PROPERTIES OF DISC-LOADED LINES

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Abstract

The microwave properties of disc-loaded cylindrical waveguides has been measured for both three and four discs per guide wavelength and normalized phase velocities from 0.5 to 1.0 A graphical presentation of the results is given. A discussion of other results quoted from the technical literature is also included.

Introduction

The properties of the disc-loaded cylindrical waveguide which are estimated by field analysis are not sufficiently accurately confirmed by experimental measurements for engineering use, presumably for the reason that the mathematical model does not represent the waveguide precisely enough and that the realization of the structure does not approximate the mathematical model closely enough, for example, in regard to surface finish. For this reason, a systematic investigation of structures was undertaken and the results of others, referenced in the technical literature, was correlated to produce the data given below.

From a few basic definitions one can derive relationships to easily measurable properties of waveguides. Specifically, shunt impedance is defined as the ratio of peak electric field intensity (squared) to power loss per unit length required to maintain it.

$$r = \frac{E^2}{-dP/dz} \tag{1}$$

The power loss per unit length is the attenuation coefficient

$$\frac{dP}{dz} = -2I \tag{2}$$

The figure of merit of a transmission system (somewhat analogously to the case of resonant cavities) is defined as the ratio of stored energy per unit length to the (time average) energy dissipated per unit length in one radian,

$$Q = \frac{\omega W}{-dP/dz} \tag{3}$$

The power flux is evidently given by the product of the linear energy density (stored energy per unit length) and the energy velocity.

$$P = W v_e \tag{4}$$

There has been some ambiguity in the past whether the group velocity (the slope of the dispersion diagram) is equal in magnitude to the energy velocity. It can be shown that in the lossless case the two are equal; when the line is lossey the group velocity, strictly speaking, is not defined,

$$v_g = \frac{d\omega}{d\beta} \tag{5}$$

In practical waveguides losses are so small that the group velocity may be taken as the energy velocity.

The above definitions are useful because they relate to easily measurable properties of a waveguide. Further, these same constants describe beam interactions with the wave including the transient regime. The only remaining property which is directly measurable is the phase velocity $v_p = \omega/\beta$ derivable as one of the constants in the solution of the wave equation in the structure.

Eqs. (1) and (3) may be combined to produce an easily measurable quantity,

$$\frac{r}{Q} = \frac{E^2}{\omega W} \tag{6}$$

Similarly, Eqs. (2) and (3) have the relation

$$I = \frac{\omega}{2 v_g Q} \tag{7}$$

From Eqs. (1) and (2) the field intensity in the waveguide may be obtained.

$$E^2 = 2IPr \tag{8}$$

This expression has limited usefulness however because the attenuation coefficient (or, indirectly, the Q of the structure) is difficult to measure accurately. From Eqs. (7) and (8) we may obtain the series impedance of the structure,

$$\frac{E}{\sqrt{P}} = \sqrt{\left(\frac{2\pi}{\lambda}\right)\left(\frac{r}{Q}\right)\left(\frac{c}{v_g}\right)} \tag{9}$$

All the terms on the RHS are easily measurable and the LHS is the information linac designers are interested in knowing.

Preliminary Investigation

In the early days of travelling wave accelerators four or more discs per guide wavelength were invariably used. When the fields in the disc-loaded structure are known, it is possible to compute the wall and disc losses (from the tangential magnetic field) and therefore the shunt impedance, since the axial electric field is also known. The result of this calculation, when maximized shows that about 3 discs per guide wavelength produces the maximum shunt impedance. Note that what is really of interest is the maximum series impedance, since that is the ratio of axial E-field to power flux. The shunt impedance alone does not relate the axial E-field to power flow; knowledge of the energy density (group velocity) is also required. Nevertheless it was of interest to confirm the above observation and a series of experimental determinations were made at Stanford University Microwave Laboratory (ref 1). The measures of r/Q and Q as a function of the number of discs per guide wavelength are shown in Figures 1 and 2 respectively. The measurements were performed at 2856 mcs for B=1 and a beam aperture of 0.822 in. for the cases of 2, 2 1/2, 3, 3 1/2 and 4 discs per wavelength. The beam aperture has unbevelled holes (ie, no rounding) in these experiments. In Figures 3 and 4 the normalized group velocity and the derived shunt impedance (R/Q • Q) are shown, respectively. From these data it was concluded that the $2\pi/3$ mode possessed properties generally more desirable than results from other degrees of loading and a general program of experimental determination of the properties of a guide was undertaken. Figure 5 shows the cylinder diameter required in each case.

Experimental Results

The two cases of most interest, the $\pi/2$ and the $2\pi/3$ modes were especially investigated in greater detail. For both cases the properties of the guide as a function of phase velocity was also investigated, the prospective use of such information being the design of bunchers. In Figure 6 is shown the cylinder diameter as a function of phase velocity and disc aperture for propagation in the $\pi/2$ mode and in Figure 7 the group velocity. Figure 8 presents the derived series impedance. In Figure 9 the cylinder diameter for propagation in the $2\pi/3$ mode as a function of phase velocity and iris aperture is shown and in Figure 10 the group velocity is shown for the same parameters. In Figures 11 and 12 the r/Q and series impedance are shown for the same conditions. In Figures 9 through 11 the experimental points (marked X) are the results of another separate investigation by SLAC, reported in ref 2.

It was not considered desirable to present the results in dimensionless form because of the peculiar numerical values which result. In any case, it is well-known that, ignoring skin depth or resistivity changes with frequency, it is valid to scale properties to other frequencies with small error. These scaling laws are: for shunt impedance $R_2/R_1 = (\lambda_1/\lambda_2)^{1/2}$ and for Q, $Q_2/Q_1 = (\lambda_2/\lambda_1)^{1/2}$. Clearly the cylinder diameter scales directly. All the experimental data reported were made at nominally 2856 mcs.

The reader interested in the properties of disc-loaded waveguides will find interesting comment in refs 3.

Experimental data for the $2\pi/3$ mode at the velocity of light is given also by Haimson, ref 4.

The experimental technique for these measurements is described in ref 5.

In the presentation of data the letters have the significance:

2b, cylinder diameter
2a, disc aperture diameter
t, disc thickness
p, periodic length

Literature Cited

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3. W. Walkinshaw, Proc. Phys. Soc. (London) 61,246 (1948)R.B.R. Shersly-Harvie, Proc. Phys. Soc. (London) 61,255 (1948) Mullet and Loach, Proc. Phys. Soc. (London) 61,271 (1948)
4. J. Haimson, IEEE Trans. Nuc. Sci. 12,499 (1965) and Microwave Beam Optics Design of the Chopper Prebuncher Injection System and Accelerator Waveguides for the Frascati 450 MeV Electron/350 MeV Positron Linear Accelerator. Varian Report, Dec. 1965
5. W. J. Gallagher, Measurement Techniques for Periodic Structures, Microwave Lab. Rep. No. 767, Stanford University (1960)

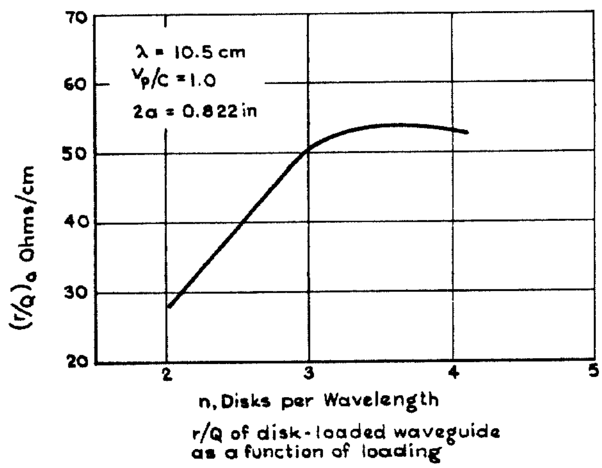


Fig. 1 r/Q of disc-loaded waveguide as a function of loading.

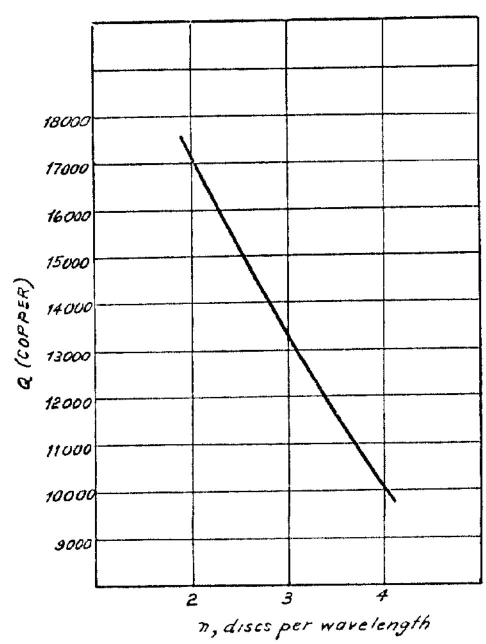


Fig. 2 Q of guide as a function of disc loading. $V_p/C = 1$, 2a = 0.822 in., f = 2856 mcs.

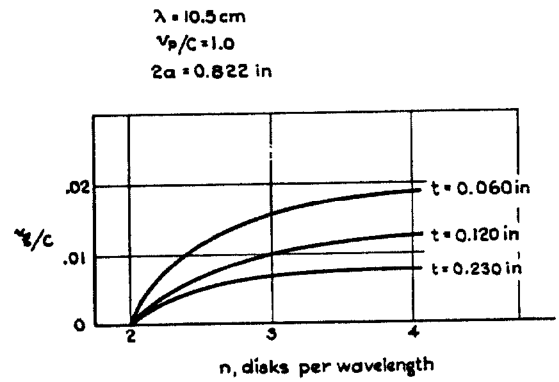


Fig. 3 Group velocity as a function of loading.

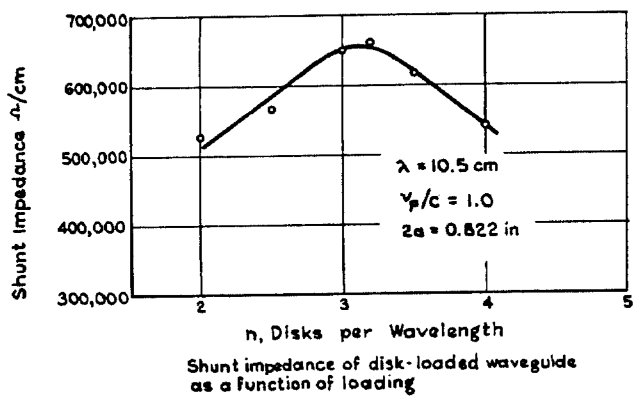


Fig. 4 Shunt impedance of a disc-loaded waveguide as a function of loading.

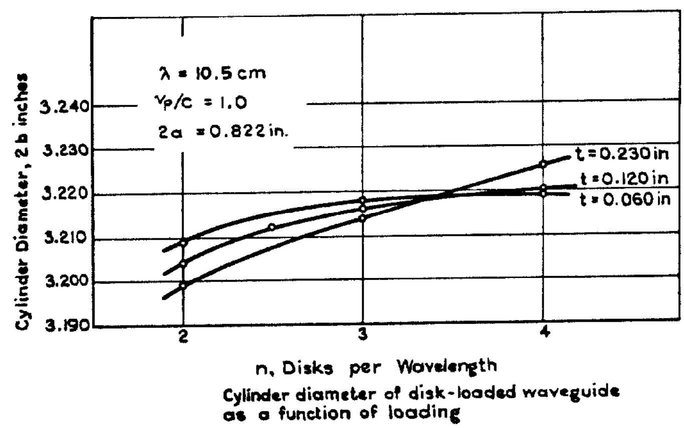


Fig. 5 Cylinder diameter of disc-loaded waveguide as a function of loading.

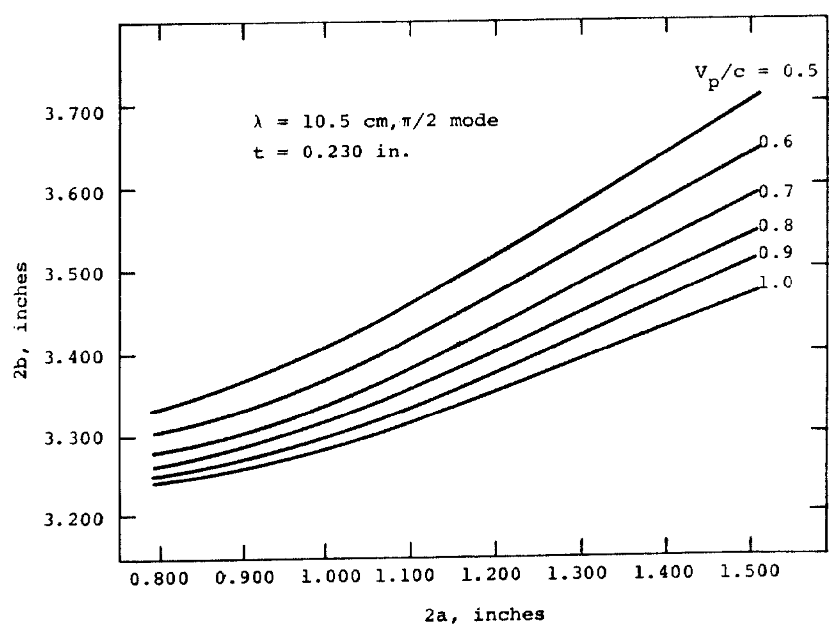


Fig. 6 Cylinder diameter as a function of phase velocity and iris aperture, π/2-mode.

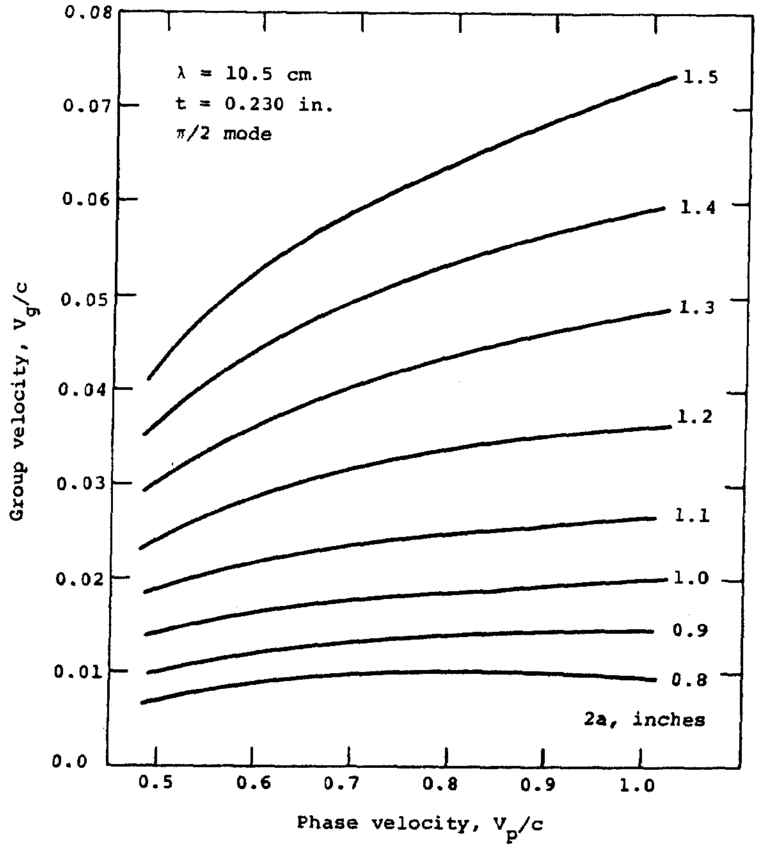


Fig. 7 Group velocity as a function of phase velocity and iris aperature, $\pi/2$-mode.

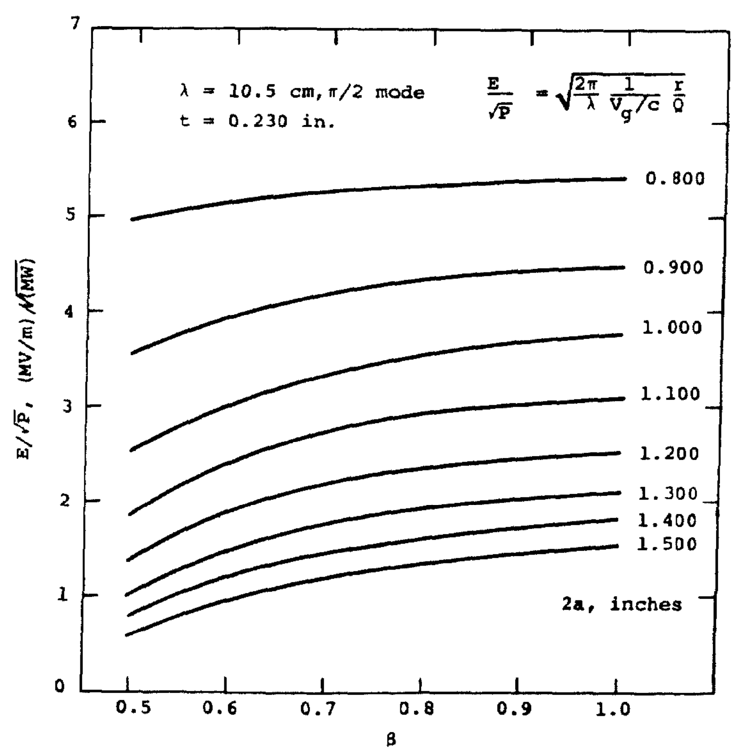
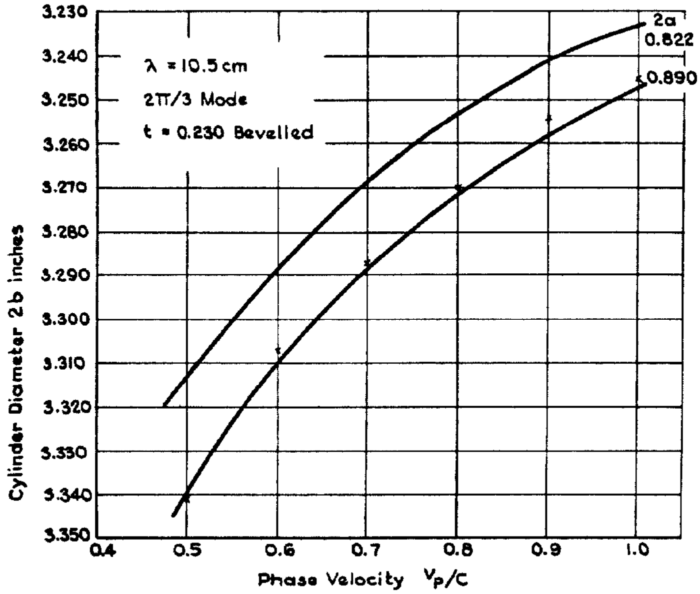
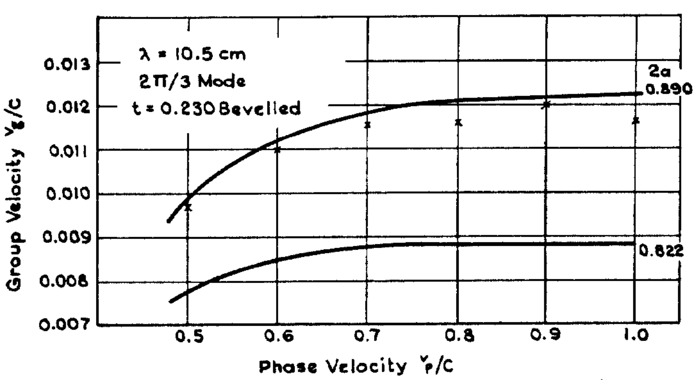


Fig. 8 Series impedance as a function of phase velocity and iris aperture, $\pi/2$-mode.



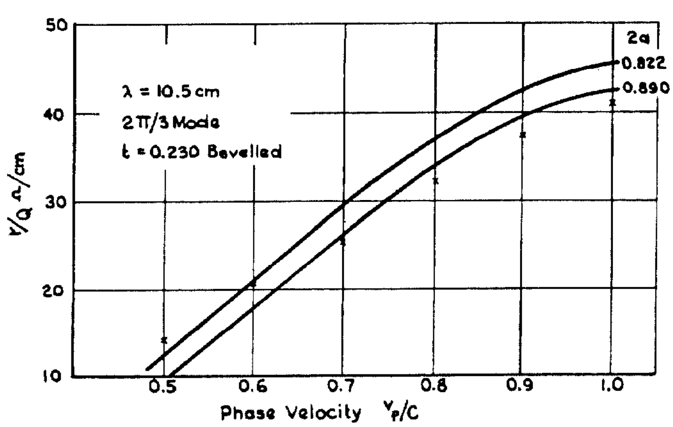
Cylinder diameter as a function of disk hole diameter & phase velocity 2π/3 mode

Fig. 9 Cylinder diameter as a function of phase velocity and iris aperture, $2\pi/3$-mode.



Group velocity as a function of disk hole diameter & phase velocity for 2π/3 mode

Fig. 10 Group velocity as a function of phase velocity and iris aperture, $2\pi/3$-mode.



r/Q as a function of disk hole diameter & phase velocity for 2π/3 mode

Fig. 11 r/Q as a function of iris aperture and phase velocity, $2\pi/3$-mode.

λ = 10.5 cm
2π/3 Mode (Bevelled)
t = 0.230 in

$$\frac{E}{\sqrt{P}} = \sqrt{\frac{2\pi}{\lambda}\ \frac{r}{Q}\ \frac{1}{v_g/c}}$$

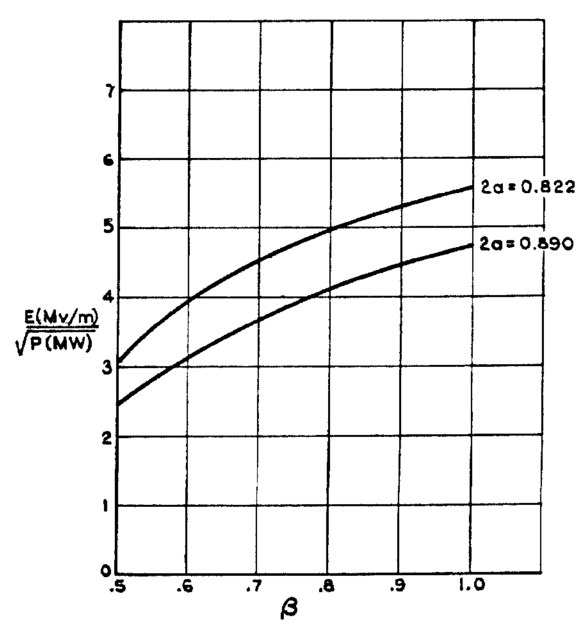


Figure 12.