ELECTRON MODEL EXPERIMENT TO STUDY INSTABILITIES IN LONG PERIODIC
FOCUSING SYSTEMS FOR INTENSE BEAMS*

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Abstract

The proposed use of intense beams of high-energy
heavy ions for pellet fusion have renewed interest in
the problem of space-charge limits in long periodic
focusing systems. Recent analytical and numerical
studies at Berkeley and NRL for beams with a K-V dis-
tribution indicate that instabilities may develop in
such systems when the beam currents exceed certain
thresholds. Since no relevant experimental data exists
with which to compare the theory, we propose an
inexpensive, small-scale experiment with a 5-20 kV
electron beam to study this problem. The electron beam
is injected into a periodic solenoid focusing system
consisting of 20 solenoid lenses initially. The per-
tinent results of beam transport theory are reviewed
and the design features of the electron beam experi-
ment are presented.

1. Introduction and Motivation

Recently, in connection with studies of heavy-ion
accelerators for pellet fusion, instabilities in beam
transport systems related to perturbations in the
space-charge potential distribution have become a focus
of interest. This particular type of instability had
first been observed a decade ago by the late
R. Chasman[1] in computer simulation studies of the
proton beam in the initial stages of the Brookhaven
200-MeV linac. Chasman found that space-charge related
effects caused a substantial growth of beam emittance
(by factors of 2 to 3), most of which occurred in the
initial sections of the quadrupole FODO lattice of the
linac. Subsequently, R. Gluckstern[2] developed an
analytical theory of the oscillation modes in a round
beam propagating through a long, uninterrupted, solen-
oidal focusing field.

The proposed use of heavy ion accelerators for
pellet fusion led to renewed interest in the problem of
high-current beam transport during the past two years.
Maschke[3] derived a formula for the maximum beam
current that can be transported in a FODO channel. An
analytic theory was developed by Reiser,[4] which clari-
fied questions concerning this formula and the scaling
laws[5] for heavy ion beam transport. In detailed
studies, Laslett, Smith, and collaborators[6] at the
Lawrence Berkeley Laboratory have been examining the
stability problem in FODO, interrupted solenoid, and
long (continuous) solenoid channels, following the
approach developed previously by Gluckstern. Parallel-
ing this work are numerical simulation studies by Haber
(in collaboration with Maschke)[7] and also by Penner.[8]
Haber studied beam propagation through FODO channels
(consisting of thin quadrupole lenses) using up to
1.6×10^4 macroparticles, and, like Chasman, he
observed emittance growth by a factor 2 to 3 for vari-
ous parameter regimes. Good agreement was found between
his results and the results for unstable modes and
growth rates obtained analytically by the Berkeley
group. The numerical simulation revealed, furthermore,
that the instabilities saturate after an initial growth.

All of this theoretical work so far is based on
the K-V distribution, the only known phase-space dis-
tribution that leads to linear force equations for the

equilibrium beam. At present, no experimental data
exists with which to compare the theory; thus, a
systematic experimental investigation of the space-
charge related instabilities is called for.

The theoretically predicted growth rates for the
instabilities depend on the beam intensity. As the
current rises above the stability threshold, the growth
rate rapidly reaches a maximum value, then decreases
with increasing current. At or near the maximum, it
typically takes only a few magnet periods of the
focusing channel to obtain an e-folding growth in
amplitude, whereas at higher beam intensities, it takes
an increasingly longer distance for the instability to
grow. Thus, for a systematic experimental study of the
instabilities, one needs a sufficiently large number of
magnet periods somewhere between 10 and 100. For ion
beams, such a channel becomes prohibitively long and
expensive. It is for this reason that we propose to
use electrons to study the physics of beam transport in
periodic magnetic fields.[9] In view of the compact size
(~3 m for 20 magnet periods) and low costs, an electron-
beam model is a natural choice for such an experiment,
and the results can be scaled to large magnet systems
for any ion species from protons to uranium ions.
Before describing the features of the proposed elec-
tron-beam facility, we will present in the next section
a brief review of the theory of beam transport and
instabilities.

2. Brief Review of Beam Transport Theory

The theory of periodic focusing was first devel-
oped by Courant and Snyder[10] and later extended by
Kapchinskij and Vladimirskij[11] to include the effects
of space charge. The latter introduced a phase-space
distribution function, now generally referred to as the
"K-V distribution," which has the property that its
projections in any two-dimensional plane (x-y, p_x-x,
etc.) are ellipses with uniform particle density.

In the absence of space-charge forces, individual
particles in a beam propagating through a periodic
focusing system perform oscillations with wavelength λ_o.
The phase shift μ_o defines the fraction of a particle
oscillation (for zero space charge) that occurs in one
period S of the lattice of periodically spaced lenses.
According to the theory, stable motion exists only for
$|\cos \mu_o| \le 1$, i.e., $\mu_o \le 180°$. Space-charge effects
result in a larger wavelength $\lambda > \lambda_o$, smaller phase
shift $\mu < \mu_o$ per lattice period. The first studies of
the Berkeley group showed that, due to the space charge,
parametric instabilities of the beam envelope occur for
$\mu_o > 90°$. These envelope instabilities (which had been
observed already in much earlier work[12] on periodic
focusing of electron beams) then limit the operation of
the system to $\mu_o \le 90°$. But even in the region below
90°, instabilities were found that are due to perturba-
tions in the potential distribution of the beam. In
the undisturbed K-V beam, the potential is quadratic in
x and y. The Berkeley group examined perturbations in
the space-charge potential, which involve terms of
higher than quadratic order in x and y and obtained the
following general results:[13,14] (1) Modes of 4th and
6th order perturbations lead to instabilities in FODO
channels, interrupted solenoid channels, and long
solenoids, which can be avoided by operating in the
range $\mu/\mu_o \gtrsim 0.4$, the maximum current being given by

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$\mu_0 = 90°$ and $\mu = 36°$. (2) Modes of third order cause instabilities in FODO and interrupted solenoid channels at even lower beam intensities. (They are absent in long solenoids.) Specifically, for $\mu_0 = 90°$, these third-order instabilities can be avoided only by operating the system at values $\mu \geq 57.5°$. On the other hand, third-order instabilities do not occur when $\mu_0 \leq 60°$. (3) Below $\mu_0 = 60°$, the intensity limit is determined by the fourth and fifth order modes, i.e., the region $\mu/\mu_0 \geq 0.4$ is stable according to (1). The Berkeley group recommends that $\mu_0 = 60°$, $\mu = 24°$ be chosen as operating parameters since the current in this case is larger than for $\mu_0 = 90°$, $\mu = 57.5°$.

Let us now briefly discuss the implications of these results for the design of a focusing channel in the framework of the smooth approximation theory,[4] which is valid in the region $\mu_0 < 90°$ for high-current beam transport. In solving the K-V envelope equations for a matched beam, it is convenient to introduce in the space-charge term the "generalized perveance" $K = (I/I_0)2\beta^{-3}\gamma^{-3}$, where I is the beam current and $I_0 = 4\pi\varepsilon_0 m_0 c^3/q$ is the Alfvén-Lawson limiting current, i.e., 3.1×10^7 A/Z amperes for ions of charge number Z and mass number A, and 1.7×10^4 amperes for electrons. One then obtains for a matched beam the relation[4]

$$K = \frac{I}{I_0}\frac{2}{\beta^3\gamma^3} = \frac{\mu_0}{S}\alpha[1 - (\varepsilon/\alpha)^2] , \qquad (1)$$

where $\alpha\pi$ is the acceptance of the channel for zero space charge and $\varepsilon\pi$ is the emittance. If ℓ denotes the length of a magnet, L the free space between magnets, the period of a FODO channel is $S = 2\ell(1+L/\ell)$, and the strength of the magnets is measured by the focusing parameter $\Theta = (qB_0/m_0 c\beta\gamma a)^{1/2}\ell$. The phase shift μ_0 is a function of Θ and L/ℓ. The acceptance parameter α, on the other hand, is defined as $\alpha = a^2/w_{0,max}^2$, where $w_{0,max}$ denotes the maximum of the characteristic amplitude function $w_0(s)$ for the periodic channel in the case of zero space charge. Like μ_0, $w_{0,max}$ is also a function of Θ and L/ℓ that can be calculated analytically. The phase shift μ of the particle oscillations and the amplitude function $w(s)$ with space charge can be calculated in the smooth-approximation theory by introducing the dimensionless space-charge parameter $u = KS/2\mu_0\varepsilon$. One obtains the expressions

$$w(s) = w_0(s)[u + \sqrt{1+u^2}]^{1/2} \qquad (2)$$

and

$$\mu = \mu_0[\sqrt{1+u^2} - u] . \qquad (3)$$

Furthermore, one finds that $\varepsilon/\alpha = \mu/\mu_0$, or, in view of (3),

$$\alpha = \frac{\varepsilon}{\mu/\mu_0} = \varepsilon[\sqrt{1+u^2} - u]^{-1} . \qquad (4)$$

If μ_0 is given and μ/μ_0 is fixed due to instabilities, then all the other parameters are uniquely determined for any given choice of the lattice ratio L/ℓ. One then obtains for the beam current with ions of charge number Z and mass number A the relation (MKS units)

$$I_{[A]} = 3.64 \times 10^6 C\left(\frac{Z}{A}\right)^{1/3}(\beta\gamma)^{7/3}B_0^{2/3}\varepsilon^{2/3} , \qquad (5)$$

which is in the form first suggested by Maschke.[3] The constant C is identical with the "figure of merit" (FM) of the Berkeley group. From the smooth-approximation theory, one obtains the result

$$C = FM = \frac{\mu_0}{1+L/\ell}\frac{1 - (\mu/\mu_0)^2}{(\mu/\mu_0)^{2/3}}\frac{1}{\Theta^{4/3}\left(\dfrac{w_{0,max}}{\sqrt{\ell}}\right)^{2/3}} . \qquad (6)$$

For a given choice of μ_0 and L/ℓ, the parameters Θ and $w_{0,max}/\sqrt{\ell}$ are fixed and the beam current in a FODO channel is seen to vary as

$$I \propto [1 - (\mu/\mu_0)^2]/(\mu/\mu_0)^{2/3} . \qquad (7)$$

In the case of interrupted solenoids, we have $S = \ell(1+L/\ell)$ and $\Theta = qB_s\ell/2m_0 c\beta\gamma$ where B_s is the solenoidal field strength. In place of (6), one obtains then for the ion current limit the relation

$$I_{[A]} = 2.48 \times 10^6 C_s \beta^2\gamma^2 B_s\varepsilon , \qquad (8)$$

$$C_s = \frac{\mu_0}{(1+L/\ell)\Theta}\frac{1 - (\mu/\mu_0)^2}{\mu/\mu_0} . \qquad (9)$$

For electrons, the numerical factor in (8) is different and we have

$$I_{[A]} = 1.36 \times 10^3 C_s \beta^2\gamma^2 B_s\varepsilon . \qquad (10)$$

3. Description of Proposed Facility

We propose to study the physics of beam transport in long periodic focusing systems for conditions where the space charge of the beam has a significant effect on the particle motion. By long systems, we mean a sufficiently large number of focusing lenses that instabilities of the type discussed above can be observed experimentally. A number of 20 periods appears to be a good initial goal, with more lenses to be added later if desired. By significant space-charge effects, we mean that the self fields of the beam increase the wavelength of the particles' oscillations by a factor 2 or more, i.e., $\mu/\mu_0 < 0.5$ or $u > 0.75$. In addition, one wants to be able to change parameters over as wide a range as possible to study, for instance, how the instabilities and the associated growth rates vary with the space-charge intensity. The proposed facility will satisfy these requirements. Electron beams generally have a very small emittance (or transverse temperature), i.e., u is significantly larger than 0.75. However, we plan to decrease u by passing the beam through a thin wire mesh, which reduces the beam current and increases the emittance due to scattering.

A major advantage of the proposed facility is the long pulse length of the beam (design goal is between 1 and 10 μs) and the absence of accelerating structures and external electric fields. In linear accelerators, the rapid acceleration of the particles and coupling effects between longitudinal and transverse motion makes an analysis of the problem very difficult, as R. Chasman pointed out already.[1] The electron beam in the proposed experiment, on the other hand, is essentially two-dimensional so that a direct comparison between experiment and theory should not be too difficult.

Figure 1 shows a schematic layout of the proposed experimental facility. The electron gun is a scaled-down version (ratio 1:5) of the 60-kV klystron gun designed by W. Herrmannsfeldt at Stanford (SLAC) for the PEP project. It has a perveance of $k = I/V^{3/2} = 0.75 \times 10^{-6}$ (amps/volt$^{3/2}$), i.e., the current is 0.75 A at 10 kV. To reduce power consumption, a pulsed high-voltage supply will be used with a tentative pulse length of a few microseconds and a repetition rate of 60 Hz. The cathode radius is 1.3 cm and the focus occurs at approximately 6.4 cm from the cathode where the beam radius is 0.57 cm. The beam then passes through an iron plate, which shields the cathode from the solenoidal magnetic field. On the upstream side of the iron plate is the emittance/current

control mesh discussed above. On the downstream side is a special coil (or coil system) to be designed for matching the beam envelope. Following that is the periodic lattice of solenoids. We have already built three test solenoids with iron shields to obtain a well-defined, pronounced field variation. These coils are shown in Fig. 1. The measured magnetic field variation on the axis is plotted in Fig. 2(a); the length of one magnet period in this case is 15 cm. However, we plan to build the system such that the spacing between magnets can be varied easily. The preliminary design parameters for the facility are summarized in Table 1. The diagnostic elements consisting of pinhole plates, Faraday cup, and fluorescent screen will be mounted inside a diagnostics chamber from where they can be moved along or perpendicular to the beam direction. At the entrance of the diagnostics chamber is a special coil to be used for focusing the beam on the screen. Our plans are to assemble the solenoidal focusing coils in steps. Starting with a few coils, the beam profile will be carefully investigated, then more coils will be added and the measurements will be repeated at the larger distance downstream. The onset of an instability should show up in the shape of the beam cross-section profile on the fluorescent screen. When such an instability is observed, a more detailed study of the beam emittance will be made.

The main objective of this experiment is to identify all effects that contribute to a significant growth of the emittance such as beam mismatch, aberrations due to nonlinear magnetic field components, alignment errors, etc. Of particular importance is the region near the gun. A major reason for choosing the PEP gun design is the fact that it produces a low-convergence, high-quality beam. This will, hopefully, avoid the "translaminar" flow problems that Brewer found in his study of beam propagation through a solenoid with uniform field.[15] Special attention must also be given to the possibility that microwaves are generated along the beam, which could lead to deterioration of beam quality.

As an aid for the design of the facility and for proper matching of the beam, we have developed a computer program that integrates the K-V envelope equations. In our solenoid case, the envelope radius R versus distance z obeys the equation

$$\frac{d^2R}{dz^2} + \kappa_s(z)R - \frac{K}{R} - \frac{\varepsilon^2}{R^3} = 0 , \qquad (11)$$

where $\kappa_s(z) = [qB_s(z)/2m_0c\beta\gamma]^2$, and $B_s(z)$ is the magnetic field on the axis. Setting $\varepsilon = 0$ and $K = 0$, the program first calculates a single-particle trajectory to determine the phase shift μ_0. With proper values of ε and K, the matched envelope radius $R(z)$ is then calculated and plotted. Fig. 2(b) shows a plot of $R(z)$ from a computer run for the experimental field shape $B_s(z)$ in Fig. 2(a). The parameter values for this case were $\mu_0 = 90°$, $\mu \approx 30°$, peak field $B_0 = 142$ G, lens period $S = 15$ cm, electron energy $T = 10$ keV.

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TABLE 1

Preliminary Design Parameters for EBTE

Phase shift per period without space charge μ_0	60°-90°
Beam current, I	0.1-2.0 A
Electron Energy, T	5-20 keV
Mean Beam Radius, R_b	0.63-0.95 cm
Axial length of test magnet, ℓ_{exp}	6.8 cm
Magnet lattice period, S	10-20 cm (adjustable)
Maximum peak B_z field, B_0	100-300 G (adjustable)
Total number of magnets	20 initially
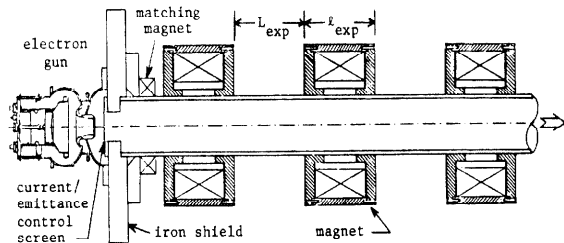
Fig. 1 Schematic of electron beam transport experiment.

(a) Axial magnetic field $B_z(z)$ [T]

Axial distance z [m] ⟶

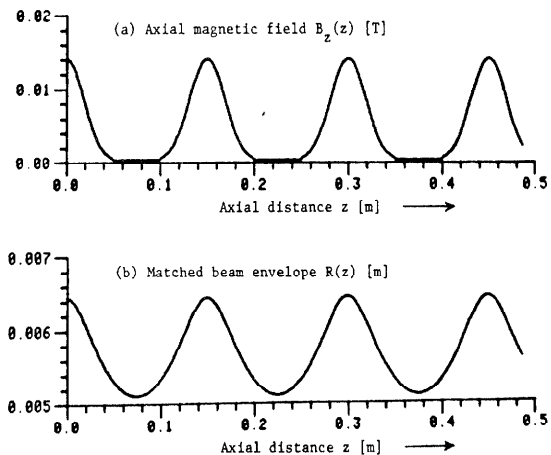(b) Matched beam envelope R(z) [m]

Axial distance z [m] ⟶

Fig. 2 The axial magnetic field along the axis for $\mu_0 = 90°$ (a) and the matched beam envelope radius for $\mu/\mu_0 \approx 0.3$ (b).