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COMPENSATION OF CHROMATIC ABERRATION IN A SINGLE PERIOD LATTICE*

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Summary

The requirement of high periodicity (even periodicity two) in a storage-ring lattice imposes a strong restriction on the number and type of insertions that can be included. On the other hand, since the insertions are matched only at one momentum, the β-function error for an off-momentum orbit is generally greater in a one-period lattice. This large β-error has several undesirable effects. All of these undesirable effects can be compensated or greatly reduced by judicious arrangement of betatron phasing and by the use of trim sextupoles. The design procedure and the resultant performance will be demonstrated for a model one-period lattice.

Introduction

Traditionally, accelerator designers have favored lattices consisting of a reasonably large number of identical periods in order to reduce the density of resonances arising from systematic errors in magnet construction and from other sources associated with the periodicity of the magnet ring. Present storage ring designs tend to have lower rotational symmetry than the synchrotrons due to the introduction of the various experimental insertions. At the same time, these rings contain features, such as beams containing a relatively broad momentum spread and regions where the amplitude functions become very large, which can make periodicity-associated effects of more concern. However, in contrast to the accelerators, a high periodicity conflicts directly with the intended use of the storage rings and so the consequences of a low symmetry structure must be examined.

The dependence of quadrupole focal length on momentum (chromatic aberration) leads to effects that may limit the performance of a low periodicity focusing system. In the standard matching procedure for a ring having a variety of insertions, one treats each insertion (including the normal cell) as a basic lattice period and associates with it an amplitude function. Where one such element joins to another, the parameters of each are adjusted so that the amplitude functions at this matching point are identical in magnitude and slope. Extension of this process to the entire ring results in a lattice the amplitude function of which is the same as that calculated for the separate elements.

The match obtained is valid for a particular set of quadrupole gradients, or equivalently, for a particular momentum. As the quadrupole current is varied from its design value, stopbands are found at a spacing in tune determined by the overall periodicity of the lattice - at every half-integer for one-fold rotational symmetry, at every integer for two-fold symmetry, and so on. An off-momentum particle finds itself in an unmatched lattice having a perturbed amplitude function. The perturbation may be regarded as a reflection of the presence of the neighboring stopbands. The change in amplitude function may lead to a significant degradation in luminosity at the intersection points of a storage ring and undesirably large beam sizes elsewhere.[1]

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In addition, the chromaticity, originating also from chromatic aberration in the quadrupoles, must be controlled to adjust the working line in the tune diagram.

In the next section, we suggest a prescription for sextupole distribution to compensate the effects noted above, and in the final section, illustrate its application to the particular lattice currently being studied for POPAE.

Sextupole Distribution

Sextupole fields must (in effect) be added to the quadrupoles in order to modify their chromatic aberration by virtue of the momentum dispersion of the orbit. In principle, one way of doing this would be to place a sextupole at each quadrupole with the sextupole strength chosen so that it is equivalent to adding a term $B'' = B'/\eta$ to the quadrupole gradient B'. Here, η is the momentum dispersion function.

But there are severe limitations to this approach. In the interaction regions, it is often desirable to set η to zero. Yet it is in these regions that a disproportionate contribution to the chromatic aberration effects arises. That this is so may be inferred as follows. The increment to the chromaticity ξ linear in $\delta p/p$ from an insertion may be written

$$\Delta\xi \equiv \delta\nu/(\delta p/p) = -(4\pi)^{-1}\int \beta(z)K(z)dz$$

$$= -(4\pi)^{-1}\int \left(\frac{d\alpha}{dz} + \gamma\right)dz = -(4\pi)^{-1}\int \gamma dz$$

where $K \equiv B'/(B\rho)$, α, β, γ are the usual Courant-Snyder parameters, and the integral is taken over the length of the insertion. The matching process requires that α be the same at either end of the insertion, hence the α' term does not contribute to the integral. So low beta insertions, where γ becomes large, exert great leverage on the chromaticity, and by extension, on the other chromatic aberration effects, however, it is precisely here that it is most inconvenient to introduce compensating sextupoles.

A second difficulty with this approach is that it provides no assurance that third-integral driving terms will be small. A valuable feature of a sextupole distribution plan is that it incorporates ab initio the requirement that third-integral resonance excitation be avoided.

The scheme we propose here places sextupoles in only the normal lattice cells, where η is inherently non-zero. A $\pi/2$ phase advance per normal cell provides a natural means for chromatic aberration compensation without excitation of third-integral resonances.

For simplicity, let us approximate quadrupoles and sextupoles by thin lens elements. For chromaticity control, we need to introduce

$$A = \Sigma_n\beta_n\eta_n S_n \quad ; \quad S \equiv B''\ell/(B\rho) = \text{sextupole strength}$$

in such a way that

$$B = \Sigma_n \beta_n \eta_n S_n e^{2i\phi_n} = 0 \quad ; \quad \phi = \text{betatron phase}$$

(so that off-momentum half-integral stopbands are not affected) and

$$C = \Sigma_n \beta_n^{3/2} S_n e^{3i\phi_n} = 0$$

(so that third-integral resonances are not excited). There is a total of six such relations, three each for the horizontal and vertical planes. Note that S has opposite signs in the two planes.

For FODO normal cells with $\pi/2$ phase advances in both planes, a simple arrangement is to place sextupoles of equal strength S_F at groups of four successive F quadrupoles and sextupoles of strength S_D at the associated D quadrupoles. For each group of four cells, we have

$$A_H = 4(\beta_{max}\eta_{max}S_F + \beta_{min}\eta_{min}S_D)$$

$$A_V = -4(\beta_{min}\eta_{max} S_F + \beta_{max}\eta_{min}S_D)$$

while for either a group of four F-sextupoles or D-sextupoles, B = C = 0. Thus, to first order in $\delta p/p$, S_F and S_D may be used to adjust the chromaticity in the horizontal and vertical planes without affecting the off-momentum half-integral or exciting third-integral resonances.

To influence the off-momentum half-integral stopbands, we want to introduce B terms in such a way that A = C = 0. This is accomplished by a modification of the foregoing procedure. Place sextupoles of strengths $+S_F$, $-S_F$, $+S_F$, $-S_F$ at the F quadrupoles and a similar set of strengths $\pm S_D$ at the D quadrupoles.

Now it is the B terms which are non-zero, while the A and C terms vanish. Total compensation of the off-momentum gradient error requires another set of B terms with a different phase. In a racetrack lattice, one possibility is to use sextupole strings in the two curved sections at the ends of the racetrack. The phase advances in the straight sections are generally not multiples of $\pi/2$. The two strings may then be tuned to produce a resultant B term of any amplitude and phase.

Application

We have applied the scheme for sextupole placement outlined in the preceding section to a provisional proton storage ring lattice developed for POPAE. This lattice is described in another paper in the proceedings of the conference;[2] for our purposes here its relevant features can be summarized as follows. The ring has the shape of a racetrack. In the arc at either end, there is a sequence of 41½ normal cells in which the amplitude and dispersion functions have the periodicity of the cell structure at the momentum for which matching was performed. These normal cells have a phase advance of $\pi/2$ for betatron oscillations. One side of the racetrack contains two low beta insertions with a minimum β^* of 1 m. The other long straight section of the racetrack contains a high angular resolution insertion with $\beta^* = 500$ m. In addition, there are three phase adjusting sections placed on both ends of the high beta insert and in between the two low beta insertions. Each phase adjusting insertion can vary the phase advance of betatron oscillations simultaneously in both planes over a range of from 105° to 205° without introducing maximum β values much higher than that in the normal cell.

Figure 1 illustrates the horizontal and vertical tunes versus momentum without introduction of sextupoles. As the periodicity of the lattice would imply, half-integral stopbands appear at both 35 and 35½. The tune spread across a beam stack having a width of 0.3% in momentum would be 0.2.

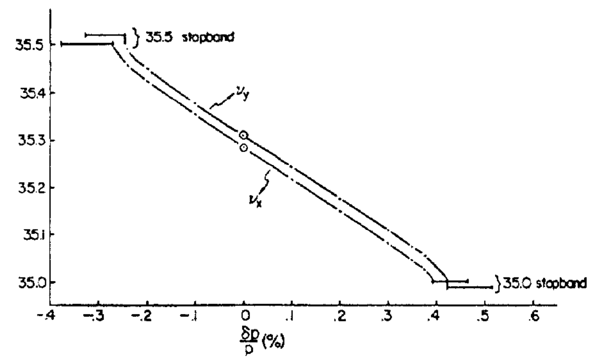

Fig. 1. Vertical (ν_y) and horizontal (ν_x) tunes versus momentum for storage ring lattice before introduction of sextupoles. Stopbands are displaced vertically from one another for clarity.

Now we place sextupoles for chromaticity compensation in 40 normal cells at each end of the racetrack - a total of 160 sextupoles -, and after adjustment of their strengths arrive at the tune versus momentum situation shown in Figure 2. The tune spread across

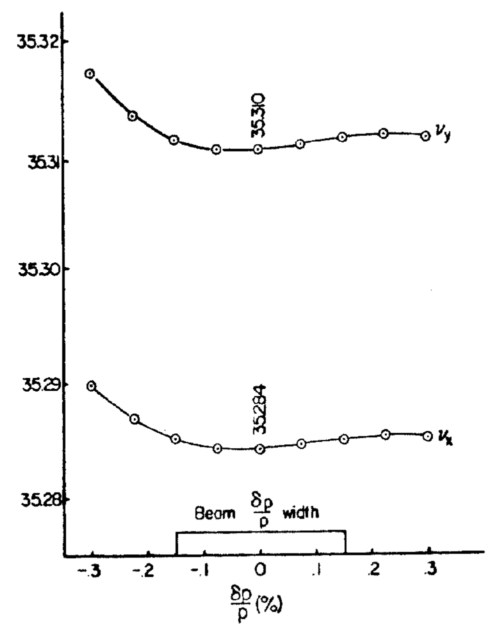

Fig. 2. Vertical and horizontal tunes versus momentum for storage ring lattice after introduction of 160 chromaticity compensating sextupoles in 40 normal cells in each of the north and south arcs.

the beam has been reduced by somewhat more than a factor of 100, and the stopbands have retreated outside the range covered in the Figure. The graphs suggest that the chromaticity can be controlled adequately by this means.

Turning now to the amplitude function variations, we have not as yet found it necessary to add sextupoles alternating in sign to yield non-vanishing B terms as proposed in the preceding section. Rather, the phase adjusting insertions may be used to advantage here. All that need be achieved is that the amplitude functions at the crossing points not be significantly momentum dependent and elsewhere the amplitude functions must remain within reason.

As suggested earlier, the low beta insertions are the major contributors to the modulation of the amplitude function. If we evaluate the amplitudes of the wave in $\delta\beta/\beta$ originating from the several insertions, we find that to first order in $\delta p/p$ the effect of each low beta insertion is larger by a factor of six than that of the high beta insertion and larger by a factor of twelve than the influence of a phase adjusting insertion when the latter is tuned for its maximum disturbance to the amplitude functions.

Since the low beta insertions are located near one another, a suitable tune of the phase adjusting insertion between them can significantly reduce the amplitude of the $\delta\beta/\beta$ variation throughout most of the rings. Figure 3 illustrates two settings of the phase

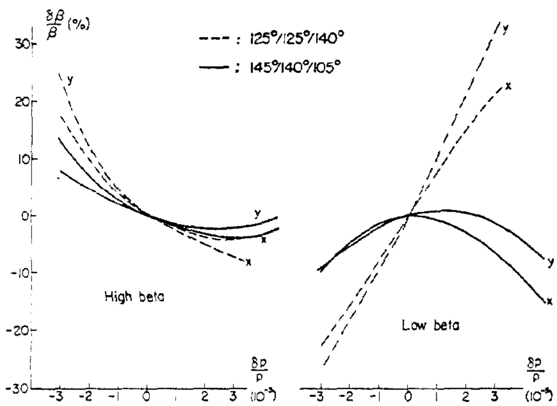

Fig. 3. Fractional change in the amplitude function as a function of momentum at the crossing point in the high angular resolution insertion and in one of the high luminosity insertions, for two settings of the phase adjusting insertions. The numbers separated by slashes at the top of the Figure are the settings of the three phase adjusting insertions in clockwise order starting from the north end of the rings.

adjusting insertions, one of which yields a relatively insensitive dependence of the amplitude functions on momentum at the intersection points. Though the situation at only one of the low beta regions is shown, the behavior at the other is similar. In the better of the two conditions presented in Figure 3, the magnitude of $\delta\beta/\beta$ for a $\delta p/p$ of 0.2% in either plane does not exceed about 11% throughout the ring exclusive of the region between the two low beta insertions; at a point between them, $\delta\beta/\beta$ becomes as large as 33%. In contrast, the tune of the phase adjusting insertions associated with the dashed lines in the Figure leads to a variation in $\delta\beta/\beta$ of 20% over most of the ring for the same $\delta p/p$.

We feel that the application to our lattice demonstrates that the scheme considered here is effective in neutralizing chromatic aberration effects. Several points must, however, be made. The compensation method outlined in this paper is attractive because of its conceptual simplicity, but a general analytical treatment for lattices having the complexity of present storage ring designs is at the very least difficult. The phase adjusting insertions have been included in the provisional POPAE lattice in order to provide readily variable parameters in this study, and they have indeed been valuable as indicated above. Insertions which vary the vertical and horizontal phases independently may also be useful. The near-contiguity (accidental, insofar as the purposes of this paper are concerned) of two identical low beta regions has also proved to be of value; were this not the case, we would have likely been led to a study of non-vanishing B terms. Finally, our approach has been to worry first about phenomena linear in $\delta p/p$; having disposed of the first order effects, one may well wish to examine the possibilities for removal of higher order terms (the curvatures of $\delta\beta/\beta$ in Figure 3) by octupoles, etc.

Acknowledgements

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References

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