ACCELERATORS FOR THE FUSION PROGRAM

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Introduction

The problems of building a successful Fusion Reactor are enormously complex but the basic requirements for "breakeven" are reasonably straightforward and were stated many years ago by Lawson. For a D-T reaction which produces a 14.1 MeV neutron and a 3.5 MeV α-particle these requirements are: (i) $T \approx 10$ keV and (ii) $n\tau \approx 10^{14}$ where T is the temperature of the reacting components (assumed to have a Maxwellian energy distribution), n is the particle density in cm^{3} and τ is the time required for the burn in seconds. Two fundamentally different approaches have been made to meet these conditions. In the first approach the charged particles are confined in a "magnetic bottle" and the confinement time scales as B^{-n} where B is the magnitude of the magnetic field and n lies between 1 and 2. Obviously the plasma heating time should be comparable but less than the confinement time. The second approach is characterized by not demanding any confinement at all. In this case $\tau = r/c_s$ is the disassembly time of a body of size r and $c_s \approx 10^{8}$ cm/sec at $T \approx 10$ keV is the free expansion velocity. Since the second approach operates on a very much shorter time scale the power requirements and the intensities of all quantities are proportionately much higher. Thus the heating energy delivery and absorption times have to be extremely short.

There are several applications for accelerated particle beams in both these approaches but it is clear that the requirements imposed and types of accelerators needed are quite different for the two cases. The main applications in magnetic confinement systems for which accelerators have found use are: -

(i) Plasma production
(ii) Plasma heating
(iii) Direct fusion between beam and target
(iv) Magnetic field shaping by internal beam currents.

In the case of inertial confinement systems the only alternative source of energy to high power pulses of charged particles are high power lasers.

The requirements on particle energy are relatively modest; 0.1 to 10 MeV appears to be the range covering most of these applications. But because power requirements are large the beam currents range from 10 - 10[illegible] A. The available accelerators fall into two categories: low power (MW level, ≥ 0.1 sec pulses) which have been primarily developed for large volume systems like tokamaks and mirrors, and high power ($\sim 10^{3}$ MW level, 10^{-7} sec pulses) sources. We will now take up some specific applications of these sources and conclude with future requirements for these accelerators dictated by the needs of the different schemes for achieving "breakeven" conditions.

Plasma Heating in Toroidal Systems (Tokamaks)[1]

The main energy loss in a system with closed lines of force e.g. the Tokamak is by diffusion across the lines of force. The characteristic times for particle and energy confinement are $\tau_p \sim a^2/D$ and $\tau_E \sim a^2/\chi$ where a is the radius of the plasma column and D and χ are the diffusivity and heat conductivity respectively. If the particles interact only through classical coulomb collisions then D scales as $(R/a)^{3/2}q^2T^{-1/2}B^{-2}$ where $q = (5\,a^2B/RI)$, I is the toroidal current (A), R and a are the major and minor radii (cm) and B is the toroidal field (G). Notice that in a toroidal system D is enhanced by the factor $(R/a)^{3/2}q^2 \sim 10^2$ over the conventional diffusivity for a system with straight lines of force. On the other hand if "microturbulence" or enhanced fluctuations driven by plasma currents, density/temperature gradients etc., are dominant a pessimistic scaling T/B could result. The actual situation is probably between these two limits. The heating power required can be roughly estimated for a plasma volume V

$$P = nTV/\tau \equiv f(R/a,\ T,\ B) \quad ,$$

a function only of the toroidal field, for fixed R/a and T, once we have recognized that $n\tau \approx 10^{14}$ and $\tau = a^2/D$. The ohmic dissipation of the externally induced toroidal current provides some fraction of this heating and typically heats the plasma to $T_e \sim 1 - 3$ keV in present day machines. However, it is too optimistic to assume that ohmic heating is sufficient to raise the temperature to ignition. This ignition occurs at a temperature when the energy of the α-particles is deposited within the plasma. Fig. 1 shows a calculation by Sweetman[2] from which it is clear that ohmic heating alone is unable to take the temperature beyond 4 keV. The ignition temperature in this case is ~ 20 keV. This gap can be crossed only with the help of additional sources of energy. Even for a "breakeven" experiment it is essential to have auxiliary energy source otherwise the poloidal $\beta_p = 8\pi nT_e/B_p^2$ is limited to $\frac{1}{2}(1 + I^2/(1{\cdot}6)^2)^{-1/2}$ which is well below the limit $\beta_p = R/2a$ for MHD stability;[1] here the poloidal field $B_p = I/5a$ MG, I is in MA.

The currently favored technique for auxiliary heating is by the injection of a beam of energetic neutral H or D atoms into a Tokamak plasma and because of their neutrality they are not inhibited from crossing the magnetic surfaces. The mean free path for charge exchange and ionization of an atom between 20 and 100 keV is given by[2]

$$\lambda = 5.5 \times 10^{13}\ E/n_e \quad ,$$

where E is the neutral energy in keV, n_e is the electron density in cm^{3}. When λ is comparable to a, maximum beam absorption is obtained by tangential injection. The heating time is roughly given by the slowing down time of the fast ion by collisions with the background. Experiments[3] on the ATC machine at Princeton University have demonstrated plasma heating by neutral beams. A 100 keV beam of H^+ injected into an ohmically heated plasma of density $n \sim 1.8 \times 10^{13}$ at $T_i \approx 220$ eV produced an incremental increase in ion temperature of $\Delta T_i \approx 110$ eV.

The neutral beams are generated[4] by extracting ions from a plasma source of which there are currently two types (i) the duo PIGatron developed at Oak Ridge and (ii) the filament developed at Berkeley LBL. The plasma meniscus is accessible through a multiple-aperture (3-4 mm dia. holes or 2 mm x 7 cm slits) plate. Thus a large plasma surface is utilized to produce many parallel ion beams with high current density using only electrostatic focusing by accel-decel electrode system. Current density of $0.4 \sim 0.5$ A/cm^2 over tens of cm^2 are presently achieved with extracting electric fields of 100 keV at total energies of 15-40 keV. After acceleration the ions are immediately injected into a gas (H_2) cell. About 60-80% of the ions are charge-exchanged into a neutral beam. The capability of the LBL source is 50 A, 20 keV, 30 msec while the Oak Ridge source is rated at 5 A, 40 keV, 300 msec.

Direct Fusion between Beam and Target

The idea of direct D-T fusion reactions between a deuterium beam and a suitable tritium target was considered and rejected a long time ago on the grounds that almost all of the beam energy will be utilized in needless heating and scattering since the coulomb cross section is so much greater than the fusion cross section. This idea was revived recently by Dawson, Furth and Tenney[5] who recognized that the ion slowing down time by coulomb collisions can be increased by maintaining the plasma electron temperature above 5 keV. If the injected ion produces Q times its initial energy in fusion reactions before slowing down then the Lawson criterion is replaced by $\eta_n Q > 1 - \eta_p$ where η_p and η_n are the efficiencies of conversion to electrical energy from plasma and fusion products respectively. The maximum value of Q is approximately 4 for injected energies 150 $\sim$ 200 keV and for this Q the $n\tau$ can be reduced by approximately a factor of 10 from the Lawson figure.

A machine based on this concept and dubbed the Tokamak Test Fusion Reactor[6] is under serious consideration by ERDA. A comparison of $n\tau$ for "breakeven" for this machine is shown in Fig. 2 . A 40 MW, 80 keV, $\sim$ 30 msec beam is used mostly to raise the plasma temperature as quickly as possible. A 20 MW, 120 keV $\sim$ 30 msec beam is injected next to provide energetic ions for the reactor phase. This stage is followed by a magnetic compression stage which further raises the plasma temperature by a factor 1.35 - 1.71 and the injected ion energy by 1.56 - 2.25. The neutral injection system consists of 16 LBL, 80 keV injectors of 40 MW output and a pulse time of 43 msec and 8 LBL, 125 keV injectors furnishing, 20 MW, with pulse time of 32 msec. An alternative system consists of 24 ORNL, 150 keV injectors of 12 MW output for 500 msec.

High-Power Electron Accelerators

Developments in pulsed high-power electrical technology have led to very powerful electron beams with energy content and power up to 3 MJ and 10^{13} W. Some aspects of the E-Beams have been covered in the sessions on "Collective Acceleration and Intense Beams". The technology is based on the principle of slow charge and rapid discharge of energy reservoirs of successively shorter time scales. In a typical system the first stage is generally a Marx high voltage generator. The output of the Marx charges a transmission line or some variant connected via a switch to a diode in which the beam is actually generated. The minimum pulse duration is determined by the jitter of the switches which close in times of the order of a few nsec. The maximum pulse duration is determined by the ability of the diode to withstand the voltage. In practice, plasma is generated at cathode and anode surfaces because of the large amount of energy deposited and the diode shorts in the time $\sim 10^2$ nsec it takes this plasma to cross the gap.

Although these high power electron beams were originally developed for the purpose of generating large dosage of flash x-rays they have now found several other uses. In controlled fusion their main application is for plasma heating at very rapid rates and for D-T pellet compression. The success of these applications depends on our understanding of the physics of interaction between a beam and plasma.

E-Beam Interaction with Dense Plasma

The nature of beam-plasma interaction is a strong function of plasma electron density n_e and temperature T , the ratio of beam to plasma density n_b/n_e and the beam energy $mc^2\gamma$. At high plasma densities and low temperatures the characteristic time for beam energy loss as determined by classical two-body coulomb collisions is $\tau = \gamma/4\pi n_e r_e^2 c \; \ell n \; \Lambda_b$ where r_e and λ_d are the classical electron and debye radii respectively and $\Lambda_b = \lambda_d/r_e$. The characteristic stopping length is expressed conveniently as $\ell \approx 0.2 \; \varepsilon/\rho$ cm where ε is the beam energy in MeV and ρ is the target density in grams/cm^3. Calculations by Rudakov and Samarsky,[7] Yonas et al.[8] and Clauser[9] indicate that a metallic target in the form of a spherical shell of radius $\sim$ 1 mm and thickness $\sim$ 0.2 mm when irradiated by a beam of $\sim$ 3 MeV electrons with a total energy/pulse of 5 MJ in times of order 3 $\sim$ 10 nsec compresses by ablation of the outside layer to yield a net output of power. Electron accelerators answering to these requirements are currently not available but one sees no serious technical constraints on attaining these particulars. Indeed such short pulse machines are being studied intensively at Sandia and have been reported on at this conference.

The direct energy loss of the beam by collisions can be supplemented by the decay of the plasma return current which normally equals the injected beam current and helps neutralize the beam's self magnetic field.[10] The decay time τ_d of this current is $\pi\sigma a^2/c^2$ where σ is the plasma conductivity and a is the beam radius. The bulk of the beam energy is absorbed in the ohmic dissipation of this current in a time $\tau \approx (\gamma/Nr_e)\,\tau_d$ (for $Nr_e \gg \gamma$) where N is the number of beam electrons per unit length. The ratio of τ to τ_c is given by

$$\tau/\tau_c \approx (n_e/n_b)(3\; T_e/mc^2)^{3/2}\,(\ell n\; \Lambda_b/\ell n\; \Lambda_e) \quad ,$$

where $\Lambda_e = n_e\lambda_d^3$. Situations may arise where the ohmic dissipation becomes comparable to the direct loss of energy by the beam.

E-Beam Interaction with Low and Medium Density Plasma

At densities much lower than solid densities and higher temperatures the classical time scales become very long and collective many-body effects emerge as the dominant mechanism for beam-plasma interaction. This interaction is a two-step process on the microscopic level. In the first stage the beam generates electrostatic Langmuir waves of wavelength $\lambda/2\pi \approx c/\omega_e$, $[\omega_e = (4\pi n_e e^2/m_e)^{1/2}$, the plasma frequency$]$ through the well known streaming instability. The growth rate Γ is given by $\frac{\sqrt{3}}{2}\,\gamma^{-1}(n_b/2\,n_e)^{1/3}\,\omega_e$ when the angular spread of the beam $<\Delta\theta^2>^{1/2}\; < \gamma^{-1}$ and in the opposite situation $\Gamma \approx \gamma^{-1}(n_b/n_e)\omega_e$. For the instability to take place at all one requires $\Gamma > \nu_{ei}$ the plasma electron collision frequency. The angular spread of the beam increases as a result of this interaction and we shall give below an estimate of this increase. The wave energy W_k builds up very fast in a time of order a few times Γ^{-1} until nonlinear processes become important. Two such processes have been identified: (i) self-trapping of a fraction of the beam electrons in the potential well of the Langmuir waves and (ii) a parametric instability driven by the Langmuir waves that excites short wavelength fluctuations $1 \geq q\lambda_d > (m_e/m_i)^{1/2}$, where $2\pi/q$ is the wavelength and m_e and m_i are the electron and ion mass respectively.

In the case that the initial angular spread of the beam at injection is small the saturated level of W_k is controlled by the trapping process and computer simulations lead to[11]

$$W_s/n_b mc^2\gamma \approx \tfrac{1}{2}\, s/(1+s)^{5/2} \ ,$$

where $s = \frac{1}{2}\beta^2\gamma(n_b/2\ n_e)^{1/3}$ and $\beta = (1 - 1/\gamma^2)^{1/2}$. This saturation level greatly exceeds the threshold $W_k/nT_e > (k\lambda_d)^2$ required for the second process, and the energy in the primary spectrum is pumped to short wavelength fluctuations. This stage can be approximately described by

$$\frac{1}{2}\frac{d}{dt} W_k = -\Gamma_p W_q \ , \qquad (1)$$

$$\frac{1}{2}\frac{d}{dt} W_q = \Gamma_p W_q - \nu W_q \ , \qquad (2)$$

where at $t = 0$, $W_k = W_s$ the saturated level, $\Gamma_p \approx (m_e/m_i)^{1/2}\ (W_k/nT)^{-1/2}\ \omega_e$ is the growth rate of the parametric instability and ν is the damping rate of the short-wavelength fluctuations. At low densities ν represents Landau damping and at higher densities the damping could be collisional. The characteristic time τ_2 for the evolution of this stage is approximately[12]

$$\tau_2^{-1} \approx (m_e/m_i)^{1/2}\ (W_s/nT)^{1/2}\ \omega_e$$

provided $\Gamma_p \gg \nu$. The case where the initial angular spread is large so that the two-stream growth rate is much weaker and comparable to Γ_p has been treated by Papadopoulos[13] by including the term $\Gamma_k W$ on the right hand side of Eq. (1).

The second stage described by the parametric instability can also be viewed as a process of condensation of the energy in the Langmuir waves into regions of diminishing size.[14,15] This occurs because the Langmuir waves exert a negative pressure $-\nabla W_k$ on the plasma. The limit of this process can be visualized as a state where all the wave energy now resides in a number of "blobs" whose spatial extent is of the order of a few times λ_d. Any further condensation probably leads to very rapid damping. The effect of these short wavelength fluctuations on the beam is to cause it to increase its angular spread. A beam electron moving at a velocity much greater than the thermal velocity "sees" these "blobs" as a stationary distribution of localized electric fields. The individual scattering events lead to a diffusion in velocity with a characteristic time $\tau_D \approx (\Delta\theta)^2/D_{\theta\theta}$. For scattering through a radian $\Delta\theta \sim 1$ we obtain

$$\tau_D^{-1} = \omega_e (W_q/nT_e)\ (T_e/m_e c^2)^{[illegible]/2} \ .$$

If this time is comparable to some of the other characteristic times for wave growth and decay estimated above there may be considerable reduction in the efficiency of energy transfer between the beam and plasma. I hope this brief discussion is a sufficient indication of the complexity of the collective interaction and why no single estimate of the stopping length is available. Elaborate computer simulations and well designed experiments are the best guide in this regime.

Application of E-Beams to Plasma Heating

Tokamaks

The use of intense E-beams for Tokamak heating has been suggested.[16] It offers the advantage of rapid heating of plasma electrons and subsequent equilibration with the ions on the time scale of $(m_i/m_e)\ \nu_{ei}^{-1}$ where ν_{ei} is the electron ion collision frequency. Since the heating time is much less than the confinement time for E-beam injection the total heating energy is minimized and the required energies are not much beyond the present capabilities of the largest machines. However, there are a number of problems that have to be investigated before E-beam heating is a serious contender:-

(i) Successful injection and trapping of E-beams into a Tokamak plasma contained in a high toroidal field. Some initial success has been obtained in injection experiments conducted by the Cornell[17] and P.I.[18] groups.

(ii) Plasma loss to the walls and increased impurity generation in the injection process.

(iii) Loss of beam electrons to the walls at injection port and elsewhere releasing impurities.

(iv) Effect of beam currents on macroscopic plasma stability; the stability may in fact be improved.

(v) Pulse repetitive rates for high power E-beams would have to be considerably improved to match the rest of the Tokamak system.

Multiple Mirrors

E-beams are inherently more suited to schemes that directly take advantage of their high power. They are strong competitors to systems that require high power lasers e.g. the D-T pellet compression[7,8] scheme and medium density, high β, CO_2 laser heated, scheme proposed by Dawson[19] et al. in which the heated plasma is confined in a very long solenoid with the ratio of length to radius of order 10^5. Budker[20] has proposed a variation of the Dawson scheme in which the magnetic field ($\sim$ 100 kG) of the solenoid is not high enough to confine the plasma but is sufficient to reduce the transverse heat conductivity to tolerable levels. The plasma pressure is transmitted to the confining rigid container walls through a layer of very dense cold plasma near the wall. The end losses are reduced by providing several magnetic mirrors in tandem on each side of the solenoid. The length ℓ of each mirror is adjusted to equal the ion mean free path λ, so that the particle motion along the field lines is a random walk instead of a free flow. The leakage velocity through the ends is decreased by a factor of $M^{[illegible]}\ell/\lambda$ compared to that through an homogeneous field, where M is the mirror ratio B_{max}/B_{min}. In Budker's design a reactor would consist of a conducting shell $\sim$ 10 m in length with an internal diameter of 10 cm with about 10 mirrors, M = 3 on either side. A plasma of density $10^{1[illegible]}\ cm^{-3}$ is heated by E-beams to a thermal energy content of $10^{[illegible]}$ MJ with a lifetime of 10^{-1} sec. The requirements on the E-beams are $10^{[illegible]}$ eV, $10^{[illegible]}$ A and pulse times of 10^{-4} sec. Present day capability of machines of this power is only for pulses of 10^{-7} sec. Thus, much further technological development is necessary for this application.

LINUS

E-beams have also been proposed for preheating of plasmas in the density range 10^{17} - 10^{18} cm^3 for reactors based on the principle of magnetic compression by imploding metallic liners (LINUS).[21] In this scheme most of the heating energy is provided by magnetic compression so that existing technology is capable of delivering the required preheat energy.

High Power Ion Accelerators

Quite recently high power, high voltage electrical technology, which is the basis of E-beam development, has been applied to create pulsed multi-kiloampere ion beams at energies ranging from 0.1 - 2.0 MeV by Humphries et al.[22] In a diode in which both cathode and anode are capable of emitting electrons and ions respectively the ratio of the space charge limited electron and ion current densities is $j_i/j_e = (Zm_e/m_i)^{1/2}$ where Z is the ion charge number. Thus electrons carry away almost all of the power delivered to the anode and the plain diode is quite inefficient for ion-beam production.

A simple device shown in Fig. 3 alleviates this problem. Two cathodes are located symmetrically on each side of an anode made of a highly transparent mesh. A fraction T of the electrons arriving at the anode pass through and are reflected by the opposite cathode. Thus, a large fraction of the emitted electrons circulate in the device and only a fraction $2(1 - T)/(1 + T)$ of the equivalent current for a solid anode constitutes the drain on the external power supply. The fraction of the electrons that actually collide with the anode deposit their energy to produce a plasma for ion emission. Successful results have been obtained from (i) coating metal anode meshes with hydrocarbons, (ii) nonconducting nylon meshes and (iii) aluminized mylar sheets of appropriate thickness for high energy $\gtrsim$ 1 MeV. Electrons produced at the cathode edge are affected strongly by the fringing electric field and follow an orbit that quickly takes them to the opaque anode holder. A magnetic field of sufficient strength parallel to the triode axis helps to prevent these electrons from drifting outwards and improves the device performance significantly. If one of the cathodes is removed to infinity the reflecting electrons form a virtual cathode at approximately the same distance as the actual cathode anode gap. Eliminating one of the cathodes allows for easy ion extraction. At the high achieved ion-current densities $\sim$ 10 A/cm^2 the propagation of unneutralized ion beams over any distance is not possible because of strong space charge repulsion. However, the ion beams in this device emerge automatically neutralized because they pick up an equal number of electrons from the surfeit of electrons around the cathode. These electrons need only a small fraction (m_e/m_i) of the ion energy to follow the beam. Thus, the beam is able to propagate in a good vacuum, unlike E-beams, with divergence properties determined by the initial emittance. Proton currents in excess of 3 kA at current density $\sim$ 10 A/cm^2, and total protons per pulse in excess of 10^{14} have been achieved. The best device efficiency, at 100 keV, achieved is 42% which is pretty close to the maximum possible of 50%. The current density is also close to the space charge limit. Table I summarizes the experimental data.

Pulsed Proton-Layer

The earliest application of relativistic electrons to a fusion device was suggested by Christofilos.[23] He proposed the "Astron" in which relativistic electrons were injected into a magnetic mirror to form a circulating layer of electrons (E-layer). When the

Table I

η = Proton Energy/Total Energy Input $\int VIdt$

Machine	100 kV Marx	500 kV, 7 Ω Blumlein	150 kV Blumlein	5 MV CREB
Ion Energy	~ 100 keV	300 keV	130 keV	2 MeV
Ion Current	2 x 250 A	2 x 2,050 A	2 x 3,300 A	~ 5,000 A
Pulse Width	~ 50 nsec	~ 50 nsec	~ 50 nsec	≤ 50 nsec
Current Density	~ 10 A/cm²	8.3 A/cm²	~ 20 A/cm²	~ 30 A/cm²
Efficiency η		9%	42%	
Type of ions	proton	proton	proton	proton & Al^{++}

density of this circulating current reached a level such that $Nr_e > \gamma$, where N is the number of electrons per unit length of the layer, the diamagnetic field of the E-layer would equal the external field. Any increase of the current density beyond this level would result in a region in which the lines of force would be closed, surrounded by a region of open lines of force of the mirror field (see Fig. 4). The plasma is confined in the region of closed lines of force which has ideal properties from the point of view of MHD stability. The plasma is heated by the energy delivered to it by the E-layer in collisions and the system was viewed as a steady state reactor. This concept ran into two difficulties. Given a realistic rate of plasma diffusion the size of the ensuing containment region demanded relativistic electrons ~ 100 MeV for a "breakeven" device. The synchrotron radiation from electrons of this energy is prohibitive and their energy is lost in a time less than that required for fusion. However, Christofilos pointed out that this difficulty can be overcome by using high energy ~ GeV protons (instead of electrons) which have negligible synchrotron loss. The second difficulty arose when it was found experimentally that successive injection of pulses of 6 MeV, 650 A, 300 nsec from an induction electron accelerator did not increase the E-layer axial current density but only increased its total length keeping the field reversal factor constant. Fortunately the Cornell group,[24] working with an intense E-beam of 500 kV, 40 kA, < 100 nsec was successful in producing field reversal in single pulse injection because the pulse had far more electrons than were needed for field reversal of the layer.

Encouraged by the success in producing intense ion beams from high power ion accelerators described earlier we can now begin to discuss reversed P-layers by single pulse injection. However, the proton energy required for a "breakeven" situation is still very much in excess of what is possible from diode technology. To overcome this limitation Sudan and Ott[25] suggested the adiabatic magnetic compression of a P-layer, formed by single pulse injection, to the required energy.

P-Layer Compression

Let the proton layer be of thickness Δ, mean radius R and length L such that $\Delta/L \ll 1$ and $\Delta/R \ll 1$ (see Fig. 4). In our calculations we assume for simplicity that $L \gg R$ but our results will be qualitatively correct even if $L \sim \pi R$. In order for the P-layer to be in equilibrium at these high current densities it has to be electrostatically neutralized by an equal number of electrons. The P-layer is located in a region of good vacuum and we exclude the possibility of electron conduction along the lines of force from external container walls. The compression time is assumed much smaller than the time taken by the flux to diffuse through the layer $4\pi\sigma\Delta^2/c^2$. In this limit both the axial flux Φ_a through the layer and the poloidal flux trapped within the layer Φ_p are conserved i.e.

$$\Phi_a = \pi R^2(B_{ex} + 4\pi I/L) = \text{const.} \quad ,$$

$$\Phi_p = 2\pi R(2\pi I\Delta/L) = \text{const.} \quad ,$$

where $I = I_B + I_e$ is the net azimuthal current composed of the beam current I_B and the electron current I_e and B_{ex} is the external magnetic field. From the conservation of flux we obtain

$$Ru_\varphi = \text{const.} \quad ,$$

where u_φ is the mean azimuthal beam velocity. Thus the beam current $I_B = Neu_\varphi/2\pi R$ scales as

$$R^2 I_B = \text{const.} \quad ,$$

where N the total number of beam ions is assumed to be conserved. The variation of Δ with compression is obtained from the constancy of the radial invariant $J_r \approx u_\perp^2/\omega_\beta$ where ω_β is the betatron frequency and the transverse beam pressure $\approx n_b m_i u_\perp^2$. An expression for ω_β including the effect of self fields is given by

$$\omega_\beta = \Omega\left[1 + \zeta u_\varphi/\Omega\Delta\right]^{1/2} \quad ,$$

where $\zeta = 4\pi I/LB_{ex}$ is the field reversal factor, $\Omega = eB_{ex}/m_i c$ is the cyclotron frequency in the external field. In the limit that $\zeta \ll \Delta/R$, $\omega_\beta \approx \Omega$ and Δ is determined by the spread in beam gyro centers, i.e.

$$\Delta/R \approx u_\perp/u_\varphi \propto B_{ex}^{1/2}/u_\varphi \quad .$$

In the opposite limit $\zeta > \Delta/R$ we have $\omega_\beta = (4\pi u_\varphi eI/m_i L\Delta)^{1/2}$. The transverse beam pressure is now balanced by the self-field i.e.

$$p_\perp = n_b m_i u_\perp^2 = B_s^2/8\pi = 2\pi I^2/L^2 \quad .$$

Since $n_b = N/2\pi RL\Delta$ and $u_\perp^2 \propto (u_\varphi I/L\Delta)^{1/2}$ we have $\Delta \propto (u_\varphi L/R^2 I^3)^{1/3}$. It is easy to see that the compression of a uniform B_{ex} does not increase the axial energy in a collisionless beam. From the conservation of the longitudinal invariant $v_\parallel^2/\omega_\parallel$ we observe that L remains constant. A consistent solution to all the constraints gives us

$$IR^2 = \text{const.} \qquad I_B R^2 = \text{const.}$$

$$B_{ex}R^2 = \text{const.} \qquad \Delta/R = \text{const.}$$

$$\zeta = \text{const.} \qquad L = \text{const.}$$

$$\text{and } u_\perp^2/u_\varphi^2 = \text{const.}$$

Defining C to be ratio of final to initial external magnetic field, the ion energy increases as C, I_B increases as C and the current density as $C^{3/2}$. It is, in principle, possible to achieve breakeven conditions by compression to high magnetic fields of a few hundred kilogauss. For field reversal the number of injected ions required is $\geq 10^{17}$; and the injected energy ~ 1 MeV. Such a layer can be formed by injection through a magnetic cusp[26] or by alternative techniques. Initial experiments already indicate that such injection is feasible.[27] Towards the end of the compression phase plasma is created within the P-layer by the introduction of D-T pellets which are rapidly ionized. Plasma heating takes place by collisions of the ion beam with the confined plasma. The characteristic slowing down time for the protons is greater than their scattering time for $T_e \sim 10$ keV and proton energy ~ 100 MeV. The topology of the magnetic field creates a natural divertor for the impurities from the walls which are swept away along the open lines of force.

Conclusion

I hope I have been able to demonstrate that particle accelerators play a key role in the fusion program even though these high power and high current accelerators have little resemblance to the accelerators of high energy physics. The target for the development of neutral beam accelerators is dictated by the heating requirements for the succession of even bigger Tokamaks leading to "breakeven" and finally reactors. The current enthusiasm about direct fusion between beam and target plasma in both closed and open systems creates a strong incentive for increasing the particle energy capability of these accelerators to 100 ~ 200 keV. This goal is probably accomplished by straightforward extrapolation of present techniques. Developments in E-beam technology take two different directions. On the one hand, inertial confinement D-T pellet approach requires highly focussed beams of short duration ~ 3 - 10 nsec while other approaches involving plasmas of medium density require high power capability over much longer durations of 10^{-6} to 10^{-4} sec. than presently achieved. This is a serious challenge. Intense ion-beam technology is still in its infancy. However, its promise should encourage rapid development and it may have applications beyond the Fusion Program.

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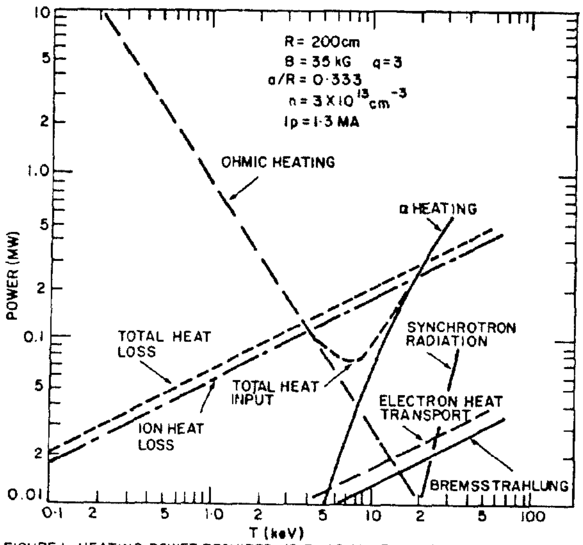

FIGURE 1. HEATING POWER REQUIRED VS PLASMA TEMPERATURE FOR A TOKAMAK REACTOR [D.R. SWEETMAN, REF(2)]

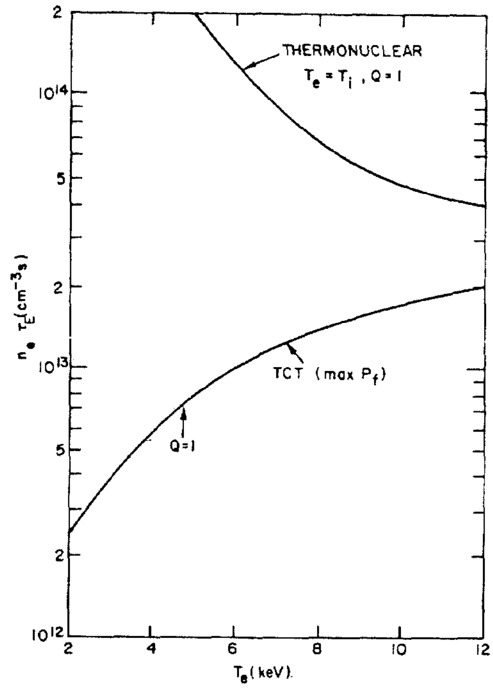

FIGURE 2. COMPARISON OF $n\tau$ VALUES FOR BREAK-EVEN IN D-T USING THERMONUCLEAR AND DIRECT FUSION (TCT) MODES FOR PROPOSED TCT-TFTR MACHINE [FROM REF (6), p 1-61]

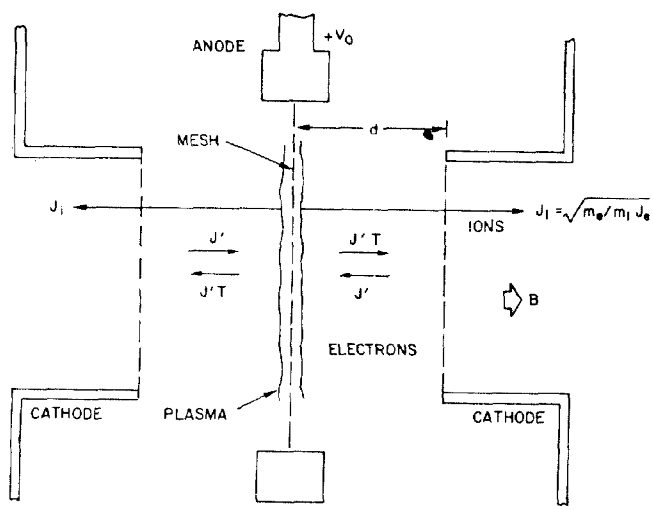

FIGURE 3 SCHEMATIC OF REFLEX TRIODE; NET ELECTRON CURRENT J_N DRAWN FROM EXTERNAL SUPPLY IS OBTAINED AS FOLLOWS: $J'(1+T) = J_{CL}$ (CHILD-LANGMUIR CURRENT), $J_N = 2(1-T)J' = 2(1-T)(1+T)J_{CL}$

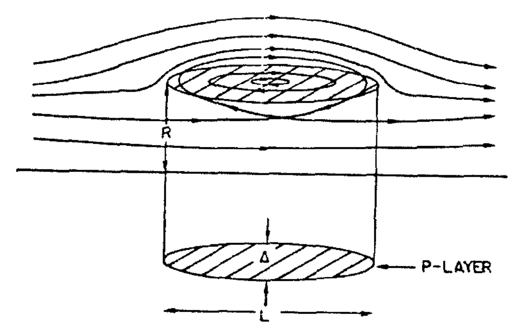

FIGURE 4. SCHEMATIC OF MAGNETIC FIELD OF AN E-LAYER OR PROTON-LAYER IN EQUILLIBRIUM IN AN EXTERNALLY GENERATED MIRROR FIELD