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INCOHERENT TRANSVERSE SPACE CHARGE EFFECTS DURING INJECTION, RF CAPTURE AND EARLY ACCELERATION IN AGS*

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Summary

After a review of the present situation in the AGS incoherent space charge effects in respectively the just injected beam, the coasting beam and the bunched beam are considered. Numerical estimates of shifts and spreads in betatron frequencies are given. The ratio of peak to average azimuthal densities in bunched beams is calculated for parabolic and uniform initial energy distributions. It is concluded that the intensity limit of the AGS is probably set by loss of particles out of vertical betatron phase via resonances and coupling in the early bunched beam.

Present Status of AGS

On several occasions dc beams of 2 \times 10¹³ protons (i.e. with a uniform azimuthal density of 2.5 \times $10^{1\,\rm C}$ protons/m) have been circulating in the AGS for up to a few hundred revolutions. All dc beams are eventually lost on the inner vacuum chamber wall because the rate of rise of the guiding field cannot be reduced to zero. From the loss patterns of beams with 2×10^{13} protons one concludes that such beams occupy most of the horizontal aperture. Attempts to accelerate them have consistently met with very poor capture and accelerating efficiencies, typically less than 10%. In general that efficiency decreases increasingly quickly with increasing circulating beam intensity (= intensity before capture begins); the highest accelerated intensity of 9.5 × 10^{12} protons/pulse was achieved with a circulating beam of ~ 1.6×10^{13} and 8-9 × 10^{12} is obtained with less than 1.4 × 10^{15} . The loss of rf efficiency with increasing intensity cannot be fully explained with the loss in betatron aperture due to bunching. Typical loss patterns show after small initial losses during the first 60 revolutions after injection terminates and totaling < 10% of initial intensity a sharply increased loss rate correlated with the gapvoltage amplitude program for some 200 revolutions. This takes of the order of 25%. The rate decreases to zero gradually during the following 10,000 or more revolutions. This takes another 5% so that the net efficiency is of the order of 60% for an initially circulating beam of 1.4 x 1013 protons.

The intensity of the beam to be injected has been as high as 70 mA (of 200 MeV protons), during normal operations it is somewhere between 40 and 60 mA, and no clear optimum has been established. The emittance increases somewhat with current, half of a 50 mA beam falling inside the 1π x 10^{-6} rad-m contour and 0.8 of it inside 3π x 10^{-6} rad-m. It is injected during 20-25 revolutions usually leaving some of the horizontal aperture free by having the injection closed orbit somewhat on the outside of the center of the aperture and by deliberately not injecting during the last several revolutions of the available injection period. After the first 25-30% of the injection period the circulating beam increases about linearly in intensity while injection progresses until the end of the process, suggesting that a constant charge is added each turn. This behavior does not depend very strongly on the shape of the horizontal emittance at the inflector exit (i.e. eccentricity and orientation of major axis).

Since the rate of closed orbit motion in the neighborhood of the inflector (= bump collapse rate) necessary for stacking in betatron phase space is nearly constant and chosen to have even the largest partial acceptance flooded by a properly adjusted horizontal emittance ellipse of the injected beam, one would expect the circulating beam to increase quadratically rather than linearly.

In the early part of the injection period in particular, the way in which the circulating beam increases is affected by the excitation of the two bump magnets, bending magnets, one about a quarter betatron wavelength upstream of the inflector exit, the other a quarter wavelength downstream of it, which together produce the variable bump in the closed orbit at inflector azimuth. These magnets are excited by half sinusoidal currents whose relative amplitudes and phases would seem to be important for the development of the partial acceptance. In practice timing appears to be more important than amplitude ratio and neither arrives at theoretical values during tuning for maximum intensity.

Although the injected beam is known to be 1.5-2 cm high at the exit of the inflector (calculated from emittance measurement and supported by flag observation) it increases by a factor 2-3 within the first few hundred revolutions. This is not caused by obvious vertical steering errors and/or maladjustment of the vertical emittance, and seems even fairly independent of these parameters.

The energy characteristics, i.e. central energy and distribution, of the linac beam appear to be of prime importance.

Space Charge Effects in Injected Beam

Space charge effects in the just injected beam tend to invalidate previous considerations on multiturn stacking in betatron phase space since these were based on linear transformations.^{1,2} The motion of particles inside an azimuthally short sample of the just injected beam is subject to external forces, i.e. the guiding field, the self-field of any already circulating beam, the self-field of any later injected circulating beam and to internal forces, i.e. the self-field of the sample itself. If the nonlinearities in the external forces are small enough to neglect the differences of their effects on individual particles in the sample for the several revolutions necessary to define the partial acceptance for that sample, the betatron motion of those particles may be described as the superposition of one with respect to the sample's axis and one of that axis with respect to the closed orbit. Each motion is a rotation in a properly normalized transverse phase space, the rate of rotation about the axis is less than the rate of rotation of the axis about the closed orbit in the presence of space charge. The instantaneous rotation rate about the axis is a function of the radial and axial excursion with respect to that axis. In Appendix A we calculate to first order the frequency shifts as functions of the amplitudes for a circular cylindrical beam matched to the external focusing system, with a uniform population in four dimensional phase space, neglecting image effects. Applying the result to the AGS for the injected beam, assuming a beam current of 0.8 x 50 mA inside two dimensional emittance of $3\pi \times 10^{-6}$ rad-m, one finds a frequency spectrum

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that stretches from v-0.31 for particles with negligible amplitudes with respect to the sample axis in either plane to v-0.08 for particles with maximum excursions in each plane. The first shift is twice as large as would be found for a beam of equal dimensions and current but uniformly populated in real space. Since the actual distribution in four dimensional phase space is peaked towards the center, space charge defocusing is more nonlinear than assumed and the maximum frequency shift therefore larger.

These tune shifts cause deviations from expected transverse positions on successive revolutions and make it impossible to fully populate the partial acceptances defined by linear theory. They also distort the shapes of the partial acceptances and make their boundaries vague. As a result, the acceptance of the accelerator cannot be populated to the density of the injected beam.

Because image effects were neglected in this calculation, the frequency shifts obtained are likely to be smaller than the real ones, perhaps by 20%.³

Space Charge Effects in Circulating Beam

Space charge effects in circulating beams have been treated extensively in the literature.³⁻⁸ In⁸ tune shifts were calculated for a 200 MeV proton beam of 2.5 x 10^{10} protons/m in approximated AGS geometry, assuming half cosine distributions, axially and radially, in real space. In four dimensional phase space such distributions are about 50% denser in the center than uniform ones. These calculations take electric and magnetic image effects into account. For beam dimensions of 10×6 cm² it was found that the space charge tune shifts follow from

$$\Delta v_{\mathbf{x}} = -0.053 \ (1 - 0.0965 \ \hat{\mathbf{x}}^2), \quad \hat{\mathbf{y}} = 0$$
$$\Delta v_{\mathbf{y}} = -0.283 \ (1 - 0.070 \ \hat{\mathbf{y}}^2), \quad \hat{\mathbf{x}} = 0$$

in the two planes of symmetry. Here \hat{x} and \hat{y} are the betatron amplitudes in cm (x = radial, y = axial). These results apply after the axial (= vertical) beam blow-up, mentioned earlier, has taken place. During and immediately after injection the beam should only be 1.5-2 cm high. Since the tune shifts are about proportional to charge density and the aspect ratio of the physical cross section is larger then assumed in the calculation one would expect the calculated maximum shifts Δv_y to reach -1. immediately after injection. Under such conditions rapid axial blow-up is to be expected.

Space Charge Effects in Bunched Beams

The image effects in bunched beams are complicated by the difference in boundary conditions for the dc and ac components of the self-fields. The vacuum chamber wall represents the boundary for both the dc and ac components of the electric field, as well as for the ac component of the magnetic field. The dc component of the magnetic field is bounded by the iron of the guiding field magnets and free space, provided there are no other iron structures nearby nor dc currents flowing in the vacuum chamber walls. This complication is especially noticeable in moderately bunched beams, where the ac component is not much larger than the dc one. For an intensity of 2×10^{13} in the AGS and a local-to-average density ratio equal to 2^{47} calculates

$$\Delta v_{\mathbf{x}} = -0.1435 (1 - 0.053 \hat{\mathbf{x}}^2), \quad \hat{\mathbf{y}} = 0$$

$$(v_{\mathbf{y}} = -0.519 (1 - 0.099 \hat{\mathbf{y}}^2), \quad \hat{\mathbf{x}} = 0$$

The zero amplitude shift in axial direction is very large, but reduces to -0.05 for motion of maximum amplitude (= 3 cm).

Azimuthal Density Distributions

In view of the importance of the azimuthal density distribution a computer program was written to study this in dependence of the energy distribution in the injected beam, the synchronous energy in the accelerator as determined by guiding field and rf frequency, peak energy gain per turn and rate of acceleration. Assuming adiabatic processes throughout, the program maps the (azimuthally uniform) energy distribution that exists before capture begins onto accelerating buckets with specified characteristics. Thereafter it projects the resulting distribution onto the phase (= azimuthal) axis, yielding the azimuthal density distribution.



In Fig. 1 the ratio $\hat{\lambda}/\lambda$ of the peak and average densities as function of the difference EO between the central energy in the coasting beam and the synchronous energy. Curve 1 refers to stationary buckets, i.e. to zero energy gain per turn, curve 2 to 15 keV/turn. The central energy in the coasting beam is 200 MeV, the energy distribution parabolic with a base width of 500 keV.

TABLE I

keV/Turn			
ES+EO	0	15	150
500 + 0	1.97	2.65	4.03
270 + 110	1.60	2.24	3.37
500 , Unif	1.57	2.21	3.34
920 + 0	1.97	2.32	3.19
500 + 210	. 1.55	1.88	2.57

Other results have been assembled in Table I where coljumns represent different accelerating rates and rows λ/λ as function of energy spread ES and energy offset EO. The first three rows refer all to beams with a maximum energy error of 500 keV, the first two with parabolic distributions, the third one with a uniform distribution. The last two rows refer to beams with a maximum energy error of 920 keV. In all cases the ω celerating voltage has been chosen to yield a captur efficiency between 0.99 and 1. Inspection of the results shows that capture of a beam with a parabolic distribution and an energy offset yields peak densities very nearly as low as that of an uniformly populated beam without offset and the same maximum error with respect to the synchronous energy. Both are rather lower than a parabolic beam without offset and the same energy spread as the uniform one. It may also be seen that larger energy spreads yield lower densities and that the densities increase with the rate of acceleration. Since the space charge induced tune shifts are proportional to $\lambda/(\sqrt{\gamma^2-1})$ it follows that for constant shift λ may increase as $\sqrt[4]{\gamma^2-1}$ in first order. The relationship differs from $\lambda/(\beta^2\gamma^3)$ which might be expected because the area of the beam cross section decreases as $\sqrt{\gamma^2-1}$. It allows to calculate the acceptable increase in the rate of acceleration with energy.

Discussion and Conclusion

It follows from the calculations presented that transverse space charge effects play already a fairly severe role in the early part of the cycle of the AGS at normal intensities (up to 8.5 $\times 10^{12}$ accelerated). In order to limit the shifts in the vertical betarron frequencies to $|\Delta y| \leq 0.3$ the azimuthal peak charge density must remain below $\sim 3 \times 10^{13}$ protons/m. Since it seems possible to limit λ/λ to < 2.5 by proper rf techniques this corresponds with an acceleratable beam of $\sim 1.2 \times 10^{13}$ protons. Assuming that these techniques yield also capture efficiencies of the order of 80%, an accelerated intensity of 1.2×10^{13} would require a coasting beam of 1.5×10^{13} . Beams with such intensities have been fairly common practice for some years now. They might still be unsuitable in terms of their cross sectional charge distributions however.

Whether beams with frequency shifts up to 0.3 can be maintained during early acceleration depends upon the amplitude growth rates as set by the stopband structure and horizontal/vertical coupling. Since both betatron frequencies of most particles are modulated by the differences of their averages and in addition by twice a synchrotron frequency, many particles will be driven repeatedly across resonances and change their betatron amplitudes. $^{7}\,$ Such changes will modify the cross sectional charge distributions and the betatron frequency distributions with them. They may also modify details of the stopband pattern because part of the pattern is generated by the beam itself via image effects⁶ in azimuthally non-uniform structures close to it (e.g. injection and extraction magnets, ferrite structure of bump magnets, pick-up electrodes, variations in vacuum chamber cross section).

It appears that maximizing the accelerated intensity depends on several factors: Control of injection and rf capture to minimize the peaks in the density distribution, reduction of stopband widths and horizontal/vertical coupling to reduce growth rates, control of acceleration such that a maximum number of particles survives the process.

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Appendix A

Betatron Tune Shifts in Cylindrical Beam

Assuming a spherical distribution in four space with uniform density d and radius ρ one obtains for the density $\bar{\lambda}(\mathbf{r})$ in real space $\bar{\lambda}(\mathbf{r}) = d\pi (\rho^2 - r^2), \quad \mathbf{r} < \rho$ = 0 $\mathbf{r} > \rho$.

Thus the charge $\lambda(\mathbf{r})$ inside a cylinder of radius r is $\lambda(\mathbf{r})=d\pi^{2}\mathbf{r}^{2}(\rho^{2}-\frac{1}{2}\mathbf{r}^{2}) = 2\Lambda(\mathbf{r}/\rho)^{2}(1-\frac{1}{2}(\mathbf{r}/\rho)^{2})$

with Λ the total charge per unit length. For the x and y components of the electric field one finds

$$E_{\mathbf{x}} = \frac{\Lambda}{\pi \varepsilon_{0}} \frac{\mathbf{x}}{\rho^{2}} \left(1 - \frac{1}{2} \frac{\mathbf{x}^{2} + \mathbf{y}^{2}}{\rho^{2}} \right)$$
$$E_{\mathbf{y}} = \frac{\Lambda}{\pi \varepsilon_{0}} \frac{\mathbf{y}}{\rho^{2}} \left(1 - \frac{1}{2} \frac{\mathbf{x}^{2} + \mathbf{y}^{2}}{\rho^{2}} \right)$$

The betatron motion of a test particle in free space (no image effects) follows from

$$\begin{aligned} \mathbf{x}'' + (v_{\mathbf{x}}/\mathbf{R})^{2} & \left(1 - \frac{\Lambda}{\pi\varepsilon\rho^{2}} \cdot \frac{1}{\beta^{2}\gamma^{3}} \cdot \frac{1}{E_{o}} \cdot \left(\frac{\mathbf{R}}{v_{\mathbf{x}}}\right)^{2}\right) \mathbf{x} = \\ &= -\frac{\Lambda}{2\pi\varepsilon\rho^{2}} \cdot \frac{1}{\beta^{2}\gamma^{3}} \cdot \frac{1}{E_{o}} \cdot \frac{\mathbf{x}^{3} + \mathbf{y}^{2}\mathbf{x}}{\sigma^{2}} \\ \mathbf{y}'' + (v_{\mathbf{y}}/\mathbf{R})^{2} & \left(1 - \frac{\Lambda}{\pi\varepsilon\rho^{2}} \cdot \frac{1}{\beta^{2}\gamma^{3}} \cdot \frac{1}{E_{o}} \cdot \left(\frac{\mathbf{R}}{v_{\mathbf{x}}}\right)^{2}\right) \mathbf{y} = \\ &= -\frac{\Lambda}{2\pi\varepsilon\rho^{2}} \cdot \frac{1}{\beta^{2}\gamma^{3}} \cdot \frac{1}{E_{o}} \cdot \frac{\mathbf{x}^{2}\mathbf{y} + \mathbf{y}^{3}}{\sigma^{2}} \quad . \end{aligned}$$

Applying the method of Krylov, Bogoljubov and Mitropolskii one finds for the frequency shifts to first order:

$$\begin{split} \Delta \upsilon_{\mathbf{x}} &= \Delta \upsilon_{\mathbf{x}o} \bigg[1 - \bigg[\frac{3}{8} (\hat{\mathbf{x}}/\mathbf{o})^2 + \frac{1}{2} \bigg\{ 1 + \frac{1}{2} \cos\left(2(\bar{\upsilon}_{\mathbf{x}} - \bar{\upsilon}_{\mathbf{y}})\frac{\mathbf{s}}{\mathbf{R}} + \theta\right) \bigg\} (\hat{\frac{\mathbf{v}}{\mathbf{p}}})^2 \bigg] \frac{\upsilon_{\mathbf{x}}}{\bar{\upsilon}_{\mathbf{x}}} \bigg] \\ \Delta \upsilon_{\mathbf{y}} &= \Delta \upsilon_{\mathbf{y}o} \bigg[1 - \bigg[\frac{3}{8} (\hat{\mathbf{y}}/\mathbf{o})^2 + \frac{1}{2} \bigg\{ 1 + \frac{1}{2} \cos\left(2(\bar{\upsilon}_{\mathbf{x}} - \bar{\upsilon}_{\mathbf{y}})\frac{\mathbf{s}}{\mathbf{R}} + \theta\right) \bigg\} (\hat{\frac{\mathbf{x}}{2}})^2 \bigg] \frac{\upsilon_{\mathbf{y}}}{\bar{\upsilon}_{\mathbf{y}}} \bigg] \\ \text{where} \quad \Delta \upsilon_{\mathbf{x}o} &= -\frac{1}{2} \frac{\Lambda}{\pi \varepsilon \rho^2} + \frac{1}{\beta^2 \gamma^3} + \frac{1}{\mathbf{E}_o} + \frac{\mathbf{R}^2}{\upsilon_{\mathbf{x}}} \\ \Delta \upsilon_{\mathbf{y}o} &= -\frac{1}{2} \frac{\Lambda}{\pi \varepsilon \rho^2} + \frac{1}{\beta^2 \gamma^3} + \frac{1}{\mathbf{E}_o} + \frac{\mathbf{R}^2}{\upsilon_{\mathbf{y}}} \end{split}$$

represent the zero amplitude shifts and where

$$\bar{v}_{x} = v_{x} + \Delta v_{xo}$$
$$\bar{v}_{y} = v_{y} + \Delta v_{yo}$$

 $2\pi R$ is the circumference of the accelerator, $\overset{}{\overset{}{\overset{}}{\overset{}}}_{x}$ and $\overset{}{\overset{}{\overset{}}{\overset{}}}_{y}$

its unperturbed betatron frequencies, β , γ the usual relativistic velocity and energy and ρ the velocity of light.

It may be seen that the space charge induced octupole causes the betatron frequencies for most particles, (i.e. those with $\hat{x} \neq 0$ and $\hat{y} \neq 0$) to be modulated with

twice the difference in betatron frequencies. In the extrema and with $\Delta \upsilon_0^{} << \upsilon$:

$$\Delta v_{\mathbf{x}} = \Delta v_{\mathbf{x}0} \left[1 - \left\{ \frac{3}{8} (\hat{\mathbf{x}}/\rho)^2 + \frac{1}{2} (1 \pm \frac{1}{2}) (\hat{\mathbf{y}}/\rho)^2 \right\} v_{\mathbf{x}} \right]$$

$$\Delta v_{\mathbf{y}} = \Delta v_{\mathbf{y}0} \left[1 - \left\{ \frac{3}{8} (\hat{\mathbf{y}}/\rho)^2 + \frac{1}{2} (1 \pm \frac{1}{2}) (\hat{\mathbf{x}}/\rho)^2 \right\} v_{\mathbf{y}} \right]$$