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ANALYSIS OF THE RESONANT TRANSFORMER ACCELERATOR IN THE GATED AND UNGATED BEAM MODE*

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INTRODUCTION

The resonant transformer accelerator has found use in both average power[1,2,3] and pulsed power[4,5] applications. In this paper an analysis is presented of some of the operational requirements, capabilities, and limitations of such systems. Many subtle engineering details (e.g. half-wavelength transformer resonances) are not considered, since they do not affect the general results. Careful attention must be given to them, however, before a compact and reliable unit can be realized.

GATED BEAM MODE

Figure 1 shows the basic electrical configuration for this version of the resonant transformer accelerator. It finds greatest use in industrial applications since it is readily adaptable to high repetition rate (average power) operation. The system consists of two transformer coupled series resonant circuits. Energy stored in the primary capacitor (C_1) is switched out through a conventional hydrogen thyratron and transiently transferred to the secondary capacitor (C_2). The transformer provides voltage gain. Near the peak of the secondary capacitor voltage cycle, the energy is used to drive a pulsed electron beam load -I(t). The beam is formed by gating a gridded thermionic cathode, and then accelerated down a graded acceleration tube. Any energy remaining in the circuit after the beam is turned off is returned to the primary capacitor through a diode string, in this way improving overall system efficiency. The system operating frequency must allow sufficient time to return the thyratron to its full hold-off voltage during the recharging of the primary capacitor.

In practical systems, series losses will occur (due to skin effect heating, capacitor losses, and thyratron and diode losses) which can be represented by equivalent lumped resistances (R_1 and R_2). Also, any primary circuit magnetic flux which does not couple into the secondary circuit is represented by a stray series inductance (L_3). Defining the quantities

$$M = k\sqrt{L_1L_2} \quad = \text{transformer mutual inductance}$$

$$W_1^2 = 1/(L_1 + L_s)\, C_1$$

$$W_2^2 = 1/L_2C_2$$

$$Q_1 = \sqrt{(L_1 + L_3)/C_1}\;/R_1$$

$$Q_2 = \sqrt{(L_1/C_2)}\;/R_2$$

$$k_{eff} = \sqrt{\frac{L_1}{(L_1 + L_3)}}\; k$$

$$G_v = \sqrt{L_2/L_1 + L_3)}$$

$$W_\pm^2 = \frac{1}{2}\,\frac{W_1^2 + W_2^2}{(1-k_{eff}^2)}\left\{1 \pm \sqrt{1 - 4(1-k_{eff}^2)\,\frac{W_1^2\,W_2^2}{(W_1^2+W_2^2)^2}}\right\}$$

$$\alpha = \frac{1}{2}\,\frac{\frac{W_1}{Q_1}(W_+^2 - W_2^2) + \frac{W_2}{Q_2}(W_+^2 - W_1^2)}{(W_+^2 - W_-^2)(1 - k_{eff}^2)}$$

$$\beta = \frac{1}{2}\,\frac{\frac{W_1}{Q_1}(W_2^2 - W_-^2) + \frac{W_2}{Q_2}(W_1^2 - W_-^2)}{(W_+^2 - W_-^2)(1 - k_{eff}^2)}$$

it can be shown that given the approximation

$$\alpha << W_+$$

$$\beta << W_-$$

the secondary capacitor voltage has the following Laplace transform:

$$\tilde{V}_L = \tilde{V}_1 + g(s) - h(s)\, g(s) \tag{1}$$

where

$$V_1 = \frac{k_{eff}}{1-k_{eff}^2}\, G_v V_o\, \frac{W_2^2 s}{(S^2+2\alpha s+W_+^2)(S^2+2\beta s+W_-^2)} \tag{2}$$

$$h(s) = \frac{W_2^2}{(1 - k_{eff}^2)}\;\frac{S^2+\frac{W_1}{Q_1}\, s + W_1^2}{(S^2+2\alpha s+W_+^2)(S^2+2\beta s+W_-^2)} \tag{3}$$

and

$$g(s) = \frac{1}{sC_2}\,\tilde{I} \tag{4}$$

The inverse transform gives

$$V_L(t) = V_1(t) + \frac{1}{C_2}\int_0^t I(t')dt' - \int_0^t H(u)G(t-u)du \tag{5}$$

where

$$V_1(t) = \frac{k_{eff}}{(1-k_{eff}^2)}\, G_v V_o\, \frac{W_2^2}{W_+^2-W_-^2}\left\{e^{-\beta t}\mathrm{Cos}W_-t - e^{-\alpha t}\mathrm{Cos}W_+t\right\} \tag{6}$$

The requirement for resonant operation can be derived from equation (6) by considering the lossles response ($\alpha = \beta = 0$). The remaining time dependence maximizes

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when

$$W_+ = 2\,W_- \tag{7}$$

which, together with the definition of W_+ and W_-, gives the following necessary relationship between effective transformer coupling ($k_{eff} \leq k$) and the primary to secondary tuning ratio ($X = W_1/W_2$):

$$\frac{1+X^2}{X} = 2.5\sqrt{1-k_{eff}^2} \tag{8}$$

Using this equation, it can be shown that resonance operation can be achieved for any value of effective coupling lying in the range

$$0 < k_{eff} \leq 0.6$$

provided the tuning ratio is selected properly. This fact is what makes resonant transformer work in practice; any departure in transformer coupling from the chosen design value can be compensated for by fine tuning the primary circuit frequency (by adjusting C_1).

For systems with losses, equation (6) gives the degradation in secondary voltage. To calculate the system efficiency, it is necessary to consider the primary capacitance voltage. For an unloaded system ($I(t) \equiv 0$), it is given by the following equation:

$$\frac{V_c(t)}{V_o} = \frac{1}{3(1+X^2)}\left\{(4-X^2)e^{-\beta t}\mathrm{Cos}W_-t+(4X^2-1)e^{-\alpha t}\mathrm{Cos}2W_-t\right\} \tag{9}$$

where it is assumed that the resonance condition (equation (8)) is satisfied. There are two sources of loss: one is due to series resistances R_1 and R_2, while the other arises from charging losses when resistive isolation of the power supply is used. The first can be expressed as

$$\frac{\Delta E_1}{E_o} = 1-\left(\frac{V_c(W_-t=2\pi)}{V_o}\right)^2$$

while the second is

$$\frac{\Delta E_2}{E_o} = \left(1-\frac{V_c(W_-t=2\pi)}{V_o}\right)^2$$

where

$$E_o = \frac{1}{2}C_1V_o^2 = \text{initial stored energy}$$

The overall efficiency is then

$$\eta = 1-\frac{\Delta E_1+\Delta E_2}{E_o} = 2\,\frac{V_c\ (W_-t=2\pi)}{V_o} - 1 \tag{10}$$

which is a function of the tuning ratio. Figure 2 shows the computed efficiency for several values of $Q_1 = Q_2$. Note that lower values of tuning ratio are preferable; however, this must be compatible with the requirement of obtaining reasonably high energy transfer efficiency from the primary to secondary capacitor. Another constraint is the desire to provide a long clearing time for the hydrogen thyratron. In practice, an optimum compromise is realized for a tuning ratio of

$$X = \sqrt{\frac{8}{17}} \tag{11}$$

In this case, overall efficiency is high and approximately two-thirds of the initially stored energy is transiently transfered to the secondary. Most importantly, the thyratron is not required to conduct current during the second half of the resonance cycle ($\pi \leq w_-t < 2\pi$). For higher tuning ratios, the thyratron must conduct current for a portion of the second half-cycle, and this strains its voltage recovery capability.

The case of interest is, of course, a lossy system which drives an electron beam load. It is then necessary to gate the beam in such a way that:

- there is little or no beam energy flunctuation (that is $I(t) \simeq C_2\,\frac{dV_2}{dt}$)
- as much energy as possible is delivered to the electron beam load.

While the first requirement can be met by making the load current ($I(t)$) a complex function of time, in practice it is a simpler engineering problem to choose the following form:

$$I(t) = I_o\left\{U(t-t_1) - U(t-t_2)\right\} \tag{12}$$

where

$$U(x) = \begin{cases} 0, & x<0 \\ 1, & x>0 \end{cases}$$

The current loading can be expressed as a dimensionless variable by defining

$$\varepsilon = \frac{I_o}{W_2C_2V_2(t=t_1)} \sim \left(\frac{I_o}{C_2\,\frac{dV_2}{dt}}\right)$$

A practical limitation on the beam energy fluctuation is $\pm$ 10%, and for illustration purposes the tuning ratio given by equation (11) will be assumed. Table 1 summarizes the results of a numerical study of this case. The main feature which emerges is that for real systems ($Q_1 \approx Q_2 \approx 30$), best performance is realized by gating the electron beam on as early as possible during the negative-going excursion of the secondary capacitor voltage. Note that even without charging losses the system efficiency is not high; circuit (Q) losses tend to dominate. This clearly emphasizes the importance of good engineering design to minimize Q.

UNGATED BEAM MODE

In this application, the resonant transformer circuit is used to pulse charge a fast transmission line store. Figure 1 is still applicable with the following modifications: the secondary capacitor is now a transmission line and, if deionized water is used as the dielectric media, the resistor R_2 shunts the secondary capacitor. Since energy recuperation is not required, the primary circuit switch can be a simple spark gap.

For this configuration,

$$Q_2 = \frac{R_2}{\sqrt{\frac{L_2}{C_2}}} = W_2\tau$$

where

$\tau = \epsilon_o\epsilon_r\rho$ = relaxation time of dielectric medium

and

ϵ_r = relative dielectric constant of water ≈ 80

ρ = volume resistivity of the deionized water

Equation (2) still applies if α and β are changed to:

$$\alpha = \frac{1}{2}\,\frac{\frac{W_1}{Q_1}(W_+^2 - W_2^2) + \frac{W_2}{Q_2}(W_+^2(1-k_{eff}^2) - W_1^2)}{(W_+^2 - W_-^2)(1 - k_{eff}^2)}$$

$$\beta = \frac{1}{2}\,\frac{\frac{W_1}{Q_1}(W_2^2 - W_-^2) + \frac{W_2}{Q_2}(W_1^2 - W_-^2(1-k_{eff}^2))}{(W_+^2 - W_-^2)(1 - k_{eff}^2)}$$

If skin resistance in the transformer primary winding is the main contributor to R_1, then Q_1 can be written as

$$Q_1 = \sqrt{\frac{\mu_o}{2\rho'}}\;\frac{R}{1+R/l}\;\sqrt{W_1} \approx \sqrt{W_1}$$

where

ρ' = volume resistivity of the winding material

R = mean radius of the primary winding (cm)

l = total length of the primary winding (cm)

This is usually a large number ($Q_1 > 100$), and for design purposes can be made infinite.

The energy transfer efficiency is given by the equation

$$\eta = \frac{C_2}{C_1}\left(\frac{V_2(Wt = \pi)}{V_o}\right)^2$$

$$= \frac{25}{9}\,k_{eff}^2\,\frac{X^2}{(1+X^2)^2}\left\{e^{-\pi\,\beta/W_-} + e^{-\pi\,\alpha/W_-}\right\}^2$$

Assuming Q_1 is very large, maximum efficiency occurs for X = 1, and to obtain at least 90% transfer requires $Q_2 \geq 20$. In terms of effective stress time (the time during which the voltage is greater than 63% of peak), this implies

$$t_{eff} \leq \frac{\rho''}{3.18}\quad \mu\text{sec}$$

where ρ'' is the volume resistivity of the water in megohm-cm.

For unity tuning ratio, the transfer efficiency can be written as

$$\eta = e^{-\frac{1.987}{Q_2}}$$

so that

$$E = \frac{1}{2}C_1 V_o^2\, e^{-\frac{1.987}{Q_2}}$$

$$= \frac{1}{2}\frac{\tau^2}{Q_2^2}\, e^{-\frac{1.987}{Q_2}}\,\frac{V_o^2}{L_1}$$

This expression has a maximum at

$$Q_2 \approx \frac{1.987}{2} \approx 1$$

so that

$$E \leq 3.5 \times 10^{-12}\,(\rho'')^2\,\frac{V_o^2}{L_1}$$

with a transfer efficiency of about 13.5%. Using the following representative numbers:

V_o = 200 KV ($\pm$ 100 kV charging)

$L_1 \approx 1\mu H$

$\rho'' \approx 10$ megohm-cm

then the maximum amount of energy which can be transferred is 14 megajoules, with 104 megajoules initially in the primary. At 50% transfer efficiency, 6.2 megajoules can be put into the secondary. If high loss is acceptable, the transformer can in prinicple pass enormous amounts of energy. For efficient operation (90% transfer efficiency),

$$E \leq 6.4 \times 10^{-14}\,(\rho'')^2\,\frac{V_o^2}{L_1}$$

which, using the same numbers as above, limits the amount of energy in the secondary to 250 kilojoules. Within a factor of roughly two, this is the limit attainable without resorting to major engineering refinements (cooling of the water, unsymmetric feeding of the transformer primary, etc.).

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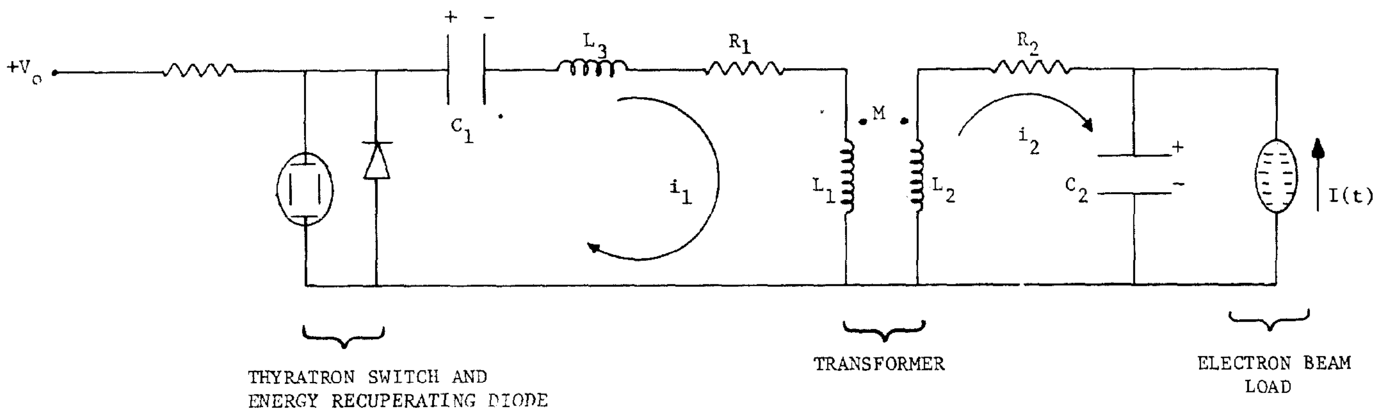

Figure 1. CIRCUIT SCHEMATIC.

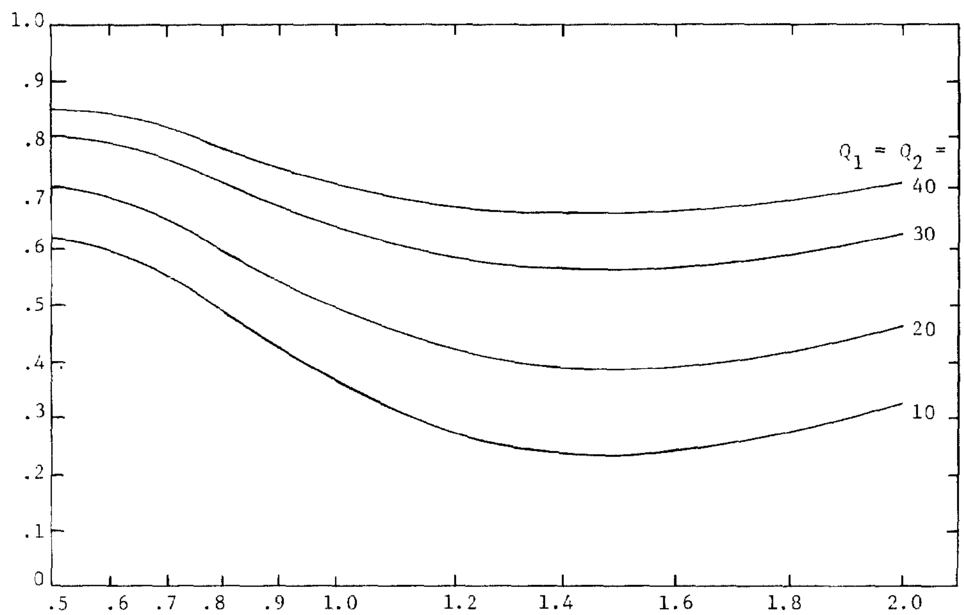

Figure 2. SYSTEM EFFICIENCY VERSUS TUNING RATIO.

TABLE 1

PERFORMANCE OF ELECTRON BEAM LOADED ACCELERATORS

$X = \sqrt{\frac{8}{17}}$ $\quad Q_1 = Q_2 = 30 \quad$ $\frac{\Delta V_B}{V_B} = \pm 10\%$

Turn-On Time $W_- t_1$	Pulse Width $W_-(t_2-t_1)$	Beam Voltage V_B/V_{MAX}	Current Loading ε	Energy Extracted E_B/E_o	η (no charging losses)
150°	60°	70%	0.5	27%	55%
155°	50°	79%	0.26	15%	40%
160°	40°	86.5%	0.1	5.5%	20%