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CALORIMETRIC DETERMINATION OF BEAM ENERGY*

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Summary

A high-precision calorimetric system has been developed for determining beam energy. The beam is square-wave (on-off) modulated and the time derivative of the calorimeter temperature is determined at a time late in the modulation half-cycle. This avoids sensitivity variations associated with beam size and location on the calorimeter. The data are collected and analyzed in a small on-line computer. The use of beryllium, with its very small backscatter coefficient for electrons, permits the use of the method for electron accelerators. Using such a calorimeter with a high-stability electron accelerator, standard deviations of the order of 0.03 percent for series of 20 measurements were achieved for energies from 0.025 to 1 MeV over several hours time period. The calorimeter system is usually calibrated with a low energy (~ 20 keV) positive ion beam whose energy is well established. Where lower accuracy is acceptable, the sensitivity may be determined from first principles to within 1 percent. The method will be described and the optimization of the various parameters discussed.

Introduction

A problem which occurs in several disciplines is that of determining the energy of charged particles from an accelerator or, under certain circumstances, the accelerator potential. In the present paper, a calorimetric method is described which has high precision and which has produced a convenient solution to this problem.

A number of methods have been used for the above purposes. These include beam analysis by static fields, both electric and magnetic; ionization, both in gas and solids; scintillation; particle range in various materials; time-of-flight measurement; and, calorimetry. Accelerator potential has not only been inferred from particle energy measurement but also measured more directly by potential dividers and generating voltmeters. These methods will not be discussed but are pointed out simply to indicate the great variety of techniques available for making such measurements.

Calorimetric measurements have been used for several purposes in connection with ion beams for accelerators. The fact that the quantity measured is deposited energy minimizes the effect of secondary processes. Thus, a calorimeter may be used to measure the intensity of an ion beam which has entered a gas-filled region, avoiding the troublesome problems of ionization and secondary electron currents. Further, neutral beam intensity can be measured equally well if the average particle energy is known. In determining cross sections for interactions of energetic neutrals with gaseous targets, such a device is very helpful. The authors[1] have used calorimeters for these purposes but certainly cannot claim that such usage is original with them.

A somewhat different application has involved the determination of electron energy deposition as a function of depth for a beam of energetic electrons incident on various metals.[2] By making a thin-foil calorimeter of the same metal under investigation and locating it at various depths in the stack of metal foils in which the measurement was made, an energy deposition profile was obtained. Precision of the order of 1 percent was achieved. In order to achieve this result, however, several innovations were required. These form the basis for a system which we have used to measure the beam energy with precision of the order of 0.1 percent. These will now be noted below.

A major difficulty with a d.c. calorimeter is the long-term temperature drift of the system. The calorimeter will, of necessity, be supported from a heat sink which can undergo long-term temperature change. A means of minimizing this effect is to modulate the energy input so that one makes an a.c. measurement. In our case we use square-wave (on-off) modulation of the particle beam. With appropriate choice of time constants, the resulting thermocouple signal is approximately triangular in shape. One would expect that the amplitude of the thermocouple signal would be a good measure of deposited power but, unfortunately, the amplitude depends on the radial profile of the particle beam. We avoid the problem by taking the time derivative of the thermocouple signal at a time late after switching in each half cycle. It will be shown in the next section that the output thus obtained is proportional to the power input and is largely independent of the radial profile of the beam.

Design Considerations

The case to be considered has been examined elsewhere[3,4] so only a brief summary is presented here in order to facilitate the understanding of the design considerations.

Assume that the calorimeter is in the form of a thin disk of radius r_o and thickness w. Let the density, thermal conductivity, and specific heat be ρ, K, and C, respectively. Let it be supported by N radial wires attached at equal arc lengths at the periphery, each wire having length L, cross sectional area A, and thermal conductivity K_w. The temperature T(r,t) is compared to the heat sink temperature T_o, so that we use the quantity

$$u(r,t) = T(r,t) - T_o \quad .$$

For a modulation period t_o, the equilibrium solutions are:

$$u(r,t') = \sum_{n=1}^{\infty} A_n J_o(k_n r)\left(1 - \mu_n e^{-t'/\tau_n}\right) \quad \text{(Beam on)}$$

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$$u(r,t'') = \sum_{n=1}^{\infty} A_n J_o(k_n r)\, \mu_n e^{-t''/\tau_n} \quad \text{(Beam off)}$$

Here t' and t" are measured from the times at which the beam is turned on and off, respectively. The k_n's are the eigenvalues of the boundary condition. The τ_n's are the characteristic time constants and are given by:

$$\tau_n = \frac{\rho C}{K\,(\alpha^2 + k_n^2)}$$

where

$$\alpha^2 = \frac{8\,\sigma\,\epsilon\,T_o^3}{wK}$$

is the thermal radiation term. σ and ϵ are the Stefan-Boltzmann constant and the emissivity, respectively. It is to be noted that the τ_n's decrease monotonically and that

$$\tau_1 \gg \tau_2 > \tau_3, \text{ etc.}$$

The μ_n's are the equilibration factors which match the two solutions at each switch time and are given by:

$$\mu_n = \left(1 + e^{-t_o/2\tau_n}\right)^{-1} .$$

The A_n's are obtained from the initial condition and are:

$$A_n = \frac{\dot{Q}\,\tau_n}{\pi r_o^2 w\rho C} \cdot \frac{1}{\left[J_o^2(k_n r_o) + J_1^2(k_n r_o)\right]} \cdot \frac{2\,J_1(k_n r_m)}{k_n r_m} .$$

In this expression it is assumed that the heat is deposited uniformly over a radius r_m and that the total rate of heat input is then $\dot{Q} = \pi r_m^2 \dot{q}$. It should be noted that any other radial distribution of heat deposition can be approximated by a summation of terms like this with appropriate $\dot{Q}$'s and r_m's. The last factor, $2\,J_1(k_n r_m)/k_n r_m$, is very dependent on r_m for all n's except $n = 1$. Thus, if one were to use the amplitude of the a.c. signal from the thermocouple as a measure of $\dot{Q}$, the coefficient would depend strongly on the radial profile of the beam. For the first term, however, the factor is relatively insensitive to $k_1 r_m$. For $k_1 r_o$ of 0.3 (a reasonable value) the factor only changes by about 1 percent as r_m is varied from zero to r_o, with most of this change occurring near r_o.

The foregoing suggests that, if there is some means of isolating the first term of the series for $u(r,t)$, the dependence on r_m could be minimized. This was accomplished by taking the time derivative of the thermocouple signal at a time t_d after switching subject to the condition:

$$t_d \gg \tau_2, \tau_3, \text{ etc.}$$

To a very good approximation, the time derivative then becomes:

$$\frac{du}{dt} = \pm \frac{\dot{Q}}{\pi r_o^2 w\rho C}\,\mu_1\, e^{-t_d/\tau_1} .$$

Note that the factors $2\,J_1(k_1 r_m)/k_1 r_m$ and $J_o^2(k_1 r_o) + J_1^2(k_1 r_o)$ are nearly unity. From the above expression one concludes that the time derivative of the thermocouple signal is proportional to the product of the particle current and the average particle energy.

We now consider factors involved in the design of such a device. First, it is clear that the areal density must be at least equal to the maximum particle range. Because a price in sensitivity must be paid for additional thermal capacity, one should not use significantly greater thickness than is required by particle range considerations. For the same reason, the radius should be no larger than required in order to easily accommodate the radial spread and uncertainty in position of the beam.

The choice of calorimeter material depends on several factors. In general, it is desirable to have fast time response, which translates into short time constants. This in turn requires a high diffusivity ($K/\rho C$). Silver, graphite, gold, copper, and aluminum are particularly good in this respect. Backscatter of particles is particularly important. For heavy particles, such as protons, backscatter does not pose a significant problem. For electrons, however, it can be very important. Electron energy backscatter increases monotonically with Z, the atomic number, so it is essential to use a low-Z material. In this regard, beryllium is the best of the metals, the backscattered energy being less than 1.2 percent for incident electrons of energy greater than 25 keV.[5]

The support wires influence both the sensitivity and the time constants. As the wires become longer, approximations which led to the expressions given for A_n and τ_n are no longer valid. A more exact analysis indicates that longer wires increase τ_n and decrease A_n, neither of which is desirable.[4]

The choice of wire material is important, also. Generally the time constants (and particularly the first one, $n = 1$) will be too great if radiation, through the α term in the demoninator of the expression for τ_n, is the major heat loss mechanism. Since the wires serve to remove heat by conduction, one wishes to find a wire material for which the conductance-to-thermal capacity ratio is a maximum. This is equivalent to having the maximum thermal diffusivity. In a practical case, silver, gold, copper, or aluminum are all suitable. The radius of the wire is then selected to provide a suitable first-time constant.

The temperature sensor most generally used is the thermocouple. A variety of combinations of dissimilar materials provide satisfactory results. The chromel-constantan combination provides good sensitivity and has been used in the present studies. The reference junction can be imbedded in the heat-sink ring from which the calorimeter disk is suspended. An obvious advantage to the present method is that one need not provide a carefully-controlled reference as long as the changes in temperature of the heat sink are very slow compared to the modulation period. The active junction itself is usually situated at the edge of the calorimeter, i.e., at r_o. However, if fast response is essential, it is advantageous to locate the thermocouple at an intermediate radius r_1. This radius is chosen so that $J_o(k_2 r_1) = 0$ so that the second term of the series for $u(r,t)$ drops out. This permits the taking of the time derivative at an earlier time after the beam is switched so that a shorter modulation period can be used. If one thermocouple does not provide adequate sensitivity, a number of junctions may be placed in series with electrical isolation between the junctions being provided by a very thin layer of a vacuum-grade epoxy.

There are three "times" in the system which must be chosen. These are the first time constant of the calorimeter, the modulation time, and the time at which the temperature derivative is taken. Of these, the calorimeter time constant is the most important and the other two will depend strongly on it. Used in the modulation scheme described previously, the calorimeter system can be shown to have an effective time constant of $2\tau_1$. Beam fluctuations having characteristic times $\gg 2\tau$ will be read as a varying beam; those much less will be averaged out. Once the material, radius, and thickness have been chosen one has only limited control over the value of τ_1. In the expression for τ_1, k_1 is varied by the choice of heat conduction by the support system. In order to change the radiative term, α^2, one might change the emissivity by appropriate treatment of the surfaces.

Examination of the time factor in the sensitivity expression shows the following.

1. For a given ratio of τ_1/t_o, there is a time at which one can take the derivative so that small changes in τ_1 do not affect the sensitivity. This time is

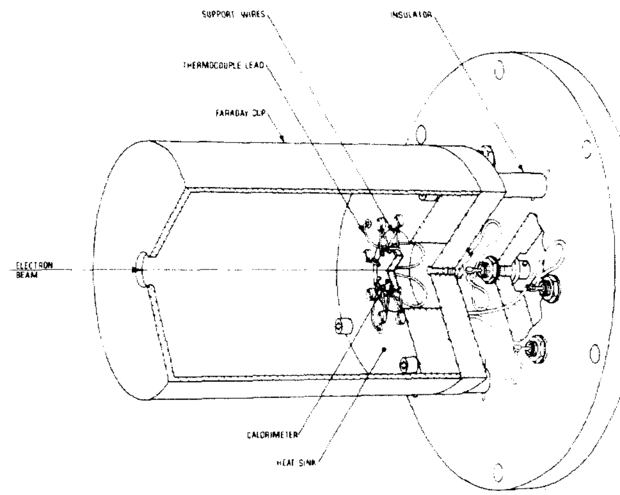
$$t_d = [t_o/2]e^{-t_o/2\tau_1}\left[1 + e^{-t_o/2\tau_1}\right]^{-1} .$$

2. As the value of t_o/τ_1 is increased, the optimum value of t_d increases, reaching a maximum of about $0.28\ \tau_1$ at $t_o/\tau_1 = 2.6$. Further increases in t_o results in a monotonic decrease in the optimum value of t_d.

As a practical matter, t_o must be great enough so that the derivative, taken at near-optimum time, is free of significant contributions from the terms in τ_2, τ_3, etc. Otherwise it is not critical.

Determination of Beam Energy for a 1 MeV Electron Accelerator

As a specific example we describe a system which has been used to determine the particle energy for an electron beam from a 1 Mv electrostatic accelerator. This system is shown in Fig. 1. Because very low backscatter was essential, the calorimeter was made of beryllium. The dimensions were 2.5 cm in diameter and 0.38 cm thick. The areal density was 0.704 gm - cm^{-2}, the equivalent of 1.31 mean ranges. This was supported by 8 copper wires, 0.025 cm in diameter and 2.31 cm long. These were attached to a heavy aluminum ring which acted as a heat sink. The resulting time constant was 120 sec. Three chromel-constantan thermocouples, connected in series, were attached at the periphery of the calorimeter disk by a thin coating of vacuum epoxy. The entire assembly was housed in a Faraday cup of aluminum having a wall



CALORIMETER ASSEMBLY

Figure 1

thickness greater than the mean electron range so that electrons entering the cup aperture could be lost only by backscattering through the entrance aperture. The probability of this was very low--of the order of 10^{-3}.

The data were collected and analyzed in an on-line digital computer. A typical run consisted of 6 cycles (t_o = 80 sec) during which the Faraday cup current and thermocouple signals were digitized and read at 0.54 sec intervals. Rather than measure the derivative only at the specific time (16.6 sec after beam-switch), the logarithm of the derivative was calculated at a number of times during the cycle. A least squares fit of log-derivative as a function of time-after-switch was made by the computer and the derivative at the desired time was then automatically calculated from this fit. This had the effect of using a very large fraction of the collected data, rather than just that at the derivative time. The result presented by the computer was the quantity $1/i\ dv/dt$, calculated at the desired time as indicated above. The use of 6 cycles of data permitted the averaging out of small beam current variations.

The system was calibrated by means of a heavy ion accelerator. The ion beam from this device was used in exactly the same manner as the electron beam. The accelerator potential, usually a few 10's of kilovolts, was measured carefully by means of a calibrated precision potential divider.

The results thus obtained for the electron accelerator are shown in Table 1. Here the calorimeter readings are compared with readings from a generating voltmeter and with an independent calibration. In the latter method, pulses from electrons from thin conversion sources produced in a solid ion chamber were compared with those from beam electrons. In order to reduce the electron beam flux to a sufficiently low value, it was necessary to scatter the beam from a thin wire into the detector. It can be seen that the agreement is generally within the estimated errors.

Table 1

Electron Beam Energy (MeV)

Generating Voltmeter[1,2]	Conversion Electrons[3]	Calorimeter[4,5]
1.203		1.153
1.071	0.998	1.033
0.9640		0.9241
0.8576		0.8241
0.7512		0.7207
0.6445		0.6188
0.5385	0.505	0.5212
0.4319		0.4140
0.4055	0.367	
0.3256	0.312	0.3140
0.2192		0.2122
0.1130	0.095	0.1093
0.0598		0.0580
0.0332		0.0324

[1]Electron gun bias voltage has been included.

[2]All readings ± 0.0005 MeV.

[3]All readings ± 5 percent.

[4]Corrected for backscattered energy (TIGER[6])

[5]All readings ± 0.1 percent.

The precision of this method is very good. For example, the standard deviation of the mean for 20 determinations at 0.5 MeV was 130 eV. Comparable results are obtained at other energies, except that the values near 1 MeV show a relatively greater standard deviation. This probably represents real fluctuations in the accelerator potential.

Conclusions

On the basis of our experience, we believe that the calorimeter system, using the methods described herein, provides an excellent measure of beam particle energy. The precision is high and is comparable or better than that attained in more commonly used methods. The apparatus is simple and does not require highly accurate machining. Although the present system uses an on-line computer for data collection and reduction, this is not a necessity.

A weakness of the present system is the desirability of frequent calibration. While no significant variations in the sensitivity with time have been observed, one would hesitate to assume that such variations do not occur. Such changes would be directly attributable to changes in thermocouple sensitivity. This problem, should it exist, could be solved by a servo system. In this scheme, the reference junction temperature would be varied by i^2R heating of an attached strip so that the thermocouple output is nulled. In this manner a drift in thermocouple sensitivity would affect slightly the gain of the servo loop but would not change the sensitivity of the system.

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References

1. J. M. Hoffman, G. J. Lockwood, and G. H. Miller, Phys. Rev. A7, 118 (1973).

2. G. H. Miller, G. J. Lockwood, and J. A. Halbleib. IEEE Trans. Nucl. Sci. NS-21, No. 6, 359 (1974).

3. G. H. Miller, Calorimetric Measurement of Dose-Depth Profiles for Incident Electrons, Sandia Laboratories Report SLA-74 0011, June 1974.

4. G. J. Lockwood and G. H. Miller, Calorimetric Determination of Energy Deposition by Electrons in Tantalum and Layered Media, Sandia Laboratories Report SAND-74 0190, September 1974.

5. G. J. Lockwood, G. H. Miller, and J. A. Halbleib, to be published.

6. J. A. Halbleib, Sr., and W. H. Vandevender, Nucl. Sci. Eng., to be published.