Properties of Contoured Cavities

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Abstract

Mesh relaxation computer programs such as the Los Alamos LALA have been developed which determine the Q, shunt resistance and resonant frequency of radially contoured, cylindrical cavities supporting a TM 01-like mode. This program has been used in particular to optimize accelerator cavity designs for the sidecoupled standing wave structure. However, it is also possible to use these cavity designs for in-line structures. In this case $\beta p = (V_p/C)\pi$, so that, for example, the longitudinal $2\pi/3$-mode at the velocity of light corresponds to the phase velocity 0.66 in the standing wave notation. It is convenient to regulate the energy velocity in this structure by means of circumferential slots between cells, leaving the beam aperture unchanged. This report presents the results of experimental measurements to determine the properties of such a structure in the $2\pi/3$-mode.

Contoured Cavities

Numerous unpublished studies have been undertaken with the object of improving the shunt impedance and Q of the simple disc-loaded line. For example, the disc thickness has been tapered in both directions, providing in one case a volume that somewhat resembled an ellipsoidal cavity which is known to have a high Q, and in the other, a drift tube-like structure which it was thought would have a high shunt resistance (1). Other studies have included the addition of fat lips resembling drift tubes on the beam aperture of an otherwise flat disc. The result of these studies has been somewhat indeterminant; negligible or no improvement was effected. The cause of failure to improve the conventional structure was undoubtedly lack of courage to completely contour the cavity; contour machining, using a template, appeared too formidable. Curiously, twenty years ago disc-loaded waveguide was fabricated by shrinking, a process that would be considered frightening today.

In the course of development of the beam-line cavities for the Los Alamos Meson Factory structure, considerable attention was devoted to cavity optimization. In particular, a mesh-relaxation code was developed to explore the effect of contour on cavity parameters. This program (LALA) calculates field distributions in cylindrical cavities, thereby permitting computation of stored energy and losses and therefore derived parameters, such as shunt impedance, Q, resonant frequency and transit time factor (2). Similar programs have been derived at BNL (JESSY) and MURA (MESSYMESH). These calculations have been confirmed closely enough for practical purposed by experimental measurements.

In view of the extraordinary properties reported for optimized cavities, we have scaled them to 3000 mcs with the aim of investigating their use as in-line structures. In such a case, to operate in the βp-mode at the velocity of light requires using a contoured cavity optimized for the particle velocity $\beta p/\pi$, since these cavities are resonant. For example, to oeprate in the $2\pi/3$-mode at the velocity of light corresponds to a phase velocity of 0.66 in the standing wave cavity notation. Several methods of coupling beteeen cavities suggest themselves. In the present case we arbitrarily decided to use four circular apertures in the septum wall on a 36 mm dia. circle.

Preliminary Observations

Before commencing the study of the chain of cavities a study was done of the properties of individual cavities (without coupling holes).

For $\beta = 1$ cavities LASL has investigated a range of cavity shapes (at 950 mcs) which are summarized here (3):

DTL, cm	1.50	2.31	2.79
TTF	0.738	0.762	0.788
ZT^2 MΩ/m	43.3	47.3	49.3
Q	34,300	33,000	31.700

DTL = Drift Tube Length
TTF = Transit Time Factor
ZT^2 = Shunt Impedance

When these structures are scaled to S-bank (2856 mcs) the above values become:

DTL, cm	0.507	0.781	0.943
TTF	0.738	0.762	0.788
ZT^2 MΩ/m	74.5	81.5	84.8
Q	20,000	19,200	18,500

The applicable scaling laws are $Q_2/Q_1 = \sqrt{\lambda_2/\lambda_1}$ and $Z_2/Z_1 = \sqrt{\lambda_1/\lambda_2}$ ignoring skin depth changes; transit time factors do not scale.

Several observations are immediately apparent: (1) the improvement in Q for the case DTL = 0.507 cm is due to coving the cavity wall since this case is identical to a disc-loaded structure in the π-mode, otherwise (no drift tube structure). (2) The cavity designs listed are not of direct interest in the proposed application because they are $\beta = 1$ cavities designed for side-soupled use. Even in that case one will not realize more than about 0.75 of the theoretical values because of the deleterious effects of the side cavities.

Nevertheless cavities of the sort listed were constructed to test the accuracy of the predictions. The cavity (composed of two halves of piece with DTL = 0.781), with circumferential coupling apertures for pick-up probes, had a resonant frequency of 3017.0190 mcs (λ = 9.936 cm) and was perturbed by a 20 mil diameter sapphire rod (ε = 9.56, a = 0.0254 cm) to a resonant frequency of 3014.3595 mcs. The perturbation produced by a small metal bead drawn along the axis was measured and the E-field pattern (square root of the perturbation), shown in Figure 3. Integrating (planimetrically) the area under these two figures gives the ratio $(\int E\cdot dz)^2/\int E^2\cdot dz$ = 3.98 cm. The periodicity of the structure is 5 cm. Spatial fourier analysis of Figure 3 (the E-field) gives for the coefficients:

$b_1 = 46.125$
$b_3 = 4.829$
$b_5 = -4.115$
$b_7 = -2.686$

Thus the shunt resistance of the resonant cavity is given by

$$\frac{R}{Q} = \frac{120\lambda}{(\epsilon/\epsilon_o - 1)\pi a^2} \frac{\Delta f}{f_o} \frac{(\int E\cdot dz)^2}{\int E^2\cdot dz} = 241\ \Omega$$

The effective shunt resistance of the cavity (taking into account transit time and field shape) is

$$\frac{R_{eff}}{R} = \left(\frac{\pi}{4}\ \frac{\mathcal{E}}{\sum \mathcal{E}_K/K}\right)^2 = 0.609$$

Estimating the Q of the cavity to be about 20,000, the shunt resistance of a cascade of these cavities would be about 58 megohms/m, neglecting excitation connections.

As a check on the accuracy of the Fourier reduction (see Appendix)

$$\frac{8d}{\pi}\left(\frac{\sum \mathcal{E}_K/K}{\sum \mathcal{E}_K}\right)^2 = 4.02\ cm.$$

Compared to

$$(\int E\cdot dz)^2/(\int E^2\cdot dz) = 3.98\ cm.$$

Clearly these cavities are useless in an in-line configuration, except for the possibility of a multiply periodic structure. Note that the value of 58 megohms/m is substantially less than the value of 70 megohms/m, used in commercial side-coupled cavity waveguide designs. We have no explanation of the discrepancy other than experimental inaccuracies, mainly in the matter of the transit time factor.

Experimental Results

Encouraged by the cavity studies, a structure was planned at 3000 mcs to observe the Brillouin diagram and r/Q of a cold-test stack of cavities. Figure 1 is an illustration of the cross-section of a septum and two half cavities, all machined as a single piece. By menas of coupling holes (4 of diameter D on the indicated circle) the group velocity of the structure was regulated. These holes are located in a region where both electric and magnetic excitation exist; the net excitation may be estimated from the field intensity integral over the area of the holes, taking into account that the magnetic polarizability of a circular hold is twice its electric polarizability. When electric coupling exceeds magnetic coupling the structure will propagate a forward wave and oppositely.

Of course, three periods of the structure were used to obtain the $2\pi/3$-mode, and one cavity removed to permit excitation in the $\pi/2$-mode in order to obtain more points on the dispersion diagram.

In Table I is shown the group velocity, r/Q, and $E_m^2/\Sigma E^2$, obtained from a spatial Fourier analysis of the axial field. The series impedance is given in the form E^2/P, taken from the formula

$$\frac{E^2}{P} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{r}{Q}\right)\left(\frac{c}{v_g}\right)$$

and shows the extraordinarily high values achievable with contoured cavities.

For the case $\beta = 0.65$ at 805 mcs Hoyt has stated a nominal value of $ZT^2/Q = 1500$ ohms/m. Although no Q is stated, a value of 25,000 would be anticipated, resulting in an estimate of 38 megohms/m for the shunt impedance. Scaled to 3000 mcs, the above value corresponds to 66 megachms/m, which is about twenty percent better than the conventional disc-loaded structure (3).

While such high values of series impedance as those given are impressive, the structure will generally support a backward wave which may be considered a disadvantage. Of course for CW, cryogenic structures there would be no objection to this mode of operation, but with ambient temperature, pulsed operation one would anticipate a loss in beam duty cycle.

It will, doubtless, not escape the reader's notice that the beam aperture in this experimental case is very small. Enlarging the beam hole and keeping the optimum contours (from the computer program) will not result in serious degradation of the properties.

Appendix

Shunt resistance is customarily defined in resonant cavities in such a way as to be a measure of the peak voltage along a specified path (usually the axis) for a given stored energy (or opwer input),

$$R = (\int E\,dz)^2/2P = (\int E\cdot dz)^2 Q/2\omega W \qquad (1)$$

since the stored energy in the steady state,

$$W = PQ/\omega$$

The factor of 2 results from filled intensity being defined as a peak value whereas power input is a mean value. Of course, this will not provide the energy gain of a particle traversing this line integral since the peak voltage only exists momentarily and the particle requires a finite transit time. Thus, one often finds the definition of an effective shunt impedance for particle interaction

$$R_{eff} = (\int E(z,t))^2/2P \qquad (2)$$

The energy gain of the particle is, however, of the form

$$V = \int E(z)\cos\left(\omega\int dz/v(z) + \varphi_s\right) dz \qquad (3)$$

where v(Z) is the velocity of the particle at z; ϕ_s is the phase of the RF field when the particle is in the center of the cavity. If the particle velocity changes sensibly in traversing the cavity the situation becomes complicated. In many cases it does not, so that, taking the axial equation of motion in the form (where a dot indicates a time derivative)

$$d(\gamma\dot{z})/dt = -(Ee/m_o)\sin\omega t \qquad (4)$$

and noting that $\gamma = (1-(\dot{z}/c)^2)^{1/2}$ we may integrate across the gap; with the boundary conditions $t = t_o$, $\gamma = \gamma_o$

$$\frac{1}{\gamma_o} - \frac{1}{\gamma} = (eE/m_o c\omega)(\cos\omega t - \cos\omega t_o) \qquad (5)$$

The exit time is approximately $t = t_o + d/\dot{z}_o$ where d is the gap length (periodicity). To optimize the gap efficiency set $d/\dot{z}_o = \pi/\omega$. Then the above becomes, where $\int E(z)dz = \hat{V}$,

$$\frac{1}{\gamma_o} - \frac{1}{\gamma} = (e\hat{V}/m_o c\dot{z}_o)(2/\pi)\cos\omega t_o \qquad (6)$$

Evidently the effectiveness of the best gap is reduced by a factor $2/\pi$. Stated somewhat differently, we may calculate, since various phases are present during transit, that the effectiveness of the gap voltage is, on the average, given by

$$\frac{\dot{z}}{d}\int_{t-d/\dot{z}}^{t+d/\dot{z}} V\sin\omega t\,dt = V\,\frac{\sin\omega d/2\dot{z}}{\omega d/2\dot{z}}\sin\omega t \qquad (7)$$

For best efficiency we set $\omega d/2\dot{z} = \pi/2$ and the transit time factor will then be $2/\pi$. Note that then the optimum periodic length of the structure has been determined, since $d = (\dot{z}/c)(\lambda/2)$.

The fundamental difficulty of expressing an exact transit time factor for a cavity is that it depends upon the excitation level of the cavity. In fact, however, there is usually so little change in the particle velocity that the intrant energy may be used. Then, the energy gain equation, assuming the particle is at mid-plane when the field intensity is maximum, $\varphi_s = -\pi/2$, and $d = (v/c)(\lambda/2)$

$$V = \int_0^d E(z)\sin(\pi z/d)\,dz \qquad (8)$$

In general, E(Z) is obtained experimentally by menas of a perturbation plot. If the function E(Z) is Fourier analyzed, assuming symmetry

$$E(z) = \sum b_k \sin\frac{k\pi z}{d} \qquad (9)$$

Then the energy gain equation may be written

$$V = \frac{d}{\pi}\int_0^d \sum b_k \sin\left(\frac{k\pi z}{d}\right)\sin\left(\frac{\pi z}{d}\right) d\,\frac{\pi z}{d} = \frac{d b_1}{2} \qquad (10)$$

since by the orthogonality condition only the fundamental component of the field analysis will contribute to energy gain. From Eqs (1) and (2) it will be obvious that the peak voltage across the cavity will be given by

$$\left(\int E\cdot dz\right)^2 = 2RP = \left(\frac{2d}{\pi}\sum\frac{b_k}{k}\right)^2 \qquad (11)$$

but the energy gain of the particle will be given by Eq (10)

$$2R_{eff}P = V^2 = \frac{d b_1}{2} \qquad (12)$$

or

$$\frac{R_{eff}}{R} = \left(\frac{\pi b_1}{4\sum b_k/k}\right)^2 \qquad (13)$$

This reduction factor now includes transit time and gap factor correction. Note also that in determining the gap (or field shape) factor correction to the experimental measure of shunt impedance (usually done with a planimeter) can also be written from the Fourier analysis

$$\left(\int E\,dz\right)^2 / \left(\int E^2 dz\right) = \frac{8d}{\pi^2}\,\frac{\left(\sum b_k/k\right)^2}{\sum b_k^2}$$

which will serve as a check on the analytic accuracy.

TABLE I

D	v_g/c	(r/Q)	$E_o^2/\Sigma E_m^2$	$E^2/P\,(MV/m)^2/MW$
12.7 mm	.0054	155 Ω/cm	0.97	178
14.3	.0092	158	0.98	108
15.9	.0133	159	0.99	75
17.5	.0164	160	0.99	61
19.1	.0213	159	0.99	47

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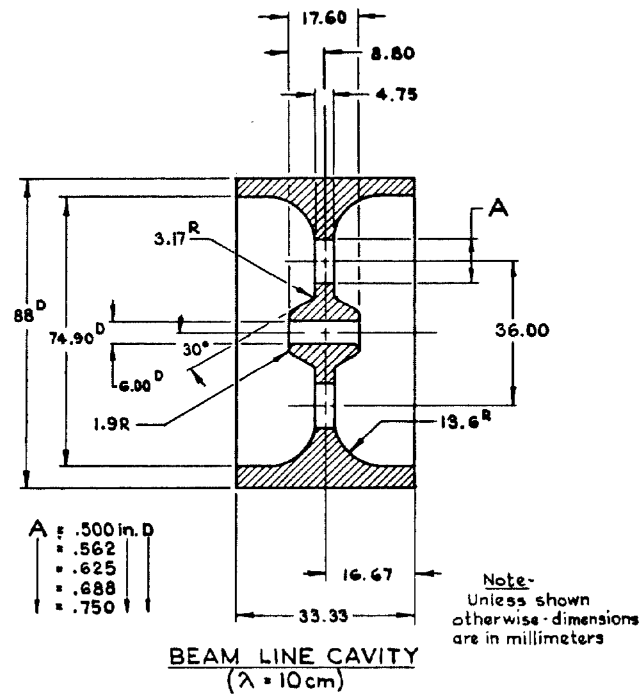

Fig. 1 Contoured cavity for 3 KMZ.

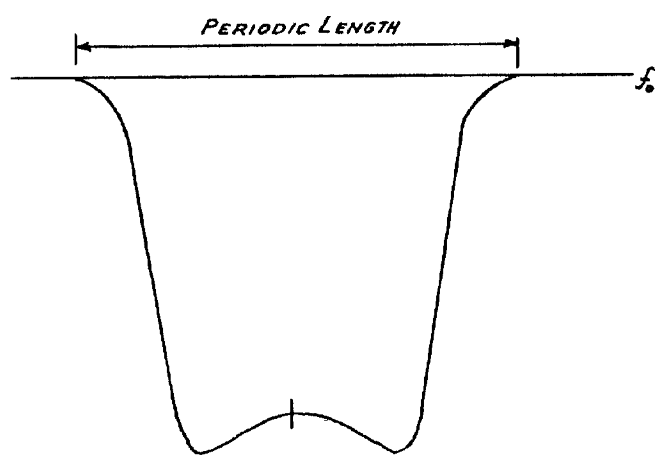

Fig. 2 Relative electric field intensity along axis of cavity.