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ENHANCED RESISTIVE WALL INSTABILITY FOR OFF-CENTERED BEAMS*

E.D. Courant,[a] M. Month, C. Pellegrini and J.M. Wang[a†]

Summary

Beam occupation of a large fraction of the available vacuum chamber, typical of high energy proton storage ring designs, results in an enhancement of the resistive wall instability. The effect is considered for ISABELLE during the current stacking procedure. Results for the coasting stack in its initial phase as well as for the injected bunches are presented.

1. INTRODUCTION

High energy proton storage rings are designed to make maximal use of the available vacuum chamber aperture. This is dictated primarily by economic considerations. The accumulation of current in a typical high energy ring creates a rather unusual beam configuration. In particular, we can have a ribbon beam in a circular chamber set off the central axis toward one side of the chamber in the median plane. It might be anticipated that such a condition could produce an enhanced resistive wall instability. Since the threshold is a strong function of the chamber radius, we could even guess, a priori, that for an off-centered beam, the threshold would significantly decrease and be roughly related to an "effective radius" which is simply the distance of closest approach to the chamber.

To use the chamber aperture optimally for high current accumulation, we would like to be able to position the beam as close as possible to the chamber on one side and to complete the ribbon with continued addition of injected pulses. The total beam within the chamber, therefore, has a variety of phases: 1) Injected bunch, 2) "Small" stack-close to wall, 3) Wide ribbon-off center, and 4) Very wide centered ribbon for bunched stack. In the latter two phases, the standard treatment of the resistive wall instability,[1] leading to a dispersion relation for the coherent frequency shift, is not adequate. Since the induced fields are sensitive to beam position within the vacuum chamber, the resulting coherent modes have variations across the beam width. In other words, there is a coupling of one part of the beam to another through the image fields. This problem which involves the solution of an integral equation for the coherent oscillation mode has been treated by E.D. Courant and M. Month.[2] They show that even for a wide ribbon there is a significant effect, but that the enhancement tends to be most severe for a narrow off-centered beam. We will, therefore, restrict ourselves to the case of a small stack.

We treat, therefore, the two phases, that of a "small" stack close to one wall, corresponding to the initial period of the stacking process, and that of the injected bunches close to the other wall. The transverse aperture is shown in Figure 1. The case of the small coasting stack is presented in section 2, while that of the injected bunched beam is detailed in section 3. ISABELLE[3] parameters are used throughout. Some implications for the ISABELLE design resulting from the resistive wall enhancement are discussed in section 4.

2. NARROW COASTING STACK

The development of coherence in the beam is inhibited by a frequency spread. We take this spread to arise from the spread in linear betatron tune.

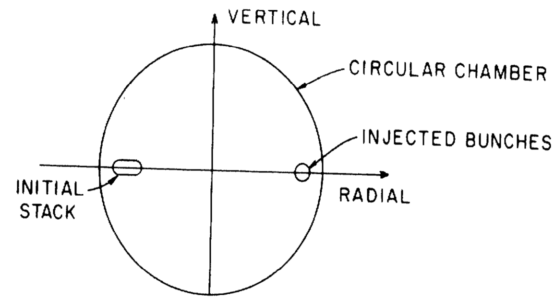

Figure 1. Geometry of stacking, transverse aperture. Bunches are injected at one edge of the circular chamber aperture and accumulated in the stack region on the other side of the chamber. The initial stack is a narrow ribbon.

In terms of a normalized tune variable, u, we can show by standard methods[1,2] that the dispersion relation for the coherent frequency, u_c, can be written

$$1 = G \int \frac{\rho(u)}{u-u_c}\, du \tag{1}$$

The variable u is related to the betatron tune ν by $u = (\nu-\nu_o)/d$, where ν_o is the central tune and d is the ½-width of the tune distribution. For a finite width, ± d, we have, range $[u] = \pm 1$. u_c is related to the coherent frequency ω by $u_c = [n-\nu_o - \omega/\omega_o]/d$, with ω_o the revolution frequency (rad/sec) and n is the azimuthal mode number [n (integer) > ν_o]. For $\rho(u)$ normalized to unity, the quantity coupling the beam to itself, G, can be expressed by

$$G = \left(\frac{I}{ed}\right)\frac{r_p R}{\gamma\omega_o\nu_o}\left\{\frac{1}{\beta^2\gamma^2}\left[-\frac{1}{\Delta h}\tan^{-1}\frac{\Delta}{h} + \frac{1}{b^2}\frac{1}{(1-t_o^2)^2}\right] + (1+i)\frac{\delta_{eff}(1+t_o^2)}{b^3(1-t_o^2)^3}\right\}. \tag{2}$$

Here, I is the coasting beam current,
d is the tune spread ½ width,
R is the average radius for the beam orbit,
β,γ are the beam velocity (with respect to c) and beam energy (with respect to the rest energy) respectively,
ν_o is the central betatron tune,
$\omega_o = 2\pi f_o$, with f_o the revolution frequency,
r_p is the classical proton radius (1.54×10^{-18}m),
Δ is the beam ½-width,
h is the beam ½-height,
b is the chamber radius,
t_o is the beam position in the vacuum chamber as a fraction of the chamber radius, and
δ_{eff} is the effective skin depth at the frequency for mode n, $f_r = (n-\nu_o) f_o$.

The beam position is at x_o, and $t_o = x_o/b$.

The time evolution of the "unstable mode" has been taken to be of the form $e^{-i\omega t}$; we thus seek solutions with $Im(\omega) \geq 0$. When it exists $Im(\omega)$ is just the e-folding growth

*Work performed under the auspices of the U.S. Department of Energy.
†Brookhaven National Laboratory, Upton, NY 11973
[a]SUNY at Stony Brook.

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rate, $Im(\omega) = 1/\tau \geq 0$. The threshold is the limit of zero growth rate, i.e., $Im(\omega) \to 0^+$. Since u_c and ω differ in sign, a threshold solution to the dispersion relation (1) is found by taking the limit $Im(u_c) \to 0^-$.

In ISABELLE, the frequency for the lowest unstable mode ($n = 23$, $\nu_o = 22.62$, $f_o = 79.64$ kHz) is f_n = 30.26 kHz. At this frequency the skin depth for stainless steel, $\delta_1 = 2.89$ mm and for copper, $\delta_2 = 0.38$ mm. These are interesting in that the ISABELLE vacuum chamber is a double walled structure, with the inner stainless steel thickness, 1 mm. Thus, the fields at this low frequency penetrate through the stainless steel into the copper where they are attenuated. In this case, the skin depth is a complex number. Satisfying the appropriate boundary conditions at the stainless steel surface as well as at the stainless steel-copper interface, we can find the effective skin depth in general:

$$\delta_{eff} = \delta_1 \left[\frac{\delta_1+\delta_2 - (\delta_1-\delta_2)\, e^{-2\kappa_1 q}}{\delta_1+\delta_2 + (\delta_1-\delta_2)\, e^{-2\kappa_1 q}} \right] \qquad (3)$$

where q is the stainless steel thickness, and $\kappa_1 = (1-i)/\delta_1$. For the ISABELLE parameters listed in Table I, we find for the lowest n = 23 mode, $\delta_{eff} = (1.414 - 0.816i)$ mm.

Table 1 General ISABELLE Parameters

Average radius,R	599.5 m	$C = 4\ 2/3\ C_{AGS}$
Central tune,ν_o	22.62	
Revolution frequency, f_o	79.64 kHz	$f_o = \beta c/2\pi R$
Beam energy,γ	31.3	$\gamma = E/mc^2$(E=29.4 GeV)
Chamber radius,b	4.4 cm	
Resistivity, ss, ρ_1	1.0×10^{-6} Ωm	
Resistivity, Cu, ρ_2	1.7×10^{-8} Ωm	Layer outside ss chamber
Thickness, ss layer,q	1.0 mm	
Azimuthal mode, n	23	n = 24,25......also possible
Resonant frequency for mode n, f_r	30.26 kHz	$f_r = (n-\nu_o)f_o$ (n=23)
Skin depth,ss,δ_1	2.89 mm	$\delta = (\rho c/\pi Z_o f_r)^{1/2}$(n=23)
Skin depth,Cu,δ_2	0.38 mm	$Z_o = 120\ \pi\ \Omega$
Thickness, Cu layer	∞	Effectively,thickness $\gg \delta_2$
Complex effective skin depth,δ_{eff}	(1.414 -0.816i) mm	Eq. (3) (n=23)

The ISABELLE beam parameters are given in Table II while the "small" beam is defined in Table III.

The threshold depends on the "current-tune" density, $i = I/2d$. This quantity has a design value essentially independent of the amount of current stacked. We can therefore use this value even in the case of a 20% stack. To be definite we define a normalized current, $\eta = i/i_o = I/I_o$, where the value $\eta = 1$ corresponds to the threshold for a small centered beam with a uniform tune distribution. Introducing η explicitly, we can write for the dispersion relation (1),

$$1 = \eta\, \tilde{G} \int \frac{\rho(u)}{u-u_c}\, du\ , \qquad (4)$$

where $\tilde{G}$ is evaluated with the nominal current and tune spread given in Table II. Thus, we can see how the threshold changes for a different distribution or as a function of where the beam is within the chamber.

Table II ISABELLE Beam Parameters

Average vertical β, β_V	28,3 m	$\beta_V = (2/3)\beta_{min} + (1/3)\beta_{max}$
Average dispersion, X_p	2.27 m	$X_p = (2/3)X_{p,max} + (1/3)X_{p,min}$
Average horizontal β,β_H	46.7 m	$\beta_H = (2/3)\beta_{max} + (1/3)\beta_{min}$
Beam emittance, $E_V = E_H$	$15\pi \times 10^{-6}$ rad-m	
Vertical ½-size,h	3.68 mm	$h = (E_V\beta_V/\pi\gamma)^{1/2}$
Horizontal,betatron ½-size,Δ_β	4.73 mm	$\Delta_\beta = (E_H\beta_H/\pi\gamma)^{1/2}$
Nominal full beam current, I	8 A	
Nominal full tune spread, ± d	$\pm 9.2 \times 10^{-3}$	
Normalized current-tune density,i_o	433.5 A/unit tune	$i_o = I/2d$

Table III Beam Parameters with Point Beam Model

Beam current, I	1.6 A	20% Nominal full current
Tune spread, ±d	$\pm 1.84 \times 10^{-3}$	20% Nominal spread
Momentum spread, $\pm \delta_p$	± 0.1%	20% Nominal stack spread
Momentum ½ size, Δ_p	2.27 mm	
Horizontal ½ size,Δ	7.0 mm	$\Delta = \Delta_\beta + \Delta_p$

In Figure 2, we plot the threshold current, $\eta(t_o)$, as a function of position in the chamber.

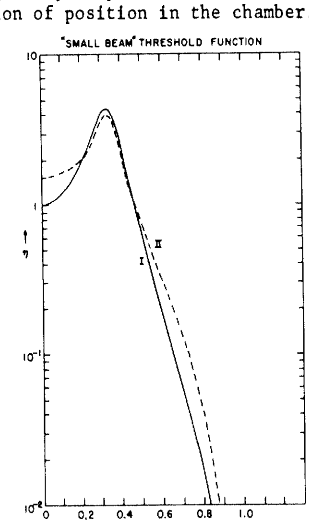

Figure 2. Current threshold function for "small beam" case. η is normalized to 1 at chamber-center with uniform distribution. t_o is ratio of distance off center to chamber radius. I. Uniform density distribution. II. Equal $\sin^2$ flanks: 80% flat.

The curves in this figure are thresholds in the sense that $Im(u_c) = 0^-$. Each solution corresponds to a frequency shift function, $u_R(t_o) = Re(u_c)$, when $Im(u_c) = 0^-$. This frequency shift function is plotted in Figure 3.

Thus, we see that for the "small" coasting stack, the threshold at the chamber center is determined by the beam self-field term (the so-called capacitive term). As we move off center, the image fields tend

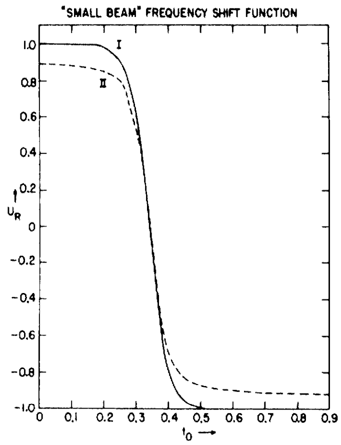

Figure 3. Frequency shift function for "small beam" case. t_o is the ratio of distance off center to chamber radius. I. Uniform density distribution. II. Equal $\sin^2$ flanks: 80% flat.

to cancel the self-field term. However, the resistive term growth lags behind the cancellation; therefore, the threshold temporarily increases. The cancellation is complete and the threshold peaks when the real frequency shift goes through zero. After this the wall contribution dominates and the threshold rapidly drops. These effects can be seen qualitatively from an examination of the coupling function, G, given in Eq. 2.

The fast decrease in the threshold is impressive, with the current down by two orders of magnitude at a point between 10% and 20% from the wall, i.e., 5 mm to 10 mm from the wall.

To estimate the growth rate, i.e., a measure of the time scale of growth that can be expected under unstable conditions, we go back to the dispersion relation (1) and take a delta function distribution, $\rho(u) = \delta(u)$. Thus, we find for the growth rate $1/\tau = \omega_o d \, Im(G)$. This is plotted in Figure 4 as a function of t_o.

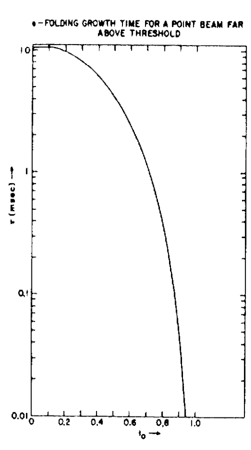

Figure 4. e-folding growth time for a point beam far above threshold. The current I = 1.6 A (20% of ISABELLE full current), τ is in units of msec. t_o is the ratio of the distance off center to the chamber radius.

This quantity gives us a feeling for the demands on a feedback system to control growing oscillations if Landau damping from tune spread is insufficient.

3. INJECTED BUNCHED BEAM

The ISABELLE current accumulation process involves the injection of a bunched beam with ~ 4.5 mHz bunch frequency from the AGS.[3] Using five pulses from the AGS, the ISABELLE injection orbit is filled with 57 equally spaced bunches. Assuming these bunches to have equal intensity, there are 57 normal modes of transverse coherent oscillations. The normal mode frequencies are given by[4] $\omega_s = \omega_o (\ell M + s - \nu_o)$ with s = 1,.......M. Here, M is the number of bunches (M=57 in the ISABELLE case treated) and s is the mode number; ν_o is the central betatron frequency for the bunch, ω_o is the revolution frequency in radians/sec, and ℓ is any integer.

For each normal mode, we have a dispersion relation similar to (1),

$$1 = G_B \int_{-1}^{1} \frac{\rho(u)}{u-u_c} \, du \tag{5}$$

Here, $u = (\nu-\nu_o)/d_B$, with d_B the half width of the tune distribution. This distribution is taken to be parabolic: $\rho(u) = 3/4\,(1-u^2)$. The normalized coherent frequency u_c is given by $u_c = [\ell M + s - \nu_o - \omega/\omega_o]/d_B$. The quantity G_B takes into account the bunch structure of the beam: $G_B = N_B\,(U_s + i\,V_s)/d_B$, where N_B is the number of particles per bunch and

$$U_s + i\,V_s = -\frac{\delta\nu}{N_B}\left(1 - \frac{h^2}{b^2(1-t_o^2)^2}\right) + (1+i)\,A \sum_{\ell=-\infty}^{\infty} \delta_{eff}(\omega)\,\frac{1+t_o^2}{b^3(1-t_o^2)^3}\,. \tag{6}$$

The direct beam term is written in terms of the beam space charge tune shift, $\delta\nu$. For an elliptical beam of semiaxes Δ, horizontal, and h, vertical and for a parabolic density distribution, the vertical tune shift is given by

$$\delta\nu = \frac{2 r_p RMN_B}{\pi\gamma^3\nu_o\, h(\Delta+h)B}\,, \tag{7}$$

where B is the bunching factor, bunch length/bunch separation, $B = ML/2\pi R$, with L the bunch length. Notice that the bunch is taken to be parabolic in density, while the stack density is almost uniform (we have taken 20% $\sin^2$ tails). The quantity A is given by,

$$A = \frac{r_p\, RM}{2\pi\,\gamma\,\nu_o}\,,$$

while δ_{eff} is the effective complex skin depth defined by (3) for the double walled chamber. It is evaluated at the unstable frequency, $\omega \approx (\ell M + s - \nu_o)\omega_o$.

In obtaining the dispersion relation (5), we have taken the betatron frequency spread to be associated with an "external" variable, the momentum spread. This provides Landau damping. In the bunched beam case this is only valid if we are dealing with a growth condition occurring on a time scale sufficiently shorter than the synchrotron oscillation period. A quantitative criterion for this is that the zero frequency spread growth period, τ_g, be larger than the synchrotron period, τ_s: that is, $\tau_g < \tau_s$. We will see that this condition is satisfied in our case.

In evaluating the image contribution to G_B [the second term on the rhs of (6)], we have taken a "point" bunch. While the imaginary part of $\sum_{\ell} \delta_{eff}(\omega)$ is convergent, the real part is divergent. For a bunch of nonzero length, the skin depth δ_{eff} must be multiplied by a phase factor which ensures convergence. We can take account of the nonzero bunch length in a simple manner by introducing a cutoff in the sum. We terminate the series at ℓ_c, corresponding to the inverse of the bunching factor: $\ell_c = 1/B$. This is equivalent to limiting the contribution to only those frequencies with wavelengths longer than the bunch length, a valid approximation for studying rigid bunch motion.

The rise time τ_g of an unstable oscillation is determined by the term V_s: $1/\tau_g = \omega_0 d_B \, \mathrm{Im}(G_B) = \omega_0 N_B V_s$. For the most unstable mode for which $s \sim \nu_0$, we have $\tau_g = 50$ msec (on axis, $t_0 = 0$). This is enhanced by a factor of 35 at $t_0 = 0.8$, giving $\tau_g = 1.4$ msec. Even the growth period at the chamber center is about four times shorter than the synchrotron period for ISABELLE at injection which is about 180 msec. See Table IV.

Table IV Injected Bunch Parameters

Energy (GeV)	$E = 29.4$
Number of protons/bunch	$N_B = 2.5 \times 10^{11}$
Horizontal emittance (rad-m)	$E_H = 15\pi \times 10^{-6}$
Vertical emittance (rad-m)	$E_V = 15\pi \times 10^{-6}$
Longitudinal emittance (eV-sec)	$E_L = 1.0$
Beam height (cm)	$2h = 0.58$
Beam width (cm)	$2\Delta = 0.78$
Momentum spread	$\Delta p/p = 10^{-3}$
Bunch length (m)	$L = 10$
Synchrotron frequency/ω_0	$\nu_s = 6.9 \times 10^{-5}$
Synchrotron period (sec)	$\tau_s = 0.18$

The dependence of τ_g on t_0 is, in fact, the same as for the coasting stack, Figure 4. However, the magnitude of τ_g should be increased by a factor of 5.

Thus, we have that Landau damping from linear tune spread will be effective, as we have assumed in writing the dispersion relation (5). This tune spread arises from bunch momentum spread and the ISABELLE chromaticity. With an overall momentum spread in the bunch of 0.1% and a chromaticity of $\xi = 2$, we have a frequency spread $d_B = \pm 1 \times 10^{-3}$. Taking a parabolic distribution, we can calculate the threshold for transverse coherent rigid bunch oscillations. The result is shown in Figure 5 where the threshold, in terms of the maximum number of particles per bunch, N_{th}, is given as a function of the ratio of x_0, the distance of the bunch center from the chamber center to b, the chamber radius: $t_0 = x_0/b$. The peak in N_{th} at $t_0 \approx 0.76$ corresponds to U_s passing through the value zero, meaning a cancellation of the direct space charge with the reactive part of the image contribution. Once this cancellation occurs, the image term dominates and the enhancement is rapid and evident in the figure. The bunch parameters used in the calculation are given in Table IV.

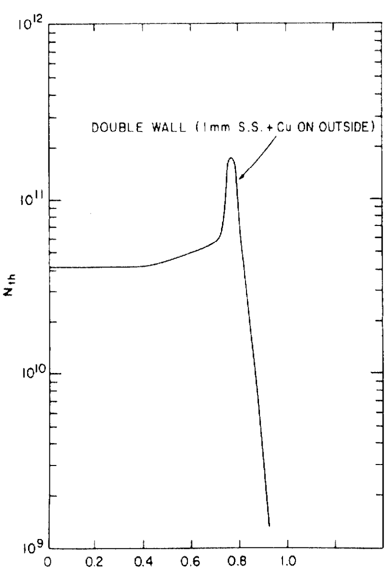

Figure 5. Threshold for injected bunched beam in I ISABELLE. N_{th} is the maximum allowable number of protons per bunch without feedback. t_0 is the ratio of the distance of the bunches off center to the chamber radius.

4. CONCLUSIONS

We have reviewed the impact of the resistive wall instability as it relates to the use of phase displacement stacking. Specific computations for ISABELLE are presented. The essential features which cause an enhancement of the instability are (1) the use of a large part of the chamber aperture to accumulate large currents and (2) the injection of bunches with high transverse density for the purpose of optimizing the luminosity of the colliding beams.

The initial coasting stack is set off the chamber center close to the wall. For the nominal allowed tune spread, a large reduction in the threshold current results with the details depending sensitively on how close to the wall the first pulses are stacked. A factor of 10 to 100 decrease in threshold occurs if 80%-90% of the aperture is to be used. e-folding growth rates tend to be high under unstable conditions ($\tau \sim 1$ msec to < 0.1 msec).

Because of the high transverse density of the injected bunches, the threshold intensity, N_{th}, the maximum number of particles per bunch, is low. Even without the wall enhancement, the threshold is $N_{th} \approx 4 \times 10^{10}$ protons/injected bunch. This is well below the design value of 2.5×10^{11} protons per bunch. At the injection orbit, about 80% to the wall, the growth periods are short, $\tau_g \sim 1$ msec.

One way to increase the threshold is to increase the betatron frequency spread. This may be accomplished by increasing the chromaticity from its design value with a sextupole adjustment or by adding an octupole term, which introduces a tune spread with betatron amplitude. This means modifying the ISABELLE design so as to extend the available betatron tune working line. We might mention that an octupole distribution could produce "curvature in the working line" and thereby cause localized beam instabilities (brick-wall effect).

Another possibility to cope with the unstable conditions is to employ a direct feedback system. Such a system should provide coherence damping on a time scale less than 1 msec and should also be capable of acting on each injected bunch independently. Furthermore, the injected bunches and coasting stack must exist simultaneously, thus necessitating a combined feedback process. Finally, the injected bunches will have finite initial coherent amplitudes due to injection errors, therefore, placing an increased demand on the feedback system in both speed and strength.

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