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CONTROL OF CLOSED ORBIT DEVIATION DUE TO SYNCHROTRON RADIATION\*

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# Introduction

The energy loss by synchrotron radiation in electron-positron storage rings occurs in every bending mag-net and is thus distributed around the ring, while the energy gain occurs at the rf cavities, which are usually lumped in only a few locations.<sup>1</sup> This type of orbit distortion is usually negligibly small compared to the design allowance in existing synchrotrons and storage rings; however, in the case of the larger storage rings now being contemplated, the distortion can be substantial in comparison to beam size and is different for electrons and positrons. This difference between the two closed orbits can produce horizontal separations and crossing angles between the two beams at the interaction regions. For example, for several of the possible PEP operating configurations concentrating the rf cavities symmetrically about one insertion region, horizontal separations are produced which are of the order of the horizontal beam size at some interaction points and are unacceptable.

There are at least three possible solutions to this problem. The first is to use a lattice in which the dispersion and its derivative are zero at the interaction regions and at the locations of the rf system. The second is to use transverse electric fields to produce the necessary corrections in the orbits of the two beams. The last is to distribute the rf accelerating system around the ring in such a way that the closed orbit deviations are within acceptable limits.

For studying this problem, the thin-lens lattice design program MAGIC has been modified to compute the closed orbit distortion for any distribution of the rf system operating at any configuration.<sup>2</sup> In the next section, we discuss the computation of the closed orbit and in the latter sections, we discuss the different options and give the reasoning that we have used to decide on the solution of distributing the rf system for PEP.

#### Orbit Computation

Consider the case where the energy change due to the radiation in the bending magnets is adiabatic; i.e.,  $\frac{E'(s)}{E_0} \ll \frac{1}{B(s)}$ , with  $\beta(s)$  the usual betatron function and  $E(s)/E_0$  the ratio of the beam energy to the design energy at a point s. The prime denotes differentiation with respect to s. The closed orbit solution can then be approximated by the sum of two terms. The first is equal to the dispersion n(s) times the quantity  $\left[\frac{E(s)-E_0}{E_0}\right]$ . Because this first term is discontinuous between the entrance and exit of an rf cavity, the second term must be a normal betatron oscillation chosen to make the closed orbit continuous. The closed orbit x(s) in the regions between cavities can be written as

$$x(s) = \eta(s) \left[ \frac{E(s) - E_0}{E_0} \right] + a \beta^{\frac{1}{2}}(s) \sin \left[ \int \frac{ds}{\beta(s)} + \psi \right], \quad (1)$$

where the values of a and  $\psi$  are chosen for each region such as to make x(s) continuous. (See Fig. 1.)

When  $\eta(s)$  is zero at the cavities, the first term is zero and hence continuous across the cavity. No betatron oscillation term is necessary; thus, if  $\eta(s)$  is

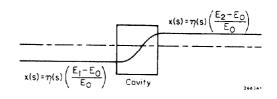


Fig. 1--Closed orbit x(s) through an rf cavity. The energy gain in the cavity is  $E_2 - E_1$ .

zero at all of the cavities, the closed orbit is given everywhere by

$$x(s) = \eta(s) \left[ \frac{E(s) - E_0}{E_0} \right]$$

For this case, the two closed orbits will coincide at any place where the dispersion is zero and in particular at the interaction points for a zero dispersion interaction design.

A more complete study of this problem has been done with the modified version of the program MAGIC referred to above. Due to synchrotron radiation, the energy loss per bending magnet is approximately given by  $\Delta E_{\rm b} \approx - U_{\rm o}/M$ , where  $U_{\rm o}$  is the total energy radiated in one revolution and M is the number of (identical) bending magnets. To make up for this loss, the particles gain energy  $\Delta E_{\rm c} \approx U_{\rm o}/N$  per cavity chain, where N is the number of identical cavity chains in the rf system distributed by some scheme along the machine circumference. Let the ratio of the beam energy to the design energy at a point s be denoted by

$$r(s) = E(s)/E_{o}$$
(2)

The parameter r(s) is determined by the operating energy and the placement of the bending magnets and the rf cavities. Due to the variation of E(s), the effective focal length for a quadrupole magnet at a point s is given by

$$f(s) \approx f_{a} r(s) \tag{3}$$

where  $f_0$  is the focal length for the design energy. Similarly, the bending angle relative to the design orbit in a bending magnet is given by

$$\Delta \theta_{b}(s) \approx \theta_{o} \left[ 1 - r(s) - \frac{U_{o}}{2ME_{o}} \right]$$
 (4)

where  $\theta_0$  is the bending angle for the particle with the design energy and r(s) is evaluated at the entrance of the bending magnet.

If one uses the vector  $\dot{X}$  with components (x,x',1) then in the thin-lens approximation, the transport matrix for a quadrupole magnet becomes

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{f(s)} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
(5)

and that, for a bending magnet, becomes

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \Delta \theta_{b}(s) \\ 0 & 0 & 1 \end{pmatrix}$$
 (6)

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The closed orbit is computed by using these matrices in MAGIC for a prescribed variation of r(s).

## Distribution of Rf System

At an energy of 15 GeV and for a typical operating configuration  $\beta_X^\star$  = 3.5 m,  $\beta_y^\star$  = 0.2 m,  $\eta^\star$  = – 0.8 m,  $\nu_X$  =  $\nu_y$  = 18.75 and an rf voltage of 45 MV , the relevant natural beam dimensions in the radial and longitudinal directions are  $\sigma_{\pi}^{*}$  = 1.2 mm and  $\sigma_{z}$  = 2.3 cm. The asterisks refer to values at interaction points. For this configuration, when the rf system is distributed symmetrically about one interaction region as shown in Fig. 2a, the orbit separation between electron and positron beams at two of the interaction points is about 1 mm. This is clearly unacceptable since the beam-beam limit and hence the maximum luminosity of  $e^+\mathchar`-e^-$  storage rings is greatly reduced when the beams are colliding at "grazing incidence". However, if the rf systems are distributed symmetrically about three interaction regions as shown in Fig. 2b, then, owing to symmetry, the separation between the beams at all of the interaction points is reduced to zero, independent of the dispersion function, and the maximum value of the half-crossing-angle  $\delta/2 \approx 10^{-4}$  rad. This condition appears acceptable, since  $\delta/2 \ll \sigma_X^*/\sigma_Z \approx 0.05$ , which implies that the beam-beam interaction whould be negligibly affected. A fur-ther advantage of distributing the rf system is the reduction in the number of synchrotron oscillations per period of the rf structure.

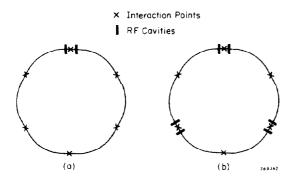


Fig. 2--A schematic layout of the central orbit of PEP showing an rf system distributed symmetrically about (a) one interaction point and (b) three interaction points.

Potential disadvantages are slightly increased cost in the rf system and possibly higher backgrounds for physics experiments at interaction regions.

### Other Options

A second option would be to provide differential beam steering in the horizontal plane by means of electric fields. This would involve at least two sets of plates located symmetrically about each of the six interaction regions and, for more flexibility in configurations, three sets. In theory, these plates could be powered in such a way as to cancel the closed orbit deviation caused by the lumped rf system. In practice, it would be very difficult to measure the closed orbit accurately to a fraction of a millimeter in order to determine the optimum electric fields for one interaction point. However, we feel it would be extremely troublesome to determine the necessary voltages operationally to optimize the luminosity at six different interaction points and that the usable high-energy physics time would be severely diminished.

The last option considered was the possibility of restricting the PEP operations only to configurations where n = 0 throughout the insertions and locating the cavities in only one of the insertions. It was found that in order to achieve the desired luminosity at 15 GeV in such configurations, it was necessary to increase the beam size in the normal cells of the ring. Basically, this is because none of the transverse beam size at the interaction points would come from the energy spread in the beam, owing to  $n^* = 0$  at the interaction point, and it is necessary to have a definite beam size at the interaction in order to achieve the desired luminosity owing to the incoherent beam-beam limit. This option then would require a larger transverse size for the beam in most of the ring, and hence would result in increased apertures and a considerable increase in the cost of the ring components.

## Discussion

We have rejected the idea of electric steering fields as a severely unattractive operational solution, the restriction of the operating configurations to  $n^* = 0$  as too costly and have decided on the option of distributing the rf systems symmetrically about three of the interaction regions. Preliminary investigation into the potential of increased backgrounds at the interaction regions where the rf is located, based on experience at SPEAR, indicates that the danger is minimal.

#### Acknowledgements

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