ELECTRON BEAM FLATTENING WITH AN ANNULAR SCATTERING FOIL

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Summary

A narrow parallel beam of electrons from an accelerator is commonly spread over a useful area by multiple scattering in a thin foil for applications such as radiation therapy and bubble chamber experiments. If the intensity at the edge of the resulting useful field is a fraction x of that in the center, then only the fraction 1-x of the incident beam is used. A secondary scattering foil which intercepts electrons outside the useful field redirects some of these into a distribution which complements that from the primary scatterer, allowing the use of a thinner primary scatterer. The resulting distribution function is developed, in a small-angle approximation, and integrated for annular secondary scatterer elements as a double series which is evaluated by a computer program in terms of the thickness and dimensions of the annulus. Results are discussed and limits of applicability of the theory are pointed out.

Mathematical Model

Definitions

Referring to figure 1, the radiation at a distance r from the axis of the electron beam can be given in terms of the projected distance s of the secondary scatterer element under consideration from the beam axis and the projected distance t from the point being evaluated, to a good approximation when the projection distance is large compared with the separation of the two scattering foils so that $D \approx D'$ and $\theta_t \approx \theta'$, the scattering angle between the ray from the primary scatterer and that to the point r from the secondary scatterer element.

To a good approximation the small-angle multiple scattering of electrons results in a two-dimensional Gaussian distribution, or a radial probability distribution[1]

$$P(s,\langle s\rangle) = \frac{2s}{\langle s\rangle^2} e^{-(s/\langle s\rangle)^2}$$

with the intensity distribution

$$I(s,\langle s\rangle) = \phi \frac{P(s,\langle s\rangle)}{2\Pi\langle s\rangle} = \phi \frac{e^{-(s/\langle s\rangle)^2}}{\Pi\langle s\rangle^2}$$

so that and

$$\int_0^\infty ds(P(s,\langle s\rangle)) = 1 \qquad \int_0^\infty ds(2\Pi s I(s,\langle s\rangle)) = \phi$$

Thus the contribution at a distance t from a secondary scatterer element at projected radius s is

$$\delta I(s,\langle s\rangle,t,\langle t\rangle) = \phi \frac{e^{-(s/\langle s\rangle)^2}}{\Pi\langle s\rangle^2} \cdot \frac{e^{-(t/\langle t\rangle)^2}}{\Pi\langle t\rangle^2}$$

where $\langle s\rangle$ and $\langle t\rangle$ are the rms scattering radius of the primary and secondary scatterers respectively.

To simplify notation we introduce the relative parameters

$$R \equiv r/\langle s\rangle \ , \ S \equiv s/\langle s\rangle \ , \ X \equiv (\langle t\rangle/\langle s\rangle)$$

where X is the ratio of thickness of the two scatterers and

$$(t/\langle t\rangle)^2 = R^2/X + S^2/X - 2RS\cos\omega/X$$

Then the relative intensity becomes

$$I = \frac{I(R,X)}{I(0,\langle s\rangle)} = \int_{S_1}^{S_2} dS \left(2S\int_0^\pi d\omega\, e^{-S^2} \cdot \frac{e^{-(R^2+S^2-2RS\cos\omega)/X}}{\pi X} \right)$$

where $I(0,\langle s\rangle) = \phi/\pi\langle s\rangle$ is the intensity on the axis of a distribution from the primary scatterer alone.

Series Integration

We can rearrange terms to get

$$I = \frac{2e^{-\frac{R^2}{X}}}{\pi X} \int_{S_1}^{S_2} dS \left(S e^{-S^2\frac{(1+X)}{X}} \int_0^\pi d\omega\, e^{+2RS\cos\omega/X} \right)$$

where the last integral becomes a Bessel function with the series representation[2]

$$\int_0^\pi d\omega\, e^{\frac{2RS\cos\omega}{X}} = I_0\left(\frac{2RS}{X}\right) = \sum_{k=0}^{\infty} \left(\frac{(RS/X)^k}{k!}\right)^2$$

Using a series representation of the exponential term in S and factoring out terms in $(R^2/X)^k$ we get

$$I = \frac{2e^{-\frac{R^2}{X}}}{X} \sum_{k=0}^{\infty} \frac{(R^2/X)^k}{k!} \sum_{l=0}^{\infty} \frac{(-1)^l(1+X)^l}{l!\,k!\,X^{k+l}} \int_{S_1}^{S_2} dS\, S^{2k+2l+1}$$

With term by term integration this becomes

$$I = \frac{e^{-\frac{R^2}{X}}}{1+X} \sum_{k=0}^{\infty} \frac{(R^2/X)^k}{k!} \frac{(S^2/X)^k}{k!} \sum_{l=0}^{\infty} \frac{(-S^2(1+X)/X)^{l+1}}{l!(k+l+1)} \Bigg|_{S_1}^{S_2}$$

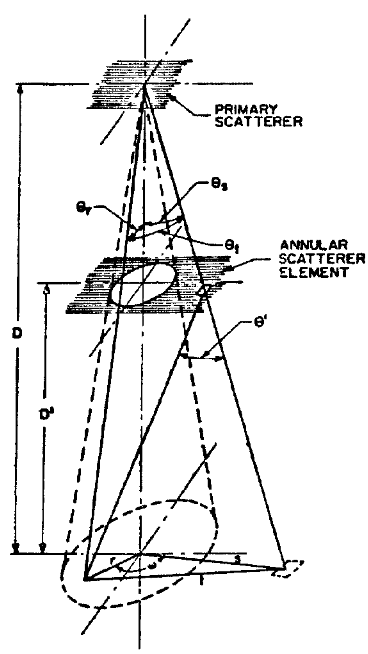

Fig. 1. Annular scatterer geometry.

Evaluation

The result of this integration is an expression which contains a negative exponential factor times a series of positive terms in R with coefficients which can be evaluated by summing the series in S.

An interactive program has been written which evaluates these terms for given values of S and X, reporting the number of iterations required to reach convergence and listing the terms on request, and then sums the series for a given range of R to produce a list or plot of the relative intensity as a function of radius measured in terms of the projected inner radius S_1 of the annular secondary scatterer.

The result is subtracted from the corresponding values using the outer radius S_2 in the determination of the series coefficients, do determine the intensity distribution from an annular zone between S_1 and S_2, or from

$$I = e^{-R^2}/(1+X)$$

to get the intensity at various radii R from a secondary scatterer extending from S_1 to S_2>><s>.

Figure 2 shows a typical plot of intensity distributions from primary and secondary scatterers separately and combined, and figure 3 shows a typical plot of an actual radiation therapy field using the annular scatterer.

Discussion

The mathematical model of multiple scattering has been and will continue to be a great help in exploring the possibilities in electron beam spreading systems, even though it is based on an idealization of the physical phenomena involved. By comparing the results with measured values one can distinguish when other phenomena than multiple scattering become important, for instance the contribution due to bremsstrahlung from the scatterer at energies above 30 MeV or the divergence due to air scattering below about 10 MeV in the geometries typical of radiation therapy installations.

References

1. E. Segre, Experimental Nuclear Physics, Wiley,1953 pages 282-287
2. H. B.Dwight, Table of Integrals, Macmillan, 1961 numbers 866.03 and 813.1

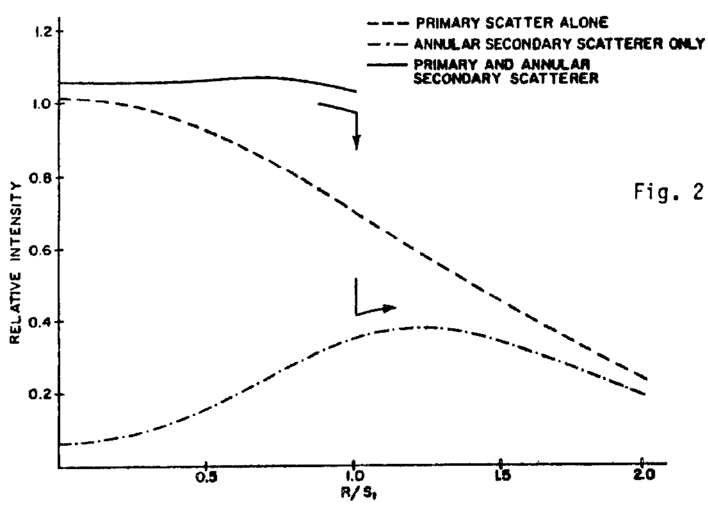

Fig. 2. Computed beam profiles for S_1 = 0.6 <s> and for X = 0.16

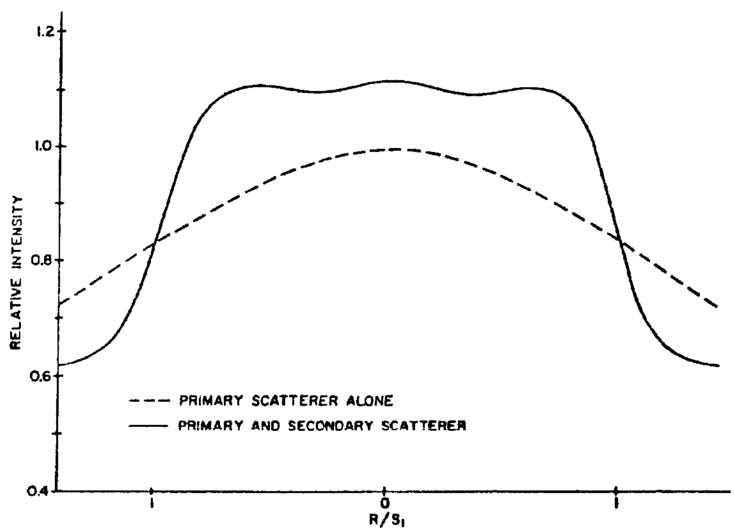

Fig. 3. 30 MeV electron therapy beam with annular scatterer, shown without collimator.