2) \qquad (2)$$

The momentum dispersion function α_p in the middle of a quadrupole is given by

$$\alpha_p = \frac{2-x}{x^2} (L/4) \, \phi_0 \qquad (3)$$

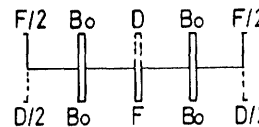ϕ_0 being the deflexion angle of a dipole.

Figure 2. Two representations of a FODO period

Schemes of dispersion suppressors

A dispersion suppressor consists of two periods which have the same focussing structure as the normal periods but with different dipole strengths. The study of the schemes of dispersion suppressors is limited to those which have a minimum of two long straight sections, one of them being the zero α_p section. Six configurations shown in Figure 3. are then possible:
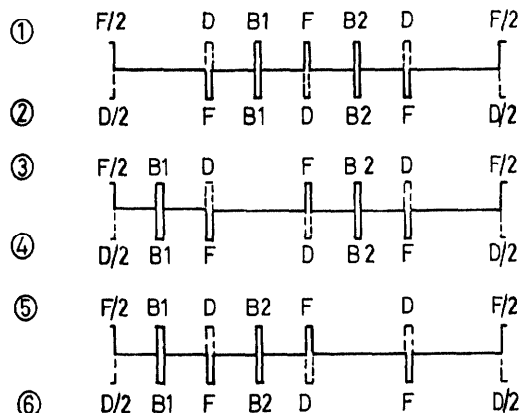

Figure 3. Six schemes of dispersion suppressors

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Calculation of the dipole strengths in a dispersion suppressor

The thin lens approximation which is used in the following is sufficient to outline the main features of the suppressor. The total transfer matrix is the product of matrices such as

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ \delta/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \phi \\ 0 & 0 & 1 \end{pmatrix}$$

which apply to a straight section, half a quadrupole and a dipole respectively.

Although these matrices are simple, their product is complicated and exceedingly tedious to calculate by hand. As an analytical insight is nevertheless important, a program which performs symbolic algebraic manipulations like matrix multiplications has been used to trace the vector $(\alpha_p, 0, 1)$ through the suppressor[3]. As a general rule, α_p and α_p' are linear functions of ϕ_1 and ϕ_2:

$$\begin{pmatrix} \alpha_p \\ \alpha_p' \end{pmatrix}_{exit} = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} + \begin{pmatrix} a_3 \\ b_3 \end{pmatrix} \phi_0 \qquad (4)$$

The coefficients a_i, b_i ($i = 1,2,3$) are polynomials in ξ, ξ being equal to x in odd numbered schemes and to -x in even numbered schemes (Table 1)

Scheme	ϕ_1/ϕ_0	ϕ_2/ϕ_0
1-2	$\dfrac{(\xi + 2)(4\xi^3 - 6\xi^2 - \xi + 2)}{2(\xi - 2)\xi^2}$	$\dfrac{(\xi + 2)(2\xi^2 + \xi - 2)}{2(\xi - 2)\xi^2}$
3-4	$\dfrac{(\xi + 2)(4\xi^3 - 6\xi^2 - \xi + 2)}{2(\xi^2 - 2)\xi^2}$	$\dfrac{(\xi + 2)(\xi - 2)}{2(\xi^2 - 2)\xi^2}$
5-6	$\dfrac{2\xi^2 + \xi - 2}{2\xi^2}$	$\dfrac{2 - \xi}{2\xi^2}$

Table 1. Bending strengths in the six schemes of dispersion suppressors

Schemes with particular values of betatron phase advance

Some arrangements of particular interest arise when the phase advances are such that the two special dipole strengths are identical or one of them vanishes. The number of dipole types is then reduced from three or two or sometimes one; schemes which were a priori different may become identical. Their characteristics are given in Table 2.

Scheme	Variant	x = sin(μ/2)	μ(degree)	ϕ_1/ϕ_0	ϕ_2/ϕ_0
1	1	.78078	102.66	1	0
1	2	.62705	77.67	0	1.42723
2	1	.55745	67.76	0	1.757
2	2	$1/\sqrt{2}$	60	.8153	.8153

Table 2. Characteristics of dispersion suppressors with a special phase advance per period

The first scheme is remarkably simple since all the dipoles of the lattice can be the same and it is the only one which has two long sections without dispersion. It turns out also that the 60 degree phase is special in this type of suppressor as it is in another category of suppressors where two dipoles are permitted in the second period. In the latter case, periods containing no dipoles could be added at the end of the suppressor to take advantage of the absence of dispersion. However, the present study is restricted to compact schemes which use at most two periods.

Application to the CERN Antiproton Accumulator

The CERN Antiproton Accumulator, described in reference 4, is designed to store 6.10^{11} antiprotons in one day. To achieve this goal, the machine needs a large momentum aperture ($\Delta p/p \sim 6\%$), a fast cooling of each injected pulse and two systems for cooling the accumulated beam.

Dispersion in revolution frequency

The dispersion η in revolution frequency f plays a central role in the theory of the stochastic cooling because it determines the rate of "mixing" of the particle samples to be cooled. Its minimum value is 0.1 From the expression

$$\eta = -\frac{df/f}{dp/p} = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} = \frac{\overline{\alpha_p}}{R} - \frac{1}{\gamma^2}$$

where γ is equal to 3.86 for a momentum p of 3.5 GeV/c it turns out that the "γ-transition" γ_t and the tune Q must be smaller than 2.45 and that the average value $\overline{\alpha_p}$ of α_p is equal to 4.2 m for a mean radius R of 25 m.

Injection, extraction and pre-cooling

As a general rule, all the devices which act upon the injected beam must be carefully shielded in order to prevent all interference with the accumulated beam. The inflector, the pre-cooling cavities and the pick-up electrodes are equipped with shutters and placed in regions of large α_p to leave the aperture as free as possible.

The inflector is a quarter betatron wavelength ($\lambda/4$) apart from the septum magnet. A symmetrical arrangement is provided for the beam extraction. The septum magnets are in sections of small α_p to minimize the deflection angle. One of these sections accommodates the accelerating cavity which thus keeps a small aperture.

The pre-cooling cavities occupy two sections $\lambda/2$ apart so that the momentum cooling does not increase the amplitude of the horizontal betatron oscillations.

Stack cooling

The density varies by a factor 10^4 from the top to the bottom of the stack. The range of the gain of the amplifiers is so wide that a universal system is not possible. The low and high density parts are cooled independently by high and low gain systems respectively. The pick-up electrodes must have a great sensitivity and a high resolution in momentum; their ideal place is in the neighbourhood of the F quadrupoles of the normal periods where the ratio $\alpha_p/\sqrt{\beta_v}$ is the highest. The cooling cavities are in

zero α_p sections. The betatron oscillations are un-affected if α_p is strictly zero and the tolerance is so tight that in addition to the two families of sex-tupoles which are normally contemplated for the correction of chromaticity, a third family has been found to be necessary to control the off-momentum orbit distortion.

While the horizontal betatron cooling can be com-bined with the momentum cooling, an extra straight section is required for the pick-ups of the betatron cooling.

Choice of the lattice

The large values of α_p and the large momentum acceptance lead to a considerable horizontal aperture. A compromise has to be kept between the number of long straight sections, the length of a period and the strengths of the dipoles.

Given a tune smaller than 2.4, the schemes of Table 2 are consistent with lattices of twofold symmetry comprising 4, 5, 6 and 7 periods per super-period. Amongst them, the scheme 2.1 is the best. The dipoles of the normal period can be made short enough and thus be moved towards the D quadrupoles, so space is left free for the stack cooling pick-ups. The new period is drawn in Figure 4.
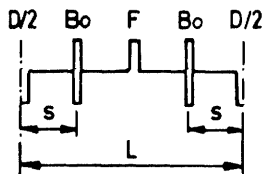

Figure 4. New configuration of the normal period

Its phase advance remains unchanged and the parameters relevant to the dispersion suppressor are:

$$\alpha_p = \frac{1 - x\left[1 - 2(s/L)\right]}{2x^2} \, L\phi_0 \, , \qquad \phi_1 = 0$$

$$\phi_2/\phi_0 = \frac{4(x + 1) \, \left[1 - x(1 - 2s/L)\right]}{x + 2}$$

The sum of the bending angles is 2π so we have a second relation to determine ϕ_0 (25.5dΩ) and ϕ_2 (39dΩ).

The parameters calculated with the thin lens approximation change only slightly for magnets of fin-ite length. The strong horizontal edge focussing of the bending magnet is avoided by making the end plates of the dipoles parallel; α_p and β_h are preserved at the expense of β_v which reaches 35 m at the maximum.

The lattice layout and the characteristic func-tions are drawn in Figures 5 and 6.

Conclusion

A review of dispersion suppressors using two periods of a FODO lattice and specific dipole strengths has been presented. The variety of possible schemes is such that the numerous require-ments imposed by the method of the stochastic cooling to the lattice of an antiproton accumulator can be met by this type of structure associated with

regular periods. As a final remark, it is worth mentioning that it was realized in the course of the design that intra-beam scattering sets a basic limit to the final momentum spread of the stack. This limit might possibly be overcome by a lattice in which the "γ transition" is imaginary. Whether the approach followed in this paper would still be suitable or not is an open question.
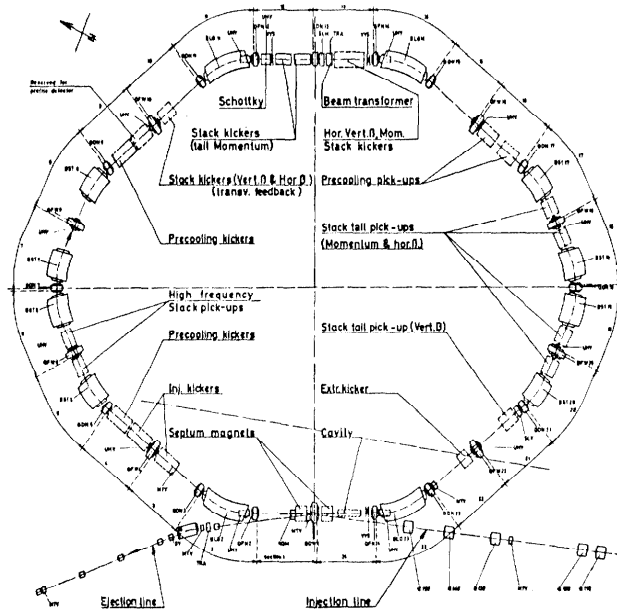

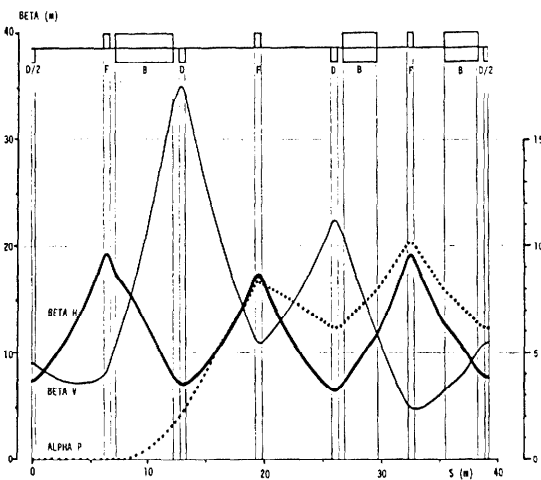Figure 5. Layout of the $\bar{p}$ accumulator

Figure 6. Lattice functions: $\beta_h, \beta_v, \alpha_p$

1. See for instance: E. Keil, Theoretical aspects of the behaviour of beams in accelerators and storage rings, CERN 77-13.

2. M. Veltman, SCHIP, CERN Program Library (R201).

3. P. M. Hanney, Private communication.

4. R. Billinge and M. Crowley-Milling, The CERN Proton-Antiproton Colliding Beam Facilities, (Proceedings of this conference).