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CAN COHERENT UV RADIATION BE OBTAINED FROM EXISTING ELECTRON STORAGE RINGS?

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Summary

Recently built electron storage rings provide very high transverse electron densities in their low-β insertions. This suggests the feasibility of laser action of an electron storage ring at optical and even shorter wavelengths. Using a semiclassical theory of induced synchrotron radiation[1], it is shown that special bending devices should in fact allow for light amplification in this spectral range. The special bending unit should be an electrostatic deflector providing an intense electric field of Coulomb type. Lower wavelength limits for laser action are estimated using reported or design performances of SPEAR I and DORIS. For the latter machine this limit is of the order of 3000 Å, provided that an electric field strength of 10 kV/cm could be achieved and the deflector is placed in a low-β insertion.

Introduction

Soon after the discovery of the laser principle, search for new laser candidates extended into numerous physical systems, and already in 1959 the criteria for amplification of electromagnetic waves by an ensemble of gyrating electrons were given[2], and subsequently experimentally verified[3]. These devices, however, made use of rather low-energy beams. In this now-relativistic regime, only microwaves could be amplified or generated. Sokolov and Ternov[4] described induced radiation processes of a relativistic electron in a homogeneous magnetic field. Some years ago the author used a semiclassical approach to investigate the influence of the field index of a weakly focusing bending field of magnetic or electrostatic type[1]. The criteria for amplification and laser action given there apply only to the case of a zero emittance beam in a weakly focusing, circular machine.

Based on some assumptions - essentially the validity of the classical description of synchrotron radiation in the domain of interest - we intend to show that laser action in the UV range of existing electron storage rings coupled to a suitable resonator could be feasible. However, this requires a special electrostatic bending element producing very high electric field strengths, to be placed in a low-β insertion. Magnetic bending seems not to allow laser action of wavelength ranges below one millimeter, and this only if they are specially designed (field index n = 1).

Principle

The basic reference[1] being in German, we want to outline briefly the argumentation given there.

A classical electron on a circular orbit emits only multiples of its revolution frequency ω_0. Quantum theory gives for the electron in a homogeneous magnetic field energy levels that differ just by $\hbar\omega_0$ and are of infinite degeneracy. However, one cannot expect a series of special lines of distance ω_0, since the levels are broadened by the interaction with the electromagnetic field to a width of

$$\Delta\omega \sim \frac{1}{\tau} \sim \frac{W_{total}}{\hbar\omega_m} \sim \frac{2}{3c}\,\frac{e^2\omega_0^2\,\gamma^4}{\hbar\omega_0\gamma^3} \sim \frac{1}{137}\,\gamma\omega_0\,,$$

where $\omega_m = \omega_0\gamma^3$ gives the magnitude of the spectral maximum; γ is of order $10^3 \div 10^4$ and therefore the spectral lines largely overlap, producing a continuous spectrum. It will turn out that this fact is crucial for the theory presented here. We have transitions to many neighbouring levels contributing to the emission and absorption of photons of given frequency ω, and hence the density of final states $1/\omega_0$ enters into the transition probability. The revolution frequency ω_0 depends on the energy of the electron in a way that is determined by the character of the guiding field.

In order to see how it enters, we consider the expression for the power spontaneously emitted into unit solid angle and unit frequency interval:

$$W(\omega,E,\psi) = \frac{1}{\omega_0}\,W_p(E,\psi)\;, \qquad p = \frac{\omega}{\omega_0}\;, \tag{1}$$

where $W_p(E,\psi)$ is the standard expression[4] for the emission into the p-th harmonic of ω_0 and with an angle ψ to the orbital plane:

$$W_p(\psi)_i = \frac{e^2p^2c}{6\;R}\begin{cases} f^2K^2_{2/3}\left(\frac{p}{3}f^{3/2}\right) & ..\; i = 1 \\ f\,\sin^2\psi\,K^2_{1/3}\left(\frac{p}{3}f^{3/2}\right) & ..\; i = 2 \end{cases} \tag{2}$$

$$f = 1 - \beta^2\cos^2\psi = \frac{1}{\gamma^2} + \beta^2\sin^2\psi.$$

(Polarization index i = 1 means the electric field vector to fall into the orbital plane; $\vec{e}_2$ is normal to $\vec{e}_1$ and $\vec{k}/k$.) In these formulae ω_0, R, p, and f depend on the energy E of the electron. Hence for the power emitted by an electron of energy E we write:

$$W(\omega,E,\psi)_{em} = W_p(E,\psi)g(E-\hbar\omega)(\bar{n}_{\vec{k}} + 1)\;, \tag{3}$$

$g(E) = 1/\omega_0(E)$ being the density of final states and $\bar{n}_{\vec{k}}$ denoting the mean number of photons per mode of the radiation field.

The leading terms (giving the classical approximation) of the squared matrix elements are identical for emitting and absorbing transitions. Hence we write for the power absorbed by the same electron

$$W(\omega,E,\psi)_{abs} = W_p(E + \hbar\omega,\psi)g(E + \hbar\omega)\bar{n}_{\vec{k}}\;. \tag{4}$$

If there are N electrons interacting with the radiation field, the gain of $\bar{n}_{\vec{k}}$ with time is given by:

*) Based on work done at the Vienna Institute of Technology.

$$\left[\frac{d\bar{n}_{\vec{k}}}{dt}\right]_{gain} = \frac{N}{V_{res}\rho(\omega)}\frac{1}{\hbar\omega} \quad W_p(E,\psi)g(E-\hbar\omega)(\bar{n}_{\vec{k}}+1) - W_p(E+\hbar\omega,\psi)g(E+\hbar\omega)\bar{n}_{\vec{k}} \quad , \quad (5)$$

$\rho(\omega) = \frac{\omega^2}{(2\pi c)^3}$ ...mode density of the radiation field,

$V_{res} = A\, L_{res}$ is the effective resonator volume. We will put A equal to the beam cross-section. The loss rate is given by:

$$\left[\frac{d\bar{n}_{\vec{k}}}{dt}\right]_{loss} = \frac{\bar{n}_{\vec{k}}}{\tau_{res}} = \frac{\bar{n}_{\vec{k}}\, c}{L_{res}}(1 - r) \quad , \quad (6)$$

where all kinds of losses are comprised in the reflection factor $r < 1$.

The criterion for laser action is then

$$\frac{N}{\hbar\omega}\left[W_p(E,\psi)g(E-\hbar\omega) - W_p(E+\hbar\omega,\psi)g(E+\hbar\omega)\right] > \frac{(1-r)A}{2\pi\ \lambda^2} \ . \quad (7)$$

In all cases of function interest $\hbar\omega << E$ holds and we can write for the expression in brackets

$$-\hbar\omega\left[\frac{\partial W_p}{\partial E}g + 2W_p\frac{\partial g}{\partial E}\right] = -\hbar\omega\left[\frac{\partial W(\omega)}{\partial\gamma} + W(\omega)\frac{1}{g}\frac{dg}{d\gamma}\right]\frac{d\gamma}{dE} \ . \quad (8)$$

In order to take into account the finite emittance of the electron beam, we should average over the angles z' of the trajectories with the orbital plane. We will, however, be interested in emission into angles $\psi > z'_{max} = z_{max}/\beta_z$ and therefore drop the integration over z'. We will equally neglect the energy spread of the beam. With these assumptions we put Eq. (7) finally into the form:

$$\lambda^2 > -\frac{(1 - r)A}{2\pi\ N\left[\frac{\partial W(\omega,\psi)}{\partial\gamma} + \frac{W(\omega,\psi)}{g}\frac{dg}{d\gamma}\right]\frac{d\gamma}{dE}} \quad . \quad (9)$$

We will use equation (9) in order to obtain lower limits for the wavelength of possible laser action for some existing machines.

Basic Assumptions and Restrictions of this Theory

The results obtained in Section above apply strictly only to the case of a homogeneous magnetic bending field, because we made use of the degeneracy of the energy eigenvalues mentioned above when putting the density of final states equal to $1/\omega_0$.

For the inhomogeneous field the degenerated energy eigenvalues split up into a double series of levels with distances $\hbar\omega_0\nu_r$ and $\hbar\omega_0\nu_z$, respectively ($\nu_r = \sqrt{1-n}$, $\nu_z = \sqrt{n}$). Note that ω_0, ν_r, ν_z do not denote revolution frequency and tunes of the actual machine, but the corresponding quantities of a virtual, weakly focusing circular machine obtained by extending the investigated bending field to the whole azimuth. The use of a simple density $1/\omega_0$ remains justified, however, as long as the major part of the photon energy goes into the change of orbital energy. An inspection of the matrix elements suggests that this is in fact the case if the horizontal betatron amplitudes are not too large. Any influence of this kind would show up in the spectrum and also in the total emitted power. The quantum corrections to the latter have been calculated for the homogeneous magnetic field by Sokolov et al. and by Schwinger[5] and for the inhomogeneous field by Gutbrod[6], who obtained:

$$W_{qu} = W_{cl}\ 1 - \frac{55}{32}\sqrt{3}\,\frac{\lambda_c}{R}\,\frac{1 - \frac{5}{3}n}{1 - n}\gamma^2 \quad , \quad (10)$$

λ_c Compton wavelength, R bending radius. So quantum effects on the radiation spectrum should be negligible as long as

$$\frac{\gamma^2}{1 - n} = \frac{\gamma^2}{\nu_r^2} << 2 \times 10^{11}\ R_{(m)} \quad , \quad (11)$$

which is certainly met in all cases to be considered. No such estimate exists for electrostatic bending fields, but for a very rough guess we may insert the corresponding value of $\nu_r = 1/\gamma$ for a Coulomb bending field. The resulting condition, using

$$R_{(m)} = \frac{5.11\gamma}{E_{b(kV/cm)}} \quad (12)$$

would then be

$$\gamma^3 << \frac{10^{12}}{E_b} \quad (13)$$

If we assume a value of 10 kV/cm for the electric bending field E_b, condition (13) yields $\gamma < 5 \times 10^3$, and the validity of the results obtained for the Coulomb bending field is restricted to electron energies below 2.5 GeV.

Properties of the deflecting fields

In order to perform the derivatives in formula (8), we need the derivatives in γ of some quantities as R, ω_0, etc. They obviously depend on the properties of the guiding field.

Magnetic bending field of field index $n = -\frac{\partial B}{\partial R}\frac{R}{B}$

From

$eBR = pc = m_0c^2\beta\gamma$,

we obtain

$$\frac{d\gamma}{dR} = \frac{1 - n}{\beta}\frac{eB}{m_0c^2} \cong (1 - n)\frac{\gamma}{R}$$

and

$$\frac{dR}{d\gamma} \cong \frac{1}{1-n}\frac{R}{\gamma}\ ,\ \frac{d\omega_0}{d\gamma} \cong -\frac{1}{1-n}\frac{\omega_0}{\gamma}\ ,\ \frac{dg}{d\gamma} \cong \frac{1}{1-n}\frac{g}{\gamma} \quad . \quad (14)$$

The total energy is $E = m_0\gamma c^2$ and

$$\frac{d\gamma}{dE} = \frac{1}{m_0c^2} \quad . \quad (15)$$

Coulomb field

The potential energy has the form $U(R) = \frac{U_0R_0}{R}$ and from

$$-m_0\gamma\frac{U}{R} = -\frac{dU}{dR} = \frac{U}{R}$$

one obtains immediately

$U = -\beta^2\gamma m_0c^2$

and the total energy is given by

$$E = m_0c^2\gamma + U = \frac{m_0c^2}{\gamma}\ ;\ \frac{d\gamma}{dE} = -\frac{\gamma^2}{m_0c^2} \ . \quad (16)$$

Noticing that the laser criterion (9) contains the factor $d\gamma/dE$ and comparing (15) and (16) one notes that one gains the enormous factor γ^2 when using a Coulomb deflection field instead of magnetic bending. We further obtain from (15)

$$\frac{dR}{d\gamma} = -\frac{R}{\gamma}, \quad \frac{d\omega_0}{d\gamma} = \frac{\omega_0}{\gamma}, \quad \frac{dg}{d\gamma} = -\frac{g}{\gamma} \quad . \tag{17}$$

In order to treat both cases at once, we unify eqs. (14) and (17) by introducing a parameter q, which stands for

$$\begin{aligned} q &= n - 1 \quad \ldots \quad \text{for magnetic bending} \\ q &= 1 \quad \ldots \quad \text{for the Coulomb field,} \end{aligned} \tag{18}$$

and we can write

$$\frac{\gamma}{R}\frac{dR}{d\gamma} = \frac{\gamma}{g}\frac{dg}{d\gamma} = -\frac{\gamma}{\omega_0}\frac{d\omega_0}{d\gamma} = -\frac{1}{q} \quad . \tag{19}$$

The values of a weakly focusing magnetic field are well known:

$$\nu_r = \sqrt{1-n}\ , \quad \nu_z = \sqrt{n}\ . \tag{20}$$

For the electric Coulomb field, they are given by (see Ref. 7 e.g.):

$$\nu_r = \frac{1}{\gamma}\ , \quad \nu_z = 1 \quad . \tag{21}$$

We evaluate now the expression in square brackets in the denominator of the r.h.s. of the criterion (9), using Eqs. (2) and (19):

$$\frac{2\pi}{m_0c^2}\left[\ldots\right] = \frac{W_0}{f\gamma}\,G_i(x) \quad . \tag{22}$$

$$W_0 = \frac{6}{\pi^2}\frac{r_e}{R}\ , \quad f = \frac{1}{\gamma^2} + \beta^2\sin^2\psi;$$

$$G_1(y) = \frac{1}{q}\left[\frac{y^2}{3}K^2_{2/3}(y) - y^3K_{1/3}(y)K_{2/3}(y)\right] - \frac{3}{f\gamma^2}\,y^3K_{1/3}(y)K_{2/3}(y) \quad ,$$

$$G_2(y) = \frac{\sin^2\psi}{f}\,\frac{1}{q}\left[\frac{2y^2}{3}K^3_{1/3}(y) - y^3K_{1/3}(y)K_{2/3}(y)\right] - \frac{3}{f\gamma^2}\,y^3K_{1/3}(y)K_{2/3}(y)\ ,$$

$$y = \frac{p}{3}\,f^{3/2}\ . \tag{23}$$

For angles $\psi > 1/\gamma$ we may neglect the second term in the G_i's, and f becomes $f \simeq \psi^2$. Both functions $G_i(y)$ are negative for $y \gtrsim 0.5$ and have significant values only for $y \sim 1$; for large values of y they behave prop. to $y\exp(-2y)$. We need the negative sign in the case of the electrostatic deflecting field for then the factor $d\gamma/dE$ is negative. In the following we will drop the polarization index, dealing only with the more favourable polarization $i = 1$. The fact that y needs to be of order one introduces an interdependence of ω, γ, R and ψ. From Eq. (23) we deduce

$$f = \left[\frac{3y}{2\pi}\frac{\lambda}{R}\right]^{2/3} = \frac{1}{\gamma^2} + \psi^2\ , \qquad y \gtrsim 1\ . \tag{24}$$

This will determine the angle ψ and we anticipate the values consistent with our results to be obtained below: for the parameters of DORIS, a wavelength of $3\ 10^{-5}$ cm and $E_b = 10$ kV/cm, $f\gamma^2 = 4.6$, $q = 1$ and $\psi = 1.9/\gamma$; $G_1(1) = -0.276$.

For a magnetic deflector $1/q = 1/(1-n)$; we will extract this factor from G_1 and will write

$$G_1(y) = G(y)/\nu_r^{\,2} \quad . \tag{25}$$

Application to existing storage rings

In order to see what one can expect we take some data from the 1974 Catalogue of High Energy Accelerators (Appendix to Ref. 8) and from Ref. 9. They are

TABLE I

Parameter	Spear I	DORIS	Unit
Energy	2.5	3	GeV
γ	4.9×10^3	5.9×10^3	
Current, per beam	0.22	0.9	A
frev	1.28	1.04	MHz
N	10^{12}	5.4×10^{12}	
β_v at interaction region	0.05	0.1	m
Bunch dimensions at interaction region	$300\times3\times0.08$	$40\times0.6\times0.03$	mm^3
Beam cross section A at interaction region	2.4×10^{-3}	1.8×10^{-4}	cm^2
$\tilde{A}$ (see Eq. 26)	5.7×10^4	6.2×10^3	cm^2

not at all up-to-date (SPEAR being transformed into SPEAR II in the meanwhile), but may still represent typical performances obtained or to be expected from recent storage rings. The parameters to be used are compiled in Table I.

In order to scale the beam cross-section A to other energies, we suppose that the vertical betatron oscillations can be decoupled from the radial ones. In this case the beam height is mainly determined by scattering on the residual gas. From Ref. 10 we take for the energy dependence of the emittances:

$$\varepsilon_h = \tilde{\varepsilon}_h \gamma^2$$

$$\varepsilon_V = \tilde{\varepsilon}_V \gamma^{-6}$$

for given machine lattice and residual gas properties. The resulting scaling low for the beam cross-section A is then

$$A = \tilde{A}\gamma^{-2} . \qquad (26)$$

We can now rewrite Eq. (9)

$$\lambda^2 > \frac{(1-r)}{N}\frac{\pi^2}{6}\frac{R}{r_e}\frac{f\gamma}{|G(1)|}\frac{\tilde{A}}{\gamma^2}\nu_r^2 , \qquad (27)$$

where we used Eqs. (16), (21) and (25) respectively, to unify the expressions for magnetic and electrostatic bending. We use Eq. (24) to eliminate f and express R by :

$$R = \tilde{R}\gamma , \quad \underset{(\mathrm{cm})}{\tilde{R}} = \frac{511}{\underset{(\mathrm{kV/cm})}{E_b}} \quad \text{for electrostatic bending}$$

$$\text{or} \quad \underset{(\mathrm{cm})}{\tilde{R}} = \frac{0.17}{\underset{(\mathrm{T})}{B}} \quad \text{for magnetic bending,}$$

and obtain

$$\lambda^{4/3} > \frac{(1-r)}{N}\frac{\pi}{4}\frac{\tilde{A}\,\tilde{R}^{1/3}}{r_e|G(1)|}\frac{\nu_r^2}{\gamma^{2/3}} = C\,\tilde{R}^{1/3}\frac{\nu_r^2}{\gamma^{2/3}} . \qquad (28)$$

Here we tried to put machine parameters (except γ) and fixed ones into C; the number of stored particles, being determined by the single beam limit, ought also to be scaled with γ, but no obvious scaling law is offered. Finally, in the wavelength range to be dealt with, good reflectors are available and we will put $(1-r) \sim 10^{-1}$. Using the data compiled in Table I we obtain the following machine constants C for the two machines:

	SPEAR I	DORIS	
C	5.8×10^4	1.2×10^3	cm
$C^{3/4}$	3.7×10^3	2×10^2	$cm^{3/4}$.

$C^{3/4}$ can now be put into

$$\lambda > C^{3/4}\tilde{R}^{1/4}\frac{\nu_r^{3/2}}{\gamma^{1/2}} . \qquad (29)$$

Equation (29) is now evaluated separately for electric and magnetic guiding field, and the resulting wavelength limits are compiled in Table II.

One notices immediately that magnetic bending allows only generation of microwaves and might rather be used to pump energy into the beam.

The orders of magnitude estimated for Coulomb bending look more promising, but they are subject to restrictions: the beam energies assumed to calculate the short-wavelength limits for light generation - 2.5 GeV and 3 GeV, respectively - violate the condition (13) for applicability of the quasi-classical theory used throughout. Remember however that condition (13) is not well-founded. For a wavelength of 3×10^{-5}cm, the adequate beam energy in DORIS would be 2.1 GeV, what looks a little safer. Of course only a fully quantum mechanical approach can tell the limits of the quasi-classical theory presented here and how the implication of quantum effects would change the results. In view of the physical possibilities offered by such a device, it might be worthwhile to study this problem more. But even in the domain of longer wavelengths, where classical theory is expected to be fully applicable, the available power and the particular properties of such a light source might be of physical interest.

TABLE II

Magnetic bending		Electrostatic(Coulomb)Bending
Assumed: $B = 1.7$ T		Assumed: $E_b = 10$ keV/cm
$\tilde{R} = 10^{-1}$		$\tilde{R} = 51$ cm
Assumed: $\nu_r^2 = \lvert 1-n \rvert = 10^{-2}$		$\nu_r^2 = \frac{1}{\gamma^2}$
$\lambda > 0.9$ cm	- SPEAR I -	$\lambda > 4.1 \times 10^{-4}$ cm
$\lambda > 0.04$cm	- DORIS -	$\lambda > 1.5 \times 10^{-5}$ cm

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